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Two proposed experiments for the Tachyon-Neutrino theory of dark matter

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Recent theory of tachyon neutrinos, providing an explanation for Dark Matter, leads to two sets of experiments for testing that theory. One predicts an enhanced signal in detection of the Cosmic Neutrino Background. The other compares models of Dark Matter used to fit data on galaxy rotation curves and also gravitational lensing. This paper starts with a semi-review of the previous theoretical work. One sees new symmetries in comparing the theory of tachyons to that of ordinary particles. In addition, we present a debunking of the false claim that tachyons would produce a causal paradox; and an apparent contradiction within the new theory, related to Dark Energy, is identified and addressed.

Keywords: Neutrinos; tachyons; dark matter; detection of cosmic neutrino background.

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1. Introduction

A series of papers of mine, published over the past dozen years show remarkable theoretical results for the hypothesis that neutrinos are tachyons (faster than light particles) with a mass around 0.1 eV. That work starts with the recognition that low energy tachyons can make strong gravitational fields through their contribution to the spatial components of the energy–momentum tensor. The Cosmic Neutrino Background (CNB) provides the physical substance for application of this theory.

These collective papers offer:

* a consistent theory of tachyons[1]
* an explanation for Dark Energy (DE)[2]
* an explanation for Dark Matter (DM)[3]
* an explanation for the chirality selection rule in weak interactions[4]

In this paper, I want to propose two sets of experiments that might verify (or negate) this theory.

In Sec. 2, I present a summary of the previous mathematical theory for tachyon-neutrinos.

In Sec. 3, I examine the Ptolemy experiment that is planned to detect the absorption of CNB neutrinos and show that its signal should be several orders of magnitude greater than expected if this Tachyon-Neutrino theory of DM is correct.

In Sec. 4, I ask about data on DM configurations in galaxies that are postulated to explain both the observations of the velocity versus radius curve and also the lensing attributed to that DM. The Tachyon-Neutrino theory predicts a ratio of 2/3 in comparing these datasets. (By comparison, the usual Newtonian theories of DM give a ratio of 2.) This section includes a discussion of how this work stands in regard to the mainstream literature about DM.

In Sec. 5, I summarize the results.

In Appendix A, I present a debunking of a widespread mistaken claim that tachyons would produce a “causal paradox”. Appendix B is a distinct but related discussion of an apparent challenge to the theory that Tachyon-Neutrinos may also explain “Dark Energy”; and I offer a qualitative explanation to resolve that challenge, which calls for further theoretical developments to become quantitative.

2. Summary of the Theory

What follows is a mini-review of my earlier theoretical work on tachyons, with some mathematics; this forms the basis for the experimental proposals to follow. Of course, there is a very large bibliography of many other peoples’ study of the idea of tachyons: a 1986 review by Recami[5] contains over 600 references; and a very recent paper by Paczos *et al.*,[6] posted in the arXiv, provides a wealth of more recent references, along with a breezy overview the many questions and controversies that have marked this field of study. At the end of this section, I shall return to discuss that interesting new paper for comparison with my own work.

My theory is all within Einstein’s Theory of Relativity, both Special and General. There will be three sections: first Classical Particle theory, then theory of Wave Equations, then, bringing those two together, Quantum Field theory.

2.1. *Classical particles*

Special Relativity places us within a 4-dimensional space–time; and there are Lorentz transformations for the basic symmetry within that manifold. The mathematics that we use to represent physical observations should be in terms of covariant quantities: scalars, vectors, higher rank tensors, in this 4-dimensional space. We also build this so as to have some connection with Newtonian physics.

Textbooks show us a 4-vector $p^\mu = (p^0, p^1, p^2, p^3) = (E, \mathbf{p})$ called the 4-momentum. E is called the energy and $\mathbf{p}$ is something like the momentum vector $\mathbf{p} = m\mathbf{v}$ of

Newtonian kinematics. A standard representation is the following:

$$E = m\gamma, \quad \mathbf{p} = m\gamma\mathbf{v}, \quad \gamma = 1/\sqrt{|1 - v^2|}. \tag{2.1}$$

Here I use units so that the velocity of light, c is the set equal to 1. If this p^μ is a covariant 4-vector then we should have the inner product with the contravariant form as a scalar.

$$p^\mu p_\mu = E^2 - |\mathbf{p}|^2 = m^2\gamma^2(1 - v^2) = \text{const.} \tag{2.2}$$

Thus, we have two distinct cases to study: if $v < 1$, that constant is written $+m^2$; and if $v > 1$, that constant is written $-m^2$. The first case we call ordinary particles; the second case we call tachyons. We do not advocate a physical symmetry here (although some other authors[7] have tried to say that tachyons must exist through some such symmetry). It is merely that the mathematics of Special Relativity is completely open to the possibility that tachyons might exist.

There is one further point to be examined. We say that the energy E of a free particle is always positive. Why is this? Think of the physics: How do we observe the motion of a particle? The Theory of Relativity comes with an observational philosophy, as follows. In any one inertial reference frame we arrange a set of local observers, situated at specified space coordinates and each having a clock, all such clocks being synchronized in that frame. A particle moves and each observer notes the time at which they see the particle at their position. This set of data points $(t_a, \mathbf{x}_a)$ is collected by the Principal Investigator and plotted on a space–time diagram. These many data points may be connected by a smooth curve, called the trajectory of the particle. This plot is commonly written mathematically as $\xi^\mu(\tau)$, where τ is some scalar parameter measured along the trajectory. We can get a 4-vector from this graph by drawing a tangent line at any point of the trajectory. This lets us define the 4-momentum, $p^\mu = m\frac{d\xi^\mu}{d\tau} = m\dot{\xi}^\mu$. But there is a small ambiguity here. The tangent line gives us the possibility of two 4-vectors: they differ by an overall minus sign.

