UC Berkeley
Fall 2007

Economics 201A - Section 3

Marina Halac

1 What we learnt this week

- Basics: mixed strategies
- Solution concepts: rationalizability, iterated weak dominance (IWD), Nash equilibrium

2 Problems

Problem 1: Does the order matter for IWD?

Last time we proved that the order of deletion does not affect the set of rationalizable strategies. Does the order of deletion affect the set of strategies that survive iterated deletion of weakly dominated strategies? Hint: consider the game below.

	L	R
U	5,1	4,0
M	6,0	3,1
D	6,4	4,4

Problem 2: Applying the definition of rationalizability

	D	E	F	G
A	2,3	3,1	1,4	4,1
B	5,4	3,0	2,1	1,0
C	3,2	5,4	2,3	1,2

Using the formal definition of rationalizability given in class, find $(A_1(t), A_2(t))$ for $t = 0, 1, ...$ When should we stop? What is the set of rationalizable actions, (R_1, R_2)?

Problem 3: Finding the Nash equilibria

Find all the Nash equilibria of the game in Problem 2.

Problem 4: Rock, Paper, Scissors

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Find all the Nash equilibria in the game *Rock, Paper, Scissors.*

Problem 5: Hotelling's law (MWG 8D5)

Consumers are uniformly distributed along a boardwalk that is 1 mile long. Ice-cream prices are regulated, so consumers go to the nearest vendor because they dislike walking (assume that at the regulated prices all consumers will purchase an ice cream even if they have to walk a full mile). If more than one vendor is at the same location, they split the business evenly.

Consider a game in which two ice-cream vendors pick their locations simultaneously. Show that there exists a unique pure-strategy Nash equilibrium and that it involves both vendors locating at the midpoint of the boardwalk.

3 Answers

Problem 1

Yes, IWD has the undesirable feature that it can depend on the order of deletion. In the game given above, if we first eliminate strategy U, we next eliminate strategy L, and we can then eliminate strategy M. Thus, in this case, (D,R) is the predicted outcome of the game. But if instead we eliminate strategy M first, we next eliminate strategy R, and we can then eliminate strategy U, so that now the predicted outcome is (D,L)!

Why do we find this result? Basically, because IWD is a theoretically "weak" concept. As discussed in class, to justify the deletion of a weakly dominated strategy for player i, we say that player i contemplates the possibility that every strategy combination of the other players occurs with positive probability. However, this is inconsistent with the logic of iterated deletion, which precisely assumes that eliminated strategies are not expected to occur.

Problem 2

$$\begin{aligned}
(A_1(0), A_2(0)) &= (\{A, B, C\}, \{D, E, F, G\}) \\
(A_1(1), A_2(1)) &= (\{A, B, C\}, \{D, E, F\}) \\
(A_1(2), A_2(2)) &= (\{B, C\}, \{D, E, F\}) \\
(A_1(3), A_2(3)) &= (\{B, C\}, \{D, E\}) \\
(R_1, R_2) &= (\{B, C\}, \{D, E\})
\end{aligned}$$

Because this is an iterative process, we stop when we obtain the same result for all players in two consecutive rounds. In the example above, $(A_1(3), A_2(3)) = (A_1(4), A_2(4)) = (\{B, C\}, \{D, E\})$, and hence the rationalizable actions are $(R_1, R_2) = (A_1(3), A_2(3)) = (\{B, C\}, \{D, E\})$.

Problem 3

Remember that not rationalizable $\Rightarrow$ not Nash! Hence, we can ignore the non-rationalizable strategies when looking for Nash equilibria. Here, this means that we can consider the reduced game,

	D	E
B	5,4	3,0
C	3,2	5,4

While it is relatively easy to find the pure-strategy Nash equilibria in 2×2 games, in general it is recommendable to proceed as follows. Consider a two-player game. Then for each player and for each feasible strategy for that player, determine the other player's best response to that strategy. We do this below by underlying the payoff to player j's best response to each of player i's feasible strategies. For example, if player 2 were to play D, then player 1's best response would be B, so we underline 5 in the first cell.

	D	E
B	$\underline{5}$,$\underline{4}$	3,0
C	3,2	$\underline{5}$,$\underline{4}$

A pair of strategies form a Nash equilibrium if each player's strategy is a best response to the other's, that is, if both payoffs are underlined in the corresponding cell. Thus, in the game above, there are two pure-strategy Nash equilibria: (B,D) and (C,E).

