UC Berkeley
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Economics 201A - Section 5

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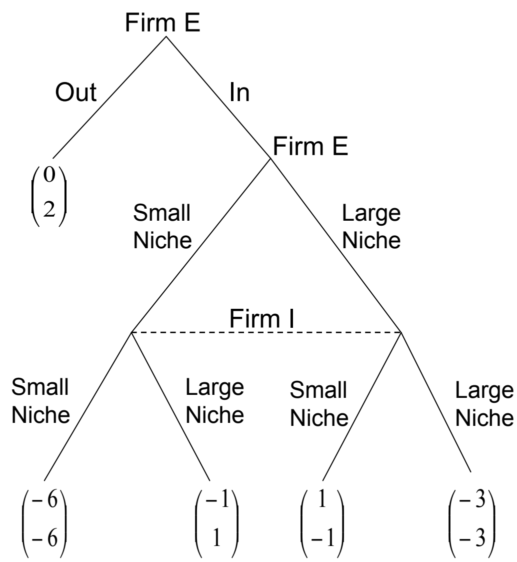
1 What we learnt this week

- Basics: subgame, continuation strategy
- Classes of games: finitely repeated games
- Solution concepts: subgame perfect Nash equilibrium (SPNE)

2 Problems

Problem 1: The niche choice game



(i) How many subgames are in this game?

(ii) Find the set of pure-strategy SPNE.

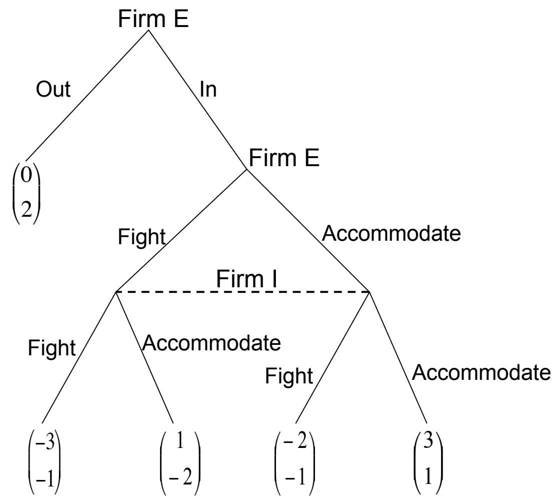
Problem 2: Predation games

Consider the predation games below.

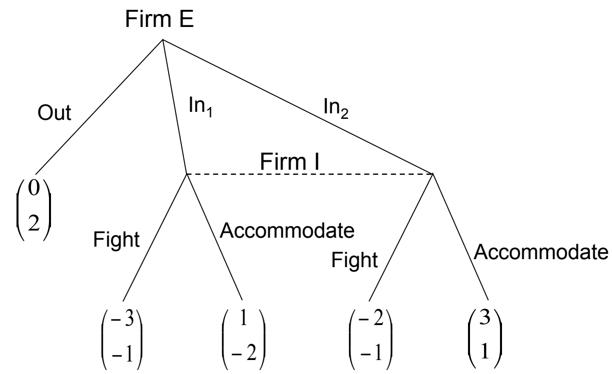
(i) Last time we showed that there are three pure-strategy Nash equilibria in Predation Game 1: (Out/Accommodate, Fight), (Out/Fight, Fight), and (In/Accommodate, Accommodate). What are the pure-strategy SPNE of this game?

(ii) Consider now Predation Game 2. Would you say that this game is different from Predation Game 1? What are the pure-strategy SPNE of this game? Are all these "reasonable"?