We physicists make a choice. (I have previously called this an axiom of physics.) We *define* the 4-momentum of any free particle as having a positive value of $p^0 = E$. (We shall see later some interesting consequences of this.) One way to explain this choice is to look at how we read the data in a space–time diagram. We say, "The particle goes from place x_1 at time t_1 to place x_2 at time t_2." From this we get the velocity $v = (x_2 - x_1)/(t_2 - t_1)$. This sounds so familiar and so elementary in physics. Yet we have built in, through our choice of language (note our use of the verb, "goes"), that we have a preferred way of reading the data in the space–time diagram. We read the data *as time goes forward.* This is the habit that leads us to choose the momentum 4-vector with always a positive time-component.

The mathematics of Special Relativity involves the Lorentz group of transformations. This is a subgroup of the larger group of symmetries called the Poincaré group. The physical symmetries are those applied to a Lagrangian formalism: invariance under translations, under 3-dimensional rotations, and under velocity boosts. The abstract operators generating those transformations are written as P^μ, J_i and $K_i i = 1, 2, 3$, respectively.

The fundamental physical principle is that laws of motion should be formulated in ways that are covariant with respect to the transformations of that group. For example, we write the conservation of energy and momentum in any reaction as

$$\sum_{\text{in}} p_i^\mu - \sum_{\text{out}} p_j^\mu = 0. \tag{2.3}$$

Here, each p^μ is an algebraic 4-vector assigned to a particular particle; and they are grouped according to whether they are seen as incoming or outgoing — relative to some interaction. When one goes from one inertial reference frame to another reference frame, each one of those 4-vectors will be seen as different, according to the Lorentz transformation.

$$p'^\mu = \Lambda^\mu_\nu p^\nu. \tag{2.4}$$

Yet, all those primed 4-momenta obey the same form of the conservation equation (2.3) in the new reference frame. This is very familiar. However there are two points that should be emphasized especially when we study tachyons. If any particle is ordinary, its 4-momentum is a time-like 4-vector and it will stay within the same (in versus out) group under any Lorentz transformation. However, a tachyon will have a space-like 4-vector for its p^μ and this means that its p^0 component, viewed algebraically under a Lorentz transformation may change sign but we have defined particles as all having a positive "energy". Looking at a space–time diagram, we immediately see that this particle's momentum should be moved from "outgoing" to "incoming" (or vice versa). The minus sign in the conservation law makes this all consistent.

Look at one more thing, the energy–momentum tensor for a classical free particle. This is a second rank tensor that serves as the fundamental source of gravitation in Einstein's General Theory of Relativity. The conventional formula, involving the trajectory $\xi(\tau)$ of a classical particle, is

$$T^{\mu\nu}(x) = m \int d\tau \dot{\xi}^\mu(\tau)\dot{\xi}^\nu(\tau)\delta^4(\xi(\tau) - x). \tag{2.5}$$

We now insert $\xi^\mu(\tau) = (\gamma\tau, \gamma\mathbf{v}\tau)$ and do the τ integral to get these results:

$$T^{00} = m\gamma\delta^3(\mathbf{x} - \mathbf{v}t), \tag{2.6}$$

$$T^{0i} = m\gamma v_i\delta^3(\mathbf{x} - \mathbf{v}t), \quad i = 1, 2, 3, \tag{2.7}$$

$$T^{ij} = m\gamma v_i v_j\delta^3(\mathbf{x} - \mathbf{v}t). \tag{2.8}$$

Newtonian gravity is for sources whose motion is negligible: T^{00} is the only important term. For tachyons, the space components of this tensor T^{ij} can be large, without limit, as the energy of the tachyon goes below the mass value m. (See Eq. (2.1).) This is a surprising new result. This leads to the suggestion that, if neutrinos are tachyons, they may have important effects in Cosmology, due to the large collection of relic neutrinos left over from the Big Bang (called the Cosmic Neutrino Background, or CNB). We will also investigate, later, a possible plus-or-minus sign in front of this formula for $T^{\mu\nu}$.

2.2. *Wave equations*

We expect neutrinos, our faves for tachyons, to be spinors and so we study the Dirac relativistic wave equation.

Here are the plane wave solutions for the ordinary Dirac equation ($v < c$):

$$\psi_o = e^{i(\mathbf{k}\cdot\mathbf{x}-s\omega t)}\frac{1}{\sqrt{2\omega}}\begin{pmatrix}(s\omega+m)|\hat{k},h\rangle\\ hk|\hat{k},h\rangle\end{pmatrix},\quad \omega=+\sqrt{k^2+m^2}. \tag{2.9}$$

Here for the tachyon Dirac equation ($v > c$):

$$\psi_t = e^{i(\mathbf{k}\cdot\mathbf{x}-s\omega t)}\frac{1}{\sqrt{2k}}\begin{pmatrix}(s\omega+im)|\hat{k},h\rangle\\ hk|\hat{k},h\rangle\end{pmatrix},\quad k=+\sqrt{\omega^2+m^2}. \tag{2.10}$$

Here $|\hat{k},h\rangle$ is a 2-component spinor, eigenfunction of helicity $\sigma\cdot\mathbf{k}|\hat{k},h\rangle = kh|\hat{k},h\rangle, h=\pm 1$. These wave functions have been normalized in a particular way, following the Lorentz invariant form $\int d^4k\delta(k^\mu k_\mu \mp m^2)$.

