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Revised theory of tachyons in general relativity

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A minus sign is inserted, for good reason, into the formula for the energy–momentum tensor for tachyons. This leads to remarkable theoretical consequences and a plausible explanation for the phenomenon called dark energy in the cosmos.

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1. Introduction

Studying tachyons, or writing research papers about them, seems always to evoke an entrenched set of negative prejudices from my esteemed colleagues. For example, the official reviewer for this paper, after noting the interesting results presented here, wrote: "However, the author would need to first explain why/how the problem of causality (due to faster than light propagation) can be ignored or avoided. Without this, tachyons would bring with them problems more serious than it is aiming to solve." Let me respond.

In my earlier paper,[1] I wrote an appendix that analyzed the familiar paradox about causality violation with hypothetical tachyons being sent between earth and a spaceship. I showed how the simple consideration of wave packets, rather than point particles, made this whole paradox disappear.

In my more recent paper,[3] I explored quantum theory for tachyon fields. Here, I was able to construct a consistent formalism in which the tachyon fields had zero commutation (or anti-commutation) relations for points that were separated by a time-like interval. In other words, the usual meaning of causality (no interaction over space-like intervals) is appropriate for slower-than-light particles, but for tachyons, causality has exactly the opposite meaning. This is all about mathematical consistency within the physical definition that tachyons, if they actually exist, are things (particles or fields) that always propagate faster than light.

2. The Older Theory

In a previous paper,[1] I studied how a classical tachyon (faster-than-light particle), if such a thing exists, would behave within the conventional theories of Special and General Relativity. If a particle follows a trajectory in spacetime described by $\xi^\mu(\tau)$, then we write the following invariant, according to Special Relativity,

$$\dot{\xi}^\mu \dot{\xi}^\nu \eta_{\mu\nu} = \epsilon\,, \tag{2.1}$$

where the dot signifies derivative with respect to τ and $\eta_{00} = +1$, $\eta_{11} = \eta_{22} = \eta_{33} = -1$. If we have $\epsilon = +1$, this is an ordinary particle, moving with speed $v < c$; and if $\epsilon = -1$, this is a tachyon, moving with speed $v > c$. A standard representation is (in units where $c = 1$)

$$\dot{\xi}^\mu = (\gamma, \gamma\mathbf{v})\,, \qquad \gamma = 1/\sqrt{|1 - v^2|}\,. \tag{2.2}$$

I then followed procedures familiar for ordinary particles and wrote down the energy–momentum tensor for any such point particle,

$$T^{\mu\nu}(x) = m \int d\tau\, \dot{\xi}^\mu(\tau)\dot{\xi}^\nu(\tau)\delta^4(x - \xi(\tau))\,. \tag{2.3}$$

The first remark was that for very low energy tachyons, which have very large velocities, the space components of this tensor would be very large.

$$T^{ii}(x) = m \int d\tau (\gamma v_i)^2 \delta(t - \gamma\tau)\delta^3(\mathbf{x} - \mathbf{v}\gamma\tau) = m\gamma v_i^2 \delta^3(\mathbf{x} - \mathbf{v}t)\,. \tag{2.4}$$

With this tensor as the source in Einstein's General Theory, I then calculated the gravitational fields produced by free moving tachyons and found that there would be a strong attractive force on other nearby tachyons moving parallel (or antiparallel). This led to a picture of rope-like structures of many tachyons that would produce strong gravitational fields in their vicinities. I thought that this structure might, if somehow attached to a galaxy, offer an explanation for the observed physical effects now attributed to Dark Matter: galactic containment and gravitational lensing.

3. The Revised Theory

Simply put, there should be a minus sign in Eq. (2.3) for a tachyon.

How I came to this idea is an amusing story; but here I shall just show the derivation.

We begin with the stationary principle for an arbitrary particle in the formalism of General Relativity, given a metric $g_{\mu\nu}(x)$ that can vary with spacetime coordinates x, replacing the flat-space metric $\eta_{\mu\nu}$.

$$A = \int d^4x\, \mathcal{L}(x)\,, \qquad \mathcal{L}(x) = m \int d\tau\, \sqrt{\epsilon \dot{\xi}^\mu \dot{\xi}^\nu g_{\mu\nu}(\xi)}\delta^4(x - \xi(\tau))\,. \tag{3.1}$$

Here, as in Eq. (2.1), $\epsilon = +1$ denotes an ordinary particle ($v/c < 1$) and $\epsilon = -1$ designates a tachyon ($v/c > 1$). The epsilon must be placed under the square root sign so that this whole argument will be positive. When we ask that the action $\mathcal{A}$ be stationary with respect to variation of the particle trajectories $\xi^\mu(\tau)$, we get the familiar geodesic equation,

$$\ddot{\xi}^\nu + \Gamma^\nu_{\alpha\beta}\dot{\xi}^\alpha\dot{\xi}^\beta = 0\,, \tag{3.2}$$

where Γ is the Christoffel symbol, involving derivatives of the metric $g_{\mu\nu}$, and we note that there is no epsilon appearing in this equation.