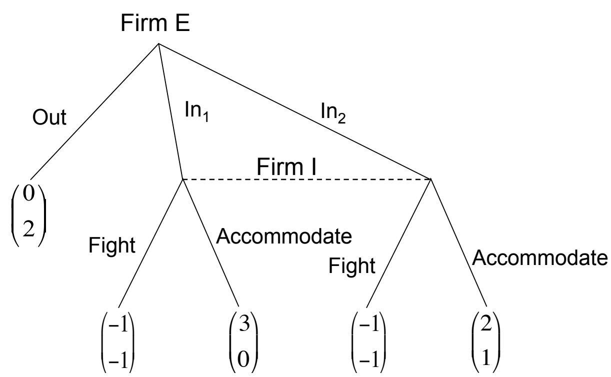
(iii) Finally, consider Predation Game 3. What are the pure-strategy SPNE of this game? Are all these "reasonable"?



Predation Game 1



Predation Game 2



Predation Game 3

Problem 3: Gibbons 12.10

Consider a game in which the simultaneous-move game below is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. The variable x is greater than 4, so that (4,4) is not an equilibrium payoff of the one-shot game. For what values of x is the following strategy (played by both players) a SPNE?

> Play Q_i in the first stage. If the first-stage outcome is (Q_1, Q_2), play P_i in the second stage. If the first-stage outcome is (y, Q_2), where $y \neq Q_1$, play R_i in the second stage. If the first-stage outcome is (Q_1, z), where $z \neq Q_2$, play S_i in the second stage. If the first-stage outcome is (y, z), where $y \neq Q_1$ and $z \neq Q_2$, play P_i in the second stage.

	P_2	Q_2	R_2	S_2
P_1	2,2	x,0	-1,0	0,0
Q_1	0,x	4,4	-1,0	0,0
R_1	0,0	0,0	0,2	0,0
S_1	0,-1	0,-1	-1,-1	2,0

Problem 4: Gibbons 12.11

Consider a game in which the simultaneous-move game below is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. Can the payoff (4,4) be achieved in the first stage in a pure-strategy SPNE? If so, give strategies that do so. If not, prove why not.

	L	C	R
T	3,1	0,0	5,0
M	2,1	1,2	3,1
B	1,2	0,1	4,4

Problem 5: MWG 9B9

Consider a game in which the simultaneous-move game below is played twice. The players observe the actions chosen in the first play of the game prior to the second play and there is no discounting. What are the pure-strategy SPNE of this game?

	b_1	b_2	b_3
a_1	10,10	2,12	0,13
a_2	12,2	5,5	0,0
a_3	13,0	0,0	1,1

Problem 6: Sustaining cooperation

Consider the following games: (1) a game in which the players play the prisoner's dilemma twice; (2) a game in which the players play the prisoner's dilemma in the first period and the coordination game in the second period; (3) a game in which the players play the prisoner's dilemma in the first period and the battle of the sexes in the second period. In all games, the players observe the outcome of the first period before the second period begins, and there is no discounting. Is it possible to have cooperation in the first period in a SPNE in any of these games?

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

(1) Prisoner's Dilemma repeated twice

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→

	A	B
A	1, 1	0, 0
B	0, 0	10, 10

(2) Prisoner's Dilemma and Coordination

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

(3) Prisoner's Dilemma and Battle of the Sexes

3 Answers

Problem 1

(i) There are two subgames: (1) starting after Firm E plays In; (2) the game itself.

(ii) Consider first the post-entry subgame. The normal-form representation of this subgame is

	Small Niche	Large Niche
Small Niche	-6,-6	-1,1
Large Niche	1,-1	-3,-3

There are two pure-strategy Nash equilibria in this simultaneous-move game: (Large Niche, Small Niche) and (Small Niche, Large Niche). In any pure-strategy SPNE, the firms' strategies must induce one of these two equilibria in the post-entry subgame. Suppose first that the firms play (Large Niche, Small Niche). In such case, the payoffs from reaching this subgame are (1,-1), and thus Firm E chooses to enter the market. Suppose instead that the firms play (Small Niche, Large Niche). In such case, the payoffs from reaching this subgame are (-1,1), and thus Firm E chooses not to enter the market. Hence, there are two pure-strategy SPNE: (In/Large Niche, Small Niche), and (Out/Small Niche, Large Niche).