Note that in the above equations, I have separated the 3-vector $\mathbf{k}=k\hat{k}$ into a magnitude and a unit vector. We know that the unit vector in three dimensions covers the surface of a sphere; and in two dimensions it covers the circumference of a circle. What is the unit vector in one dimension? It is the set of points at unit distance from a designated origin: the two points $+1$ and -1. This is the meaning of the symbol s used above; it is defined from $k^0 = s\omega, s=\pm 1$.

It is conventional to define the Dirac adjoint wave function $\bar{\psi}=\psi^\dagger\gamma^0$ and to construct covariant bilinear forms. The first two are Scalar and Pseudoscalar, made with the above wave functions for the two cases, Ordinary ($v < c$) and Tachyon ($v > c$)

$$\text{Scalar}: \bar{\psi_o}\psi_o = sm \quad \bar{\psi_t}\psi_t = 0, \tag{2.11}$$

$$\text{Pseudoscalar}: i\bar{\psi_o}\gamma_5\psi_o = 0, \quad i\bar{\psi_t}\gamma_5\psi_t = hm. \tag{2.12}$$

There is an interesting symmetry in the above results. The two labels, s and h, reverse their roles as one switches between ordinary particles and tachyons.

Here are two more, that are generally described as currents.

$$\text{Vector}: \bar{\psi_o}\gamma^\mu\psi_o = Csk^\mu \quad \bar{\psi_t}\gamma^\mu\psi_t = (k, k^0\hat{k}), \tag{2.13}$$

$$\text{Axial vector}: \bar{\psi_o}\gamma_5\gamma^\mu\psi_o = -hsC(k,k^0\hat{k}), \quad \bar{\psi_t}\gamma_5\gamma^\mu\psi_t = hk^\mu. \tag{2.14}$$

Here $C = |(k^0+m)/\omega|$. (One could make this constant C disappear by choosing a different normalization of the wave function for ordinary particles.) Again, one sees some symmetry here. The simplest results are the Vector, for ordinary particles, and the Axial vector, for tachyons. We note that those two bilinear forms come from the generally conserved currents, $\partial_\mu \bar{\psi}_o(x)\gamma^\mu\psi_o(x) = \partial_\mu \bar{\psi}_t(x)\gamma_5\gamma^\mu\psi_t(x) = 0$, while the formulas given above are for the simple plane wave solutions.

Let's read what these equations tell us. For the ordinary particles, the Scalar and Vector quantities say that the number s is Lorentz invariant. We knew that already: the mass shell for ordinary particles has the positive and negative frequency solutions as separate, and this is Lorentz invariant. But now look at the Pseudoscalar and Axial Vector quantities for tachyons: they say that the number h, which we call the helicity, is a Lorentz invariant. That is something new.

The Lagrangian density, a Lorentz invariant, looks like this for each case.

$$\mathcal{L}_o = \bar{\psi}_o(i\gamma^\mu\partial_\mu - m)\psi_o, \quad \mathcal{L}_t = \bar{\psi}_t\gamma_5(i\gamma^\mu\partial_\mu - im)\psi_t. \tag{2.15}$$

Finally, we want to look at the energy–momentum tensor for Dirac wave functions. This should be a symmetric second rank tensor that is conserved: $\partial_\mu T^{\mu\nu} = 0$. It can be derived from the Lagrangian density by taking the variational derivative with respect to the Minkowski metric.

$$T_o^{\mu\nu} = \frac{i}{4}\bar{\psi}_o(\overleftrightarrow{\partial^\mu}\gamma^\nu + \overleftrightarrow{\partial^\nu}\gamma^\mu)\psi_o = sCk^\mu k^\nu, \tag{2.16}$$

$$T_t^{\mu\nu} = \frac{i}{4}\bar{\psi}_t\gamma_5(\overleftrightarrow{\partial^\mu}\gamma^\nu + \overleftrightarrow{\partial^\nu}\gamma^\mu)\psi_t = hk^\mu k^\nu. \tag{2.17}$$

Here $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$.

In the limit of $m = 0$, the Dirac equation separates into two uncoupled Weyl equations, each of two components. One equation has $s = h$ and the other has $s = -h$.

There is also a question of language commonly used for the pair of particles from a relativistic theory: one is called the "particle" and the other the "anti-particle". For ordinary particles it is s, the sign of the frequency, which designates this label; and this is a Lorentz invariant label. For tachyons, I can choose to make this particle versus anti-particle nomenclature also Lorentz invariant: so it is the helicity h that marks this label.

2.3. *Quantum field theory*

The plane wave solutions of the previous section $e^{-ik_\mu x^\mu}$ are completely unlocalized: they are equally spread over all time and space. The physical concept of a "particle" is something rather well localized in space and time. We connect these two in the construction of a wave packet. It looks (approximately) like this

$$\psi = e^{i\mathbf{k}\cdot\mathbf{x} - is\omega t}F(\mathbf{x} - \mathbf{v}t), \quad \mathbf{v} = \mathbf{k}/(s\omega). \tag{2.18}$$

So, we can write a relation between the 4-vector k^μ, which designates a wave, and the 4-vector p^μ, which describes a particle, as

$$p^\mu = sk^\mu. \tag{2.19}$$

This is perfectly familiar for positive frequency waves ($s = +1$). It is perhaps unfamiliar for negative frequency waves ($s = -1$) but it correctly does two things: the direction of motion of the particle/packet is opposite the direction of propagation of the wave; and the energy $E = p^0$ is kept positive.

