Math 1A
Final Review

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8-10pm
Unit 3

Suggested Study Strategy and Schedule

The 20 hr plan:

The 20-hour plan is actually a substantial amount of studying. It is equivalent to about 3 full days of studying, or about 5 nights.

Topic	Hours
Fundamentals review	2
Skim textbook (work examples and theorems)	3
Intermediate problems review	4
Quiz and midterm review (redo quizzes and midterms)	2
Theorems review (special cases)	2
Theorem proofs	1
Challenge problems	4

The 10 hr plan:

Most students will elect to choose this one, especially students who have other finals to study for.

Topic	Hours
Fundamentals review	1
Skim textbook (work examples and theorems)	2
Intermediate problems review	3
Quiz and midterm review (redo quizzes and midterms)	2
Theorems review and proofs (special cases)	1
Challenge problems	1

The panic plan:

It's the night before, and you haven't started. No suggested times are given here. Take the amount of time you have, and allocate accordingly. It is unlikely that rereading the book will help you now, so it is suggested that you simply do as many problems as you can...

Topic	Hours
Fundamentals review	25%
Theorems review	10%
Intermediate problems review	35%
Quiz and midterm review (redo quizzes and midterms)	30%

- All problems should be attempted without referring to the material, and without a calculator. In other words, try to simulate test conditions.

- In all cases, to review means to do the problems again, not to read about doing the problems. You didn't learn to drive a car by reading the DMV manual, did you?

Summary Sheet of Topics for Math 1A

Functions
Types of functions
Domain and range
Transformations: Translation, Scaling, Algebraic combinations
Transformations: Function composition
Logarithms and exponentials

$$e^a = \lim_{n \to \infty} \left(1 + \frac{a}{n}\right)^n, \text{ ln a = area under curve of } f(x) = \frac{1}{x} \text{ from 1 to a}$$

Limits
Informal definition of limit
Infinite limits, one sided limits
When does a limit exist?
Squeeze theorem; other theorems for calculating limits
Epsilon-delta proof
L'Hopitals Rule

$$\lim_{x \to 0} \frac{\sin x}{x} = 1, \text{ and } \lim_{x \to 0} \frac{\cos x - 1}{x} = 0$$

Continuity
Definition of continuity

Derivatives
Definition of derivative; alternative definition of derivative
When does a derivative exist?
Chain rule
Implicit differentiation
Linear approximation and differentials
Newtonian approximation

Word problems
Related rates
Optimization
Position, velocity, acceleration problems

Minimization and maximization
First/second derivative tests
Curve sketching
Slant asymptotes

Integrals
    Riemann Sum
    Antiderivatives
    Interpretation as area
    u-substitution

Volumes
    Disk method
    Shell method

Major theorems of Math 1A
    Rolle's Theorem
    Mean Value Theorem
    Fundamental Theorem of Calculus, Part I
    Fundamental Theorem of Calculus, Part II

Suggested Problems for Review

Topic	Fundamental	Intermediate	Difficult
Functions	(1.1) 6,22,36,38,53 (1.2) 10 (1.3) 6,37,46	(1.1) 60 (1.2) 20 (1.3) 63,64	(1P+) 12
Limits	(2.2) 4,14 (2.4) 16,20 (2.6) 18,24 (4.4) 8,11,34,56	(2.2) 24 (2.3) 32,36,58 (2.4) 27,35,36 (2.6) 41,61 (4.4) 72	(2.3) 47 (2.4) 37 (2P+) 5 (4.4) 73,77,78,79
Continuity	(2.5) 10,16	(2.5) 39,53	(2.5) 57,61
Derivative	(2.8) 2,21 (2.9) 12,22 (3.1) 22,28 (3.2) 5,23 (3.4) 11,29 (3.5) 13,21,23,38,55 (3.6) 6,28 (3.8) 44 (3.11) 22,40,41 (4.9) 16	(2.9) 40,43 (3.1) 61 (3.2) 32,42 (3.4) 36,43 (3.5) 73,77 (3.6) 21,36,38,63 (3.7) 30 (3.8) 23,36,45 (P3+) 7 (4.9) 27	(2.8) 33,34 (3.1) 58 (3.4) 46 (3.6) 54 (3.7) 63 (3.8) 51,52 (P3+) 8,9,25 (4.3) 68,69,73 (4.9) 32
Word problems	(3.10) 7,21 (4.7) 4,21,38	(3.10) 11,14 (4.7) 30,31,40	(3.10) 32,36 (4.7) 50,52,54
Min/Max/Curves	(4.1) 39,43,54 (4.3) 8,17 (4.5) 20,42	(4.1) 76 (4.3) 26,65 (4.5) 32,46	(4.5) 59,60 (P4+) 1,3,13
Integrals	(4.10) 4,19,23,29 (5.1) 4,17 (5.2) 5 (5.3) 7,21,34 (5.4) 26,41 (5.5) 9,31,53,58,59 (6.1) 11	(4.10) 40,72 (5.1) 18,20 (5.2) 17,31,36 (5.5) 78 (5.6) 3,4 (6.1) 40	(5.1) 24 (5.2) 47,58,62,65 (5.3) 57,60,62 (5.5) 79,81,82
Volume	(6.2) 12,47 (6.3) 3,16,37	(6.2) 50 (6.3) 40	(6.2) 59,62,69 (6.3) 44
Theorems	(4.2) 4,5 (5.3) 12,15	(4.2) 17,19,20 (5.3) 49,60	(4.2) 25,29,36 (5.3) 58 (P5+) 12,15

Px+ refers to the Problems Plus section from Chapter x.

True/False

1. If $f(a) = f(b)$, then $a = b$

2. If $f(x) > 0$ for all x, then $\int_a^b f(x)dx > 0$ for any a and b.

3. If $f(c) = L$, then $\lim_{x \to c} f(x) = L$

4. If $f'(x) = g'(x)$, then $f(x) = g(x)$

5. $f(x) = \cos x$ is an even function

6. $f(x) = |x|$ is continuous for all x

7. A rational function can have infinitely many asymptotes

8. If $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = \infty$, then $\lim_{x \to c} [f(x) - g(x)] = 0$

9. The slope of $f(x) = \ln x$ at $x = -2$ is $f'(-2) = -\frac{1}{2}$

10. $\int_{-1}^{1} x^{-2} dx = \left[-x^{-1} \right]_{-1}^{1} = (-1) - 1 = -2$

11. If $F'(x) = G'(x)$ on [a,b], then $F(b) - F(a) = G(b) = G(a)$

12. The average value of sin x from 0 to 2π is 0

13. The derivative of $f(x) = \ln \pi$ is $f'(x) = \frac{1}{\pi}$

14. For x > 0, if $f(x) = \ln(ax)$, a > 0, and $g(x) = \ln(bx)$, b > 0, then $f'(x) = g'(x)$

15. The natural log function has an inverse by the horizontal line test

16. $\int \frac{1}{x} dx = \ln|cx|$

17. No even function has an inverse

18. $\frac{d}{dx} \int_a^b F'(x)dx = F'(b)$

19. If $f(x) = g(x)e^x$, then the only zeroes of f(x) are the zeroes of g(x).

20. If $\int_a^b f(x) - g(x)dx = A$, then $\int_a^b g(x) - f(x)dx = -A$