Problem 2

(i) The unique SPNE in Predation Game 1 is (In/Accommodate, Accommodate). To see this, note that the post-entry subgame is

	Accommodate	Fight
Accommodate	3,1	-2,-1
Fight	1,-2	-3,-1

The unique Nash equilibrium of this simultaneous-move game is (Accommodate, Accommodate). Hence, any SPNE must have both player playing Accommodate after entry. And given that, Firm E's optimal strategy is to play In. As discussed last time, this is also the only reasonable Nash equilibrium.

(ii) This game looks similar to Predation Game 1 but it is not the same. As in Predation Game 1, there are two classes of Nash equilibria, one in which Firm E stays out and Firm I fights if Firm E enters, and one in which Firm E enters and Firm I accommodates. More precisely, the two pure-strategy Nash equilibria in Predation Game 2 are (Out, Fight) and (In_2, Accommodate). Now note that in this game, both pure-strategy Nash equilibria are subgame perfect: since the game has only one subgame (the game itself), the sets of Nash equilibria and SPNE are the same.
Still, only (In_2, Accommodate) looks sensible. The reason is that for Firm I to play Fight, it must be that Firm I believes that Firm E played In_1. But In_1 is strictly dominated for Firm E by In_2. Thus, it seems reasonable to think that if Firm E decided to enter, then it must have used strategy In_2. Subgame perfection cannot eliminate (Out, Fight) because this concept combines backwards induction with equilibrium, while the argument that we make to say that (Out, Fight) is not reasonable is one of *forward induction.*

(iii) There are two pure-strategy Nash equilibria in Predation Game 3, (Out, Fight) and (In_1, Accommodate). Since the only subgame is the game as a whole, both equilibria are subgame perfect. However, we may want to say that (Out, Fight) is not reasonable, since regardless of what entry strategy Firm E uses, Firm I prefers to play Accommodate once Firm E has entered the market. But subgame perfection is not useful to rule out the unreasonable equilibrium in this case. To eliminate this equilibrium, we could require that Firm I's action after entry be optimal for some belief that she might have about Firm E's entry strategy (note that in this game, Fight is not optimal for any belief that Firm I might have). Other refinements of Nash equilibrium introduce beliefs formally and rule out this type of equilibria.

Problem 3

For the proposed strategies to be a SPNE, they must induce a Nash equilibrium in the second-period stage game and in the two-stage game. The first condition is met because the strategies specify that the players play $(P_1, P_2), (R_1, R_2), (S_1, S_2)$ in period 2, all of which are stage-game Nash equilibria. When do the proposed strategies constitute a Nash equilibrium in the repeated game? Given that the other player plays the proposed strategy, a player's payoff from the proposed strategy is $4 + 2 = 6$. Her payoff from the best deviation is $x + 0 = x$. Thus, a player will have no incentive to deviate if $x \leq 6$. The proposed strategies are then a SPNE if $x \leq 6$.

Problem 4

The stage game has two pure-strategy Nash equilibria, (T,L) and (M,C). To support (B,R) in period 1, we need a credible threat to deter deviation. Since player 2 does not have incentives to deviate, we only need a credible threat to deter player 1 from deviating. Consider the following strategies:

> Player 1: Play B in period 1. In period 2, play T if outcome is (B,R) in period 1 and M otherwise.
>
> Player 2: Play R in period 1. In period 2, play L if outcome is (B,R) in period 1 and C otherwise.

The proposed strategies are a SPNE of this repeated game. First note that they specify a stage-game Nash equilibrium in period 2, because both (T,L) and (M,C) are Nash equilibria. Second, note that they constitute a Nash equilibrium in the repeated game: given that the other player plays the proposed strategy, player 1 gets $4+3 = 7$ from the proposed strategy, and $5+1 = 6$ from the best deviation; and player 2 gets $4+1 = 5$ from the proposed strategy, and $2+2 = 4$ from the best deviation. So neither has an incentive to deviate.