Quantum Field Theory is a mathematical invention (by physicists) to represent the quantum picture of discrete states built from the wave functions described above. For ordinary ($v < c$) particles, this is a well-established theory. Some other authors[9–11] have followed too closely the path, called "canonical quantization" to go from a classical field theory to a quantum field theory. For tachyons that path leads to difficulties that are met with the abandonment of Lorentz invariance. I have chosen to stay fully within Relativity Theory; and thus I create quantum field theory free of that old habit, as follows.

Physical states — of free particles — are represented by vectors in a Fock space. A one-particle state is written as $|p^\mu, s, h\rangle$. This is read as one particle, with 4-momentum p^μ (understood to be restricted by $p^0 = E > 0$), coming as a wave packet from a plane wave with frequency $k^0 = sE$, and having helicity sh. The full relation is $p^\mu = sk^\mu$; and the definition of helicity for the particle is

$$\sigma \cdot \hat{p}|p^\mu, s, h\rangle = h|p^\mu, s, h\rangle. \tag{2.20}$$

The quantity $\Psi(x)$ is an operator in some abstract vector space and also a function of space–time coordinates. First, for ordinary particles, writing the Dirac spinor for plane wave solutions Eq. (2.9) as $u(s, h, \mathbf{k})$:

$$\Psi(x) = \int d^3k \sum_h [e^{-ik_\mu x^\mu} u(s = +1, h, \mathbf{k}) a_{h,s=+1}(\mathbf{k}) + e^{ik_\mu x^\mu} u(s = -1, -h, -\mathbf{k}) a^\dagger_{h,s=-1}(\mathbf{k})], \tag{2.21}$$

$$\{a_{h,s}(\mathbf{k}), a_{h',s'}(\mathbf{k}')\} = 0, \quad \{a_{h,s}(\mathbf{k}), a^\dagger_{h',s'}(\mathbf{k}')\} = \delta^3(\mathbf{k} - \mathbf{k}')\delta_{h,h'}\delta_{s,s'}. \tag{2.22}$$

The operators a are annihilation operators $a_{h,s}(\mathbf{k})|0\rangle = 0$; the curly brackets are anticommutators $\{X, Y\} = XY + YX$; and $|0>$ is the vacuum state.

A physical state of one particle is made by the creation operator $a^\dagger$:

$$|\mathbf{p}, s, h\rangle = a^\dagger_{h,s}(\mathbf{k} = s\mathbf{p})|0\rangle. \tag{2.23}$$

It is emphasized here that the particle states are labeled by p^μ and the wave functions are labeled by k^μ. The quantized field operator connects these two sets of things. We calculate

$$\langle 0|\Psi(x)|\mathbf{p}, s, h\rangle = \delta_{s,+1} e^{-ip_\mu x^\mu} u(s = +1, h, \mathbf{p}). \tag{2.24}$$

We read this as, Ψ annihilates positive frequency particles; and $\Psi^\dagger$ will create such particles. Alternatively,

$$\langle \mathbf{p}, s, h|\Psi(x)|0\rangle = \delta_{s,-1} e^{-ip_\mu x^\mu} u(s = -1, -h, -\mathbf{p}). \tag{2.25}$$

The field operator Ψ creates negative frequency particles. But note that both types of particles are described by a wave function with positive energy p^0.

If we want to construct a state of several free particles, we write $a_1^\dagger a_2^\dagger a_3^\dagger \ldots |0\rangle$. Note that this is antisymmetric under the interchange of any pair of particles: the Pauli principle. If we let several field operators $\Psi(x)$ act on this state, and then take the inner product with $\langle 0|$, the result will have the coordinate dependence

$$\langle 0|\Psi\Psi\Psi|p_1, p_2, p_3\rangle \sim e^{-iP_\mu x^\mu}, \quad P^\mu = p_1^\mu + p_2^\mu + p_3^\mu. \tag{2.26}$$

This will lead to the physical law conserving total energy and momentum (Eq. (2.3)).

We now work similarly for the Dirac-tachyon particle. The quantity $\Psi(x)$ is an operator in some abstract vector space and also a function of space–time coordinates. The question is: which quantum numbers of the wave functions are used to pair with the annihilation versus creation operators? In my earliest work it was just the helicity h[8]; in more recent work I chose the product sh[4]; now I shall use only the label s. Using the Dirac spinors for plane wave solutions (Eq. (2.10)), we write for tachyons:

$$\begin{aligned}\Psi(x) = \int d^2\hat{k}\int_0^\infty d\omega k^2 \textstyle\sum_h [e^{-ik_\mu x^\mu}\begin{pmatrix}(\omega + im)|\hat{k}, h\rangle \\ hk|\hat{k}, h\rangle\end{pmatrix} b_{h,s=+1}(\hat{k}, \omega) \\ + e^{+ik_\mu x^\mu}\begin{pmatrix}(-\omega + im)|-\hat{k}, h\rangle \\ -hk|-\hat{k}, h\rangle\end{pmatrix} b^\dagger_{h,s=-1}(\hat{k}, \omega)],\end{aligned} \tag{2.27}$$

$$\{b_{h,s}(\hat{k}, \omega), b_{h',s'}(\hat{k}', \omega')\} = 0, \tag{2.28}$$

$$\{b_{h,s}(\hat{k}, \omega), b^\dagger_{h',s'}(\hat{k}', \omega')\} = \delta^2(\hat{k} - \hat{k}')k^{-2}\delta(\omega - \omega')\delta_{h,h'}\delta_{s,s'}. \tag{2.29}$$