However, when we calculate the energy–momentum tensor for this Lagrangian density, either by Noether's theorem or by varying the action (3.1) with respect to the metric $g_{\mu\nu}$, we do get the epsilon out in front,

$$T^{\mu\nu}(x) = \epsilon m \int d\tau\, \dot{\xi}^\mu \dot{\xi}^\nu \delta^4(x - \xi(\tau))\,. \tag{3.3}$$

If we calculate the divergence of this $T^{\mu\nu}$, using (3.2) and (3.3), we get

$$\partial_\mu T^{\mu\nu} = -\Gamma^\nu_{\alpha\beta} T^{\alpha\beta}\,, \tag{3.4}$$

which is the usual result (without any epsilon) and we can then go on to put in the factor of $\det(g)$ and go to covariant derivatives for the full structure of Einstein's equation.

Given many particles, we can simply add their contributions to $\mathcal{A}$, each one with its own mass and epsilon. They each obey their own geodesic equation (3.2). Similarly, they each contribute additively to $T^{\mu\nu}$ as in (3.3), but here each epsilon makes an important contribution. There is, however, the alternative choice of putting a minus sign (the same epsilon) in front of the mass parameter for each tachyon in the action. This will return us to the original formula (2.3) for tachyons in the energy–momentum tensor. In this paper, we wish to explore the first choice: keep all the contributions to the action as positive quantities.

With this new $\epsilon = -1$ factor entering the tachyons' energy–momentum tensor, we will have repulsive, not attractive forces between colinear flows of tachyons. That interesting physical picture and mention of Dark Matter — given in my earlier paper — blows away.

There is something else big and new here. If we have lots of tachyons in the universe, they will all be flying about repulsed by one another but they still may contribute something important to the overall $T^{\mu\nu}$ as it is used to construct the Robertson–Walker metric of the cosmos. There will be NEGATIVE pressure, and it may be large. That is what the idea of Dark Energy was invented to provide.

4. Neutrinos and Numbers

Rather than imagining some unknown things to be tachyons, it is more conservative to suggest that some physical things, known but not completely known to us, may actually be tachyonic. Neutrinos are the best candidate. They interact very weakly with ordinary matter or radiation and they are plentiful.

From various experimental results, it is believed that neutrinos have a mass that is very small but not zero. An estimate is of the order of $m \sim 0.1\ \mathrm{eV}/c^2$.

From reigning cosmological models[2] it is thought that there is a vast sea of low energy neutrinos, called Cosmic Neutrino Background, with an average density around $340/\mathrm{cm}^3$ and an average energy E of around 1.7×10^{-4} eV per particle.

If these are actually tachyons, then we would say they have a typical velocity of $v/c \sim mc^2/E \sim 6 \times 10^2$.

We take the formula from Eq. (2.4) and multiply it by 1/3 to average over directions; then, following the previous discussion, this gives a negative pressure amounting to about $mc^2(v/c)/3$ times the density of such particles which is, $P \sim$ 6,800 eV/cm^3. This is of the correct order of magnitude to replace the current postulates of Dark Energy.

Obviously, there are a number of more detailed experimental and theoretical questions to be asked and answered before this hypothesis should be accepted as true. It will require others, more broadly experienced than I am, to become engaged.

5. Further Questions

My older theory offered a possible explanation for Dark Matter; but with $\epsilon = -1$, the force binding together flows of tachyons is gone. We now envision a universe filled with a sea of tachyonic neutrinos, which repulse each other (gravitationally) and also repulse ordinary matter. If galaxies form, as a fairly tight collection of ordinary matter, they might push a hole in that background sea of tachyon neutrinos; but then the surrounding sea would exert a "compressive" gravitational force on all that matter in the galaxy. Could this be an alternative explanation for "Dark Matter"? This compressive gravitational force would keep the outer parts of the galaxy rotating at a faster velocity, and it could also be a major factor in gravitational lensing.

While this theory gives repulsive forces between tachyons and attractive forces between ordinary particles, other combinations appear strange. The gravitational force from tachyons repels ordinary particles while the gravitational force from ordinary particles attracts tachyons. This appears to violate Newton's third law. That law comes from non-relativistic theory and is often summarized as guaranteeing overall conservation of momentum. In our relativistic theory that conservation is held by the vanishing divergence of the energy–momentum tensor. Still, this is puzzling.