To find the mixed-strategy Nash equilibria, remember that a player plays a mixed strategy iff: (i) the player is indifferent between all pure strategies played with positive probability in the mixture, given other players' strategies, and (ii) the player weakly prefers all pure strategies played with positive probability in the mixture to all pure strategies not in the mixture, given other players' strategies. Then note that, in the game above, a mixed-strategy Nash equilibrium requires that both players mix between the two strategies.

Let player 1 play B with probability p and C with probability $1-p$, and player 2 play D with probability q and E with probability $1-q$. Then player 1 is indifferent between B and C if

$$5q + 3(1-q) = 3q + 5(1-q) \;\Leftrightarrow\; q = \frac{1}{2}$$

Similarly, player 2 is indifferent between D and E if

$$4p + 2(1-p) = 0p + 4(1-p) \;\Leftrightarrow\; p = \frac{1}{3}$$

Hence, the unique mixed-strategy Nash equilibrium is $(p, q) = (\frac{1}{3}, \frac{1}{2})$.

Problem 4

We first try to find the pure-strategy Nash equilibria as we did in Problem 3. We take a pure strategy for one player, and look at what the pure-strategy best response for the other player is to that strategy. The matrix with the corresponding payoffs underlined is given below.

	R	P	S
R	0,0	-1,$\underline{1}$	$\underline{1}$,-1
P	$\underline{1}$,-1	0,0	-1,$\underline{1}$
S	-1,$\underline{1}$	$\underline{1}$,-1	0,0

Since there is no outcome where both payoffs are underlined, we find that there is no pure-strategy Nash equilibrium. This result is not surprising. This game is what is called a zero-sum game (the sum of the players' payoffs is zero). If one player wins, the other player loses. Thus, in this game, if players are playing pure strategies, some player always has an incentive to deviate.

Consider now mixed-strategy Nash equilibria. Recall the two conditions that must hold for a player to play a mixed strategy (see the solution to Problem 3 above). What would it take for a player to be indifferent between the pure strategies he plays with positive probability and to weakly prefer these strategies to all other pure strategies, given the other player's strategy? First, note that if player j is playing a pure strategy, then player i always strictly prefers one strategy over the others (if player j is playing Rock, player i wants to play Paper, etc.). Hence, the other player must also be mixing. Next, note that the players cannot be mixing between only two strategies. If player j is mixing between two strategies, then player i always strictly prefers one strategy over the others (if player j is mixing between Rock and Paper, player i wants to play Paper, etc.). Hence, player i will not be willing to mix.

It follows that a player will be indifferent and hence willing to mix only if the other player is mixing between the three strategies. More precisely, we find that there is a unique mixed-strategy Nash equilibrium: the players play each of the three strategies with equal probability (1/3).

Problem 5

Let x_1 be the location of vendor 1 and x_2 the location of vendor 2. We can associate a strategy for player i with $x_i \in [0, 1]$. We first find the payoff function for each of the vendors. Since the price of the ice cream is regulated, we can identify the profit of each vendor with the number of customers he gets. Suppose that $x_1 < x_2$. In this case, all consumers located to the left of $(x_1 + x_2)/2$ will purchase from vendor 1, while all consumers located to the right of $(x_1 + x_2)/2$ will purchase from vendor 2. Thus,

$$u_1(x_1, x_2) = \frac{x_1 + x_2}{2} \qquad u_2(x_1, x_2) = 1 - \frac{x_1 + x_2}{2}$$

Similarly, for $x_1 > x_2$,

$$u_1(x_1, x_2) = 1 - \frac{x_1 + x_2}{2} \qquad u_2(x_1, x_2) = \frac{x_1 + x_2}{2}$$

Now, if $x_1 = x_2$, the vendors split the business so that $u_1(x_1, x_2) = u_2(x_1, x_2) = 1/2$.

It is then straightforward to check that $x_1 = x_2 = 1/2$ constitutes a Nash equilibrium. No vendor can do better by deviating.

To show uniqueness, suppose first that $x_1 = x_2 < 1/2$. Then any vendor can do better by moving $\varepsilon > 0$ to the right, since he will sell almost $1 - x_1 > 1/2$ units rather than $1/2$ units. Similarly, it can be shown that $x_1 = x_2 > 1/2$ does not constitute a Nash equilibrium. Suppose now that $x_1 < x_2$. Then vendor 1 can do better by moving to $x_2 - \varepsilon$, with $\varepsilon > 0$, so this cannot be a Nash equilibrium. Similarly, it can be shown that $x_1 > x_2$ does not constitute a Nash equilibrium.