The operators b are annihilation operators $b_{h,s}(\hat{k}, \omega)|0\rangle = 0$; the curly brackets are anticommutators; and $|0>$ is the vacuum state. The single particle tachyon states are constructed as

$$|\hat{p}, E, s, h\rangle = \frac{1}{\sqrt{2p}} b^\dagger_{h,s}(\hat{p}, E)|0\rangle, \quad p = \sqrt{E^2 + m^2}. \tag{2.30}$$

An essential calculation involves the anticommutator of the field and its adjoint at two different space–time points. We find

$$\{\Psi(x), \Psi^\dagger(x')\} = (\ldots)\int_0^\infty d\omega k^2 \frac{\sin kr}{kr}\cos\omega(t - t'), \quad r = |\mathbf{x} - \mathbf{x}'|. \tag{2.31}$$

The symbol $(\ldots)$ stands for a 4×4 matrix of derivatives acting on the space–time coordinates x. That integral over ω equals zero if $|t - t'| > r$. This is the physical statement that tachyon waves always propagate faster than light. This result affirms that we have a good representation of the quantized tachyon field.

The strict separation of the positive and negative frequency wave functions shown above can be understood in another way, reverting to the previous study of wave packets. Suppose one builds an initial wave function that is a linear combination of both positive and negative frequency components: how will that packet evolve in time? One can carry out a numerical calculation of just this sort with any relativistic wave equation and see that the single packet separates into two packets, traveling in opposite directions. That is a purely mathematical exercise; Physics sees no such phenomena.

Another important step in constructing a Quantum Field Theory is to study the Wigner "Little Group". For tachyons, this turns out to be the nonunitary group O (2,1). We introduce an indefinite metric $\mathcal{I}$ in the Fock space for one-particle states and ask that the matrix elements transform properly when the states are transformed $|\phi> \to |U\phi>$.

$$\langle U\phi|\mathcal{I}O|U\chi\rangle = \langle\phi|U^\dagger\mathcal{I}OU|\chi\rangle = \langle\phi|\mathcal{I}U^{-1}OU|\chi\rangle. \tag{2.32}$$

The metric should transform the adjoint into the inverse. For tachyons, with the Little Group O(2,1), we found that the metric could be simply the helicity of the particle. (One might explore an alternative choice, perhaps involving the quantum number s. But I will not do that here.)

Finally, we want to calculate the energy–momentum tensor for quantized tachyons. Of primary interest is the appearance of plus-or-minus signs. For ordinary Dirac particles, we see, from Eq. (2.16), a factor s. When we work with the Quantized field operator Ψ instead of the wave function ψ there is one more critical step: Put the annihilation/creation operators into Normal Order. This means that the vacuum state will have zero value for $T^{\mu\nu}$. It also means that a factor of s will be introduced in the one particle expectation value. The net result, for ordinary ($v < c$) particles, is that $T^{00} > 0$. This is a famous result for ordinary matter. However, for tachyons there is a different story: from the wave functions (Eq. (2.17)) there is a factor h; from Normal Ordering there is a factor s; and from the indefinite metric there is another factor h. The net result is

$$\langle p^\mu, h, s|\mathcal{I} : T^{\mu\nu} : |p^\mu, h, s\rangle = s p^\mu p^\nu. \tag{2.33}$$

In my earlier work, there was a factor h, not s, in this important formula. Either way, we find that, if the Cosmic Neutrino Background consists of low energy tachyons, they split into two types: one type creates "positive" gravity just like ordinary matter; the other type creates "negative" gravity. This is something very new and unique coming from this tachyon-neutrino theory.

The physical model for the CNB follows from these results. The two types of tachyons separate gravitationally at some intermediate phase of cosmic evolution. One type condenses around ordinary matter in galaxies, forms self-binding clouds that provide the gravitational fields now ascribed to an unknown "Dark Matter". The other type remains distributed throughout the Universe and, through its "negative gravity" creates the pressure now ascribed to an unknown "Dark Energy." As detailed in my earlier papers,[2,3] these two theoretical predictions are both qualitatively and quantitatively in agreement with the presently accepted data.

This concludes the summary of my previous theoretical work on tachyon-neutrinos.

2.4. *An alternative formalism*

The very recent paper by Paczos *et al.* [6] noted previously, presents a new mathematical formalism for the quantization of a tachyonic field theory. They expand the usual Fock space for tachyon particles, called $\mathcal{F}$, to be a twin set of Fock spaces, written as $\mathcal{F} \otimes \mathcal{F}^*$, where the first carries "input" particles and the second is for "output" particles. With this they are able to handle particles described by a spacelike 4-momentum (e.g. tachyons) and avoid the problems of negative frequency wave functions encountered by other authors.

I find this a very interesting work; and I note that those authors say that the starting point for their innovation comes from an earlier paper of mine[18] published in 2018. The question I am unable to answer is how does their theory compare to mine, as outlined above (and first published in 2022[4])? Are they equivalent theories or are they different in some fundamental way? What distinguishes my theory from the previous authors is the identification of the quantum number s, which marks whether a given physical particle is derived from a wave function of positive ($s = +1$) frequency or negative frequency ($s = -1$). This labeling is a kind of doubling of the Fock space.

There are some significant differences in their approach compared to my own. I make a big point of distinguishing wave functions, and their 4-vector k^μ, from particles, and their 4-vector p^μ. The mapping between these two, $p^\mu = sk^\mu$, is based upon the very physical exercise of building a wave packet out of plane waves. Another difference is that I focus mostly on spinors, while they deal mostly with scalar fields. The Dirac equation shows a symmetric arrangement with the two quantum numbers h (helicity) and s (sign of the frequency) when one sets ordinary particles theory next to tachyon theory.