Problem 5

The pure-strategy Nash equilibria of the one-shot game are (a_2, b_2) and (a_3, b_3). Any SPNE involves playing one of these in the second period. Thus, playing either of these strategies in both periods constitutes a SPNE. Also, the players could use these strategies in any combination in the two periods. That gives four possible SPNE. In addition, there are other SPNE where player 1 (player 2) punishes the other player by playing a_3 (b_3) in the second period if the other player did not cooperate in the first period. This gives five other possible SPNE. In sum, there is a total of nine SPNE, which fall into four classes:

(1) Player 1 plays a_i unconditionally in both periods and Player 2 plays b_i unconditionally in both periods, $i \in \{2, 3\}$.

(2) Player 1 plays a_i in the first period and a_j unconditionally in the second period; Player 2 plays b_i in the first period and b_j unconditionally in the second period, $i, j \in \{2,3\}, i \neq j$.

(3) Player 1 plays a_i, $i \in \{1,2,3\}$, in period 1. In period 2, Player 1 plays a_2 if the outcome was (a_i, b_1) in period 1, and a_3 otherwise.
Player 2 plays b_1 in period 1. In period 2, Player 2 plays b_2 if the outcome was (a_i, b_1) in period 1, and b_3 otherwise.

(4) Player 2 plays b_i, $i \in \{2,3\}$, in period 1. In period 2, Player 2 plays b_2 if the outcome was (a_1, b_i) in period 1, and b_3 otherwise.
Player 1 plays a_1 in period 1. In period 2, Player 1 plays a_2 if the outcome was (a_1, b_i) in period 1, and a_3 otherwise.

It is clear that the four equilibria described in (1) and (2) are SPNE. To check that the five equilibria described in (3) and (4) are SPNE, note that by deviating a player loses 4 in the second period and no player can gain more than 3 by deviating in any of the described strategy profiles. Finally, to check that other pure-strategy SPNE do not exist, note that (a_2, b_3) cannot be sustained in period 1 as Player 2 would deviate, and (a_3, b_2) cannot be sustained in period 1 either as Player 1 would deviate.

Problem 6

In the prisoner's dilemma repeated twice, cooperation can never be sustained in equilibrium. The unique Nash equilibrium of the stage game is (D,D). Hence, in any SPNE, the players play (D,D) in the second period, no matter what they played in the first period. But then, in the first period, the players know that their actions will not affect what they play in the second period, and hence this is like a one-shot game. Thus, the players play (D,D) in the first period as well.

In the game in which the players play the prisoner's dilemma in the first period and the coordination game in the second period, a SPNE with cooperation in the first period does exist. Consider the following strategy (for both players):

Play C in period 1. In period 2, play B if outcome is (C,C) in period 1, and A otherwise.

Note that (A,A) and (B,B) are both Nash equilibria of the stage game. Moreover, these strategies constitute an equilibrium of the two-period game: a player's payoff from the proposed strategy is $1+10=11$, and her payoff from the best deviation is $2+1=3$. So neither player wants to deviate.

Finally, in the game in which the players play the prisoner's dilemma in the first period and the battle of the sexes in the second period, cooperation cannot be sustained in the first period in a SPNE. There are three Nash equilibria in the battle of the sexes: (B,B), (F,F), and a mixed-strategy Nash equilibrium with (p,q)=(2/3,1/3), where p is the probability with which player 1 plays B and q the probability with which player 2 plays B. So there are three possibilities for the second period, namely any of the Nash equilibria. The highest reward for a player is 2, and the worst punishment is 2/3. But then note that in all possible cases, some player wants to deviate. If (C,C) is followed by (B,B) (with the players playing the mixed-strategy Nash equilibrium with payoffs (2/3,2/3) if some player deviates), player 2 wants to deviate. If (C,C) is followed by (F,F), player 1 wants to deviate. And if (C,C) is followed by (p,q)=(2/3,1/3), both players want to deviate.