The numbers cited above describing the Cosmic Neutrino Background come from the application of classical theory of statistical mechanics. Is that correct for tachyons? I do not know.

Everything in this paper has been formulated in terms of classical (but relativistic) theory of particles. What about quantum mechanics and quantum field theory? I have written a little about that,[3] but questions remain to be explored. The following section is a brief sortie into field theory.

6. Complex Scalar Fields

Let us look at a Lagrangian for a system that involves two types of particles, each described by a complex scalar field, and they interact. The first field will describe an ordinary particle and the second will describe a tachyon.

$$\mathcal{L} = (\partial^\mu \phi_1^* \partial_\mu \phi_1 - m_1^2 \phi_1^* \phi_1) + \epsilon(\partial^\mu \phi_2^* \partial_\mu \phi_2 + m_2^2 \phi_2^* \phi_2) - V(\phi_1, \phi_1^*, \phi_2, \phi_2^*)\,, \quad (6.1)$$

where I have introduced a factor ϵ that may be chosen as $+1$ or -1. The equations of motion are now

$$\partial^\mu \partial_\mu \phi_1 + m_1^2 \phi_1 = -\frac{\partial V}{\partial \phi_1^*}\,, \qquad \partial^\mu \partial_\mu \phi_2 - m_2^2 \phi_2 = -\epsilon \frac{\partial V}{\partial \phi_2^*}\,. \quad (6.2)$$

We construct the energy–momentum tensor for the interacting system.

$$T^{\mu\nu} = (\partial^\mu \phi_1^* \partial^\nu \phi_1 + \partial^\nu \phi_1^* \partial^\mu \phi_1) + \epsilon'(\partial^\mu \phi_2^* \partial^\nu \phi_2 + \partial^\nu \phi_2^* \partial^\mu \phi_2) - \eta^{\mu\nu} \mathcal{L} \quad (6.3)$$

and we find that $\partial_\mu T^{\mu\nu} = 0$ with only the conditions $\epsilon = \epsilon'$ and $V^* = V$. (This follows Noether's theorem.)

Conventional theoretical work would have led me to write the Lagrangian with $\epsilon = +1$. The new possibility here is the choice $\epsilon = -1$. If we look at a space–space component of the tensor for a tachyon, dropping the interaction potential, we find,

$$T^{11} = \epsilon[|\partial_0 \phi|^2 + |\partial_1 \phi|^2 - |\partial_2 \phi|^2 - |\partial_3 \phi|^2 + m^2 |\phi|^2]\,. \quad (6.4)$$

For a plane wave solution this is just $2\epsilon|\partial_1 \phi|^2$, which will give a negative pressure if we choose $\epsilon = -1$.

Here the reviewer has objected that with a negative Hamiltonian density, due to $\epsilon = -1$, the energy is not bounded from below and thus such a universe is unstable. This illustrates another common mistake of applying familiar rules derived for ordinary particles to theoretical studies of tachyons. In a much earlier paper,[4] I showed how to handle the energy–momentum tensor for systems that contained tachyons as well as ordinary particles. From that, one sees that focusing on the Hamiltonian is a fine habit for ordinary particles, but for tachyons one must also include flows through time-like surfaces. A simpler example, looking at possible negative energies of tachyon states, is discussed in the Appendix below.

Appendix A. Negative Energy Tachyons?

Look at the simplest interaction one can imagine concerning a tachyon: the decay of a π meson into a μ meson and a neutrino — with the latter treated as a tachyon. Conservation of energy and momentum, which may be derived from classical particle dynamics or from quantum mechanical calculation of a transition matrix element, give the following equations, in the frame where the original particle is at rest.

$$\mathbf{p}_\pi = 0 = \mathbf{p}_\mu + \mathbf{p}_\nu\,, \qquad E_\pi = m_\pi = E_\mu + E_\nu\,, \quad (A.1)$$

$$E_\mu = +\sqrt{p_\mu^2 + m_\mu^2}\,, \qquad p_\nu^2 = E_\nu^2 + m_\nu^2\,. \tag{A.2}$$

The energy of the tachyon E_ν can be a negative quantity and so one might worry that there could be states in which the tachyon comes out with large negative energy and the μ meson comes out with large positive energy. That would be a very unstable situation. But let us look at the whole set of conservation equations written out above and see if they allow such a thing.

$$m_\pi = +\sqrt{p_\mu^2 + m_\mu^2} + E_\nu = +\sqrt{E_\nu^2 + m_\nu^2 + m_\mu^2} + E_\nu\,. \tag{A.3}$$

The last expression on the right-hand side is a monotonic function of E_ν and the only place where it can equal m_π, assuming $m_\pi > \sqrt{m_\mu^2 + m_\nu^2}$, is for positive E_ν.

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