Now, we are ready for the new proposed experiments.

3. Ptolemy

This name (for Princeton Tritium Observatory...) is a proposal to detect the Cosmic Neutrino Background (CNB) through the induced beta decay of Tritium: $\nu + T \rightarrow {}^3\mathrm{He} + e^-$. According to the standard cosmological theory, there is a great sea of neutrinos left over from the big bang: their density is $56/\mathrm{cm}^3$ (for each type of neutrino) and their temperature is around 1 K. It is expected to find a very sharp spike in the emitted electron energies at the endpoint of the usual beta decay spectrum. If the neutrinos are ordinary particles with a small mass, the location of this spike will be slightly beyond the expected end point. If they are tachyons, there is no such displacement.

Ptolemy[12] is expected to be a very difficult experiment: seeing a small effect with a high resolution electron detector. The KATRIN[13] experiment now in progress is not likely to reach the sensitivity needed to see the CNB.

3.1. *My surprise*

My recent paper[3] on DM proposes that a large portion of the CNB, rather than being dispersed throughout the universe, is condensed about most galaxies. This increased density — by a factor of the order of 10^3 — provides the extra gravitational field to explain DM effects in galaxies.

If true, this increase in neutrino density within a galaxy (our galaxy in particular) would make the expected signal in the Ptolemy experiment considerably larger than originally expected.

But wait, there is more!

My theory separates neutrinos from antineutrinos according to their helicity. The two helicity types behave differently in General Relativity — only one type explains DM. They also behave differently in beta decay (with lepton number conservation, as in my latest paper[4]). I do not know which way these two markings are connected. That is a question to be answered by experiments. So, the Ptolemy experiment should be conducted with two types of target materials: one with Tritium, an e- emitter, and the other with an $e+$ emitter. One of these is predicted to show the large enhancement, the other not.

A recent paper by Bauer and Shergold[14] analyzes the overdensity of neutrinos that may be detected by Ptolemy but they do not consider tachyon-neutrinos. This seems like an exciting possibility. I look forward to reactions from other physicists.

4. Dark Matter Data

The most common type of theory offered to explain Dark Matter — called CDM — is based upon Newton's theory of gravitation. I want to start with Einstein's theory of gravitation. To make things easy, I'll use the linear approximation to General Relativity. The metric tensor $g_{\mu\nu}(x)$ is expanded in powers of the gravitational constant G.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} - 1/2\eta_{\mu\nu}h + O(G^2). \tag{4.1}$$

Here $\eta_{\mu\nu}$ is the Minkowski metric tensor $(1, -1, -1, -1)$ and h is the contraction $h^\mu_\mu = h_{00} - \Sigma_{i=1,3}h_{ii}$. The tensor $h_{\mu\nu}$ obeys a wave equation with $T_{\mu\nu}$, the energy–momentum tensor, as its source. If we have a static situation (not interested in gravitational radiation here), then we have

$$h_{\mu\nu}(\mathbf{x}) = 4G\int d^3x' T_{\mu\nu}(\mathbf{x}')/|\mathbf{x} - \mathbf{x}'|. \tag{4.2}$$

This gives us the gravitational field, the metric $g_{\mu\nu}$, in terms of the sources $T_{\mu\nu}$. Now we need the geodesic equation to see how particles move in this field.

$$\ddot{\xi}^\mu = -\Gamma^\mu_{\alpha\beta}\dot{\xi}^\alpha\dot{\xi}^\beta, \tag{4.3}$$

where $\xi^\mu(\tau)$ is the trajectory of the particle in space–time, the dot means $d/d\tau$, and Γ is the connection, given in terms of derivatives of the metric $g_{\mu\nu}$.

4.1. *Slow matter*

Let's first ask about the motion of nonrelativistic masses in this gravitational field — motion of stars in galaxies or clustering of galaxies. We start by approximating the right-hand side of Eq. (4.3): $\dot{\xi}^\alpha = (1, 0, 0, 0)$. Then the geodesic equation becomes simply

$$d^2\mathbf{x}/dt^2 = -1/2\nabla g_{00}, \tag{4.4}$$

which we recognize as Newton's second law of motion ($F = ma$).

We usually write the F/m as the negative gradient of a potential, V. So we have

$$V(\mathbf{x}) = -G\int d^3x' S(\mathbf{x}')/|\mathbf{x} - \mathbf{x}'|, \tag{4.5}$$

$$S = 2T_{00} - [T_{00} - \Sigma_i T_{ii}) = \rho + p. \tag{4.6}$$

Here we introduce the familiar terms ρ (energy density or rest mass) and p (pressure) for the two major parts of the energy–momentum tensor.

If you forget about p, then you have Newtonian gravity. What could give you a large pressure? Low energy Tachyons; neutrinos present in large numbers throughout the universe. And they look just like a Newtonian source of gravity in the equation above but they are physically very different.

4.2. *Light*

Now we look at gravitational lensing — the deflection of light passing through a given gravitational field. Here we start the geodesic equation (4.3) with the input velocity 4-vector $\dot{\xi}^\alpha = (1, 0, 0, 1)$, for the light beam traveling initially in the z-direction. The equation of motion in the transverse directions is then

$$\ddot{x}^i = -\Gamma^i_{00} - \Gamma^i_{33} = -1/2\partial_i[g_{00} + g_{33}], \quad i = 1, 2. \tag{4.7}$$

This leads to an effective potential, as seen above, with the source

$$S = 2[T_{00} - 1/2T + T_{33} + 1/2T] = 2(\rho + p/3), \tag{4.8}$$

assuming that the pressure is isotropic. This again shows how tachyon-neutrinos can mimic (through their "pressure") the effects of Newtonian gravity.

4.3. *A glance at the DM literature*

DM has been a major issue for physics researchers for some time and readers may ask how the work presented above fits into that literature. I cannot offer a comprehensive review but will take a quick look at two books that represent milestones in the history of this subject. The first is Weinberg's "Cosmology" (2008),[15] which is a deep theoretical review of the whole subject with much mathematics and physical analysis; the second is Fisher's "What is Dark Matter" (2022),[16] an experimentalist's review of the most recent ideas and experiments on this particular topic, written for a general audience.

I stated at the outset of Sec. 4 that the prevailing approach to Dark Matter was based upon Newtonian theory of Gravity. My departure is to look at Einstein's theory of gravitation, include the effects of "pressure" in the energy–momentum tensor, and consider the possibility of tachyons, especially ascribing this property to neutrinos. So I ask: Do either of those two words, "pressure" and "tachyon" appear in those books?

Fisher's book is a small printed volume; and the subject index shows neither word present. Weinberg's book is a huge tome, which I have on my computer; so I can do a full text search and find no mention of tachyon. Weinberg does use the word "pressure" on several occasions but it is never in connection with discussions about DM.

Chapter 9 of Weinberg's book is all about Gravitational Lenses; and one sees from the start that he considers only gravitational fields produced by stationary masses — Newtonian model of gravity.

Chapter 4 of Fisher's book, entitled, "What Dark Matter is Not", discusses neutrinos, especially those comprising the Cosmic Neutrino Background. He tells readers that the neutrino mass is much too small to account for the gravitational fields attributed to DM. This is Newtonian thinking.

There are other speculative theories offered to explain DM, often based upon ideas outside of Einstein/Newton theories of gravitation. Some of those are called Modification of Newtonian Dynamics (MOND). Weinberg makes no mention of MOND theories; and Fisher discusses a detailed analysis of astronomical observations (on the Bullet Cluster) that leads to the rejection of MOND.

There is also a theory about Axions, another imagined particle that might be the source of DM. That theory gets a lot of attention from theorists and experimentalists. The tachyon-neutrino theory stands as a major independent set of physical ideas and mathematics.

The mathematics I have presented in the earlier sections is absolutely standard work within Einstein's theory of gravitation; and it shows how the "pressure" terms in the energy–momentum tensor can exactly mimic the mass (or energy density) term that we identify with Newton's theory. That calculation is easy — although it appears to be something not previously acknowledged. The challenge is to find a large source of matter-in-motion to be that source of pressure — and here the idea of tachyons is essential because at low energies they can produce surprisingly strong gravitational fields. My recent paper[3] provides the detailed quantitative calculations for this model.

Why does the theory of tachyon-neutrinos get no attention from mainstream physics researchers? Tachyons have a bad reputation. There have been a few famous occasions when some big experiment reported to have detected some neutrinos traveling faster than the speed of light — only to have those experimental results retracted due to some false readings on their equipment.[a] Those are all experiments

[a] The OPERA experiment of 2011 is the most famous case; I have discussed this quantitatively in Ref. 1.

at high energies; and the CNB that we focus on here is very low energy, where no experiments have as yet been able to detect them. (So we wait for Ptolemy.)

But there is more baggage to be noted. If you look up "tachyon" on your favorite computer search engine and follow the link to Wikipedia, you find a morass of misinformation: "Physicists believe that faster-than-light particles cannot exist because they are inconsistent with the known laws of physics. ..." In Sec. 3 of my paper,[1] I provide detailed debunking of numerous anti-tachyon myths. In Appendix A of this paper, I dissect the familiar canard that if tachyons exist, they would lead to an untenable "causal paradox".

4.4. *Data quest*

The above analysis presents a clear test of the "Newtonian"-based theories versus the tachyon-based theory about the origin of DM. This is not just a qualitative challenge ("don't ignore the pressure effects") but a precise quantitative challenge. Comparing the strength of the source as it affects the velocity curve relative to lensing observations [Eq. (4.6) versus Eq. (4.8)]. Newtonian theory shows a simple factor of 2 while the tachyon theory shows a factor of 2/3. This should be something to check out and I ask for other experts on this subject to see if the relevant data exists, or could be readily collected, to check this prediction.

5. Conclusions

Two sets of measurements are here proposed to check the tachyon-neutrino theory for DM.

Section 3 presented the proposal that experiments for the direct detection of Cosmic Background Neutrinos should see a signal several orders of magnitude greater than what is expected from the conventional theory.

Section 4 presents the proposal for comparing data on quantitative models for DM to explain two types of astronomical measurements: the galaxy rotation curves versus the gravitational lensing by individual galaxies.

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Appendix A. Do Tachyons Lead to a Causal Paradox?

A familiar claim among theoretical physicists is that if tachyons (faster-than-light particles) exist, one would encounter a causal paradox: "sending a signal into the past". This claim is examined below within the Special Theory of Relativity along with a careful attention to the words used. We find that there is no paradox.

Examining the Paradox

The textbook scenario for a paradox due to tachyons involves signals sent via faster-than-light mechanisms between Earth (E) and a spaceship (SS) traveling away from Earth. In order to get the desired effects one has the velocity of the spaceship v and the velocity of the tachyons V obeying the relation $vV > c^2$.

Event A: A first tachyon signal is sent from Earth toward the SS
Event B: That first signal is received at the SS
Event C: A second tachyon signal is sent from the SS toward Earth
Event D: That second signal is received at Earth

The claimed paradox arises if C occurs after B and D occurs before A. It appears that the rules of Special Relativity, the Lorentz Transformation, allow such a situation. This looks like "a signal sent into the past" $(A \to D)$.

I will want to critique the words used in the sentences above. And it will be important to remember the following result from the mathematics of Lorentz Transformations: If two events are separated by a spacelike interval, $\Delta x > c\Delta t$, then the time sequence of those two events may be reversed in a different reference frame. Here is the equation:

$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2) = \gamma\Delta t(1 - vV/c^2), \quad \gamma = 1/\sqrt{1 - v^2/c^2}. \tag{A.1}$$

The condition for that reversal is $vV > c^2$, where v is the velocity of the Lorentz transformation and $V = \Delta x/\Delta t$.

Let's examine the first tachyon signal. As observed in reference frame of the Earth, the time of event A is earlier than the time of event B, so it is correct to say that this signal is "sent" from Earth to SS. However, this same tachyon signal observed in the reference frame of the SS would show that the time of event A is later than the time of event B, so they would say that the signal was **not** sent from Earth to SS but rather the opposite: it was "sent" from SS to Earth!

A similar analysis of the second tachyon signal leads to the following conclusion: *Observers in the reference frame of Earth say that they "sent" two signals to the SS; while observers on the SS say that they "sent" two signals to Earth.*

There is quite a disagreement but there is **no paradox.** Special Relativity is all about recognizing that there will be different descriptions of any experiment viewed from different reference frames. The error made in promulgating the above idea of a "causal paradox" was to assume that the words "sent" and "received" have an invariant meaning.

Asserting the Paradox

I go on my computer and Google the word Tachyon. I get the following: "A tachyon (...) or tachyonic particle is a hypothetical particle that always travels faster than light. Physicists believe that faster-than-light particles cannot exist because they are inconsistent with the known laws of physics."

Next is a link to Wikipedia, its article titled, Tachyon. There I read the above two sentences, followed by, "If such particles did exist they could be used to send signals faster than light. According to the theory of relativity this would violate causality, leading to logical paradoxes such as the grandfather paradox."

The Wikipedia article cites several references for these claims. They include peer reviewed papers in scientific journals, articles in Physics Today, and widely used physics monographs and textbooks.

Something is wrong in tinseltown. How to set it right?

I found the "talk"page of this Wikipedia article and entered the following.

"**Debunking anti-tachyon myths**

"The opening paragraph of Wikipedia's article on Tachyons needs to be challenged. Causality is not a fundamental law of physics. It is a consequence of Special Relativity applicable only to particles that travel slower than the speed of light. The whole idea of "grandfather paradox" is based upon careless use of language (by many respected physicists). A detailed debunking of several anti-tachyon myths is published in Charles Schwartz, "A Consistent Theory of Tachyons with Interesting Physics for Neutrinos" Symmetry 14, 1172 (2202) Charlieschwartz (talk) 15:28, 25 February 2024 (UTC)"

There has been no response from Wikipedia editors.

Let me look a bit back in the history of this idea. The 1969 paper by Bilaniuk and Sudarshan [17] presented the theoretical idea of tachyons to a broad audience of physicists. They attribute the original idea of a causal paradox to work of Tolman in 1917. They dispense with the idea of a paradox by introducing a "reinterpretation principle" — that a momentum 4-vector with a negative energy component should be reinterpreted as a positive energy particle moving in the opposite direction. In my opinion, this was an unfortunate choice of words: it gives the appearance that they are making up a new rule in order to get out of an embarrassing situation. My analysis, shown above, uses only the standard principles of Special Relativity (with no "reinterpretation" needed); and concludes that the false idea of a "causal paradox" comes simply from careless use of language in describing the thought experiment. (In fact, a side remark in the B and S paper says, as I say above, that each set of observers reports that they sent two tachyon signals: contradicting the idea of a "signal sent back in time.") It is strange that the idea of a causal paradox due to tachyons appears to remain widely accepted among physicists.

Appendix B. Challenge Regarding Dark Energy

My theory says: Two types of tachyons — particle and antiparticle — distinguished by helicity, have opposite sign in front of their energy–momentum tensors.

My model has one type condensed around galaxies, giving the large gravitational fields commonly attributed to a mysterious DM. The other type remains distributed throughout the universe and produces the negative pressure commonly attributed to a mysterious DE.

In earlier epochs the two were mixed in the hot stuff that came out of the Big Bang; and their gravitational effects just canceled out. As the universe expanded, everything cooled, and at some time the first type condensed out, onto the galaxies, leaving the other type spread out.

The Problem: When we look at the whole of the universe (the FLRW model) then we should add up the effects of both types; and it seems that the net gravitational field claimed to produce DE cancels itself out.

A Possible Solution: Maybe, when the one type condenses onto galaxies, the increase in particle density (which I estimated in my paper to be a factor of about 10^3) implies and increase in "energy" and thus a decrease in the "pressure", due to the particular kinematics of tachyons. Thus, the condensed portion of the CNB becomes a weaker gravitational source than the portion left out there; and so the cancellation is not complete, perhaps only a mild diminution. Along with this would be a mild diminution of the DM gravitational field.

This needs a reliable mathematical model to see if the detailed calculations leave both stories credible. My earlier attempt at Statistical Mechanics for Tachyons[18] needs to be reinvestigated.

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