Modern Physics Letters A
Vol. 26, No. 30 (2011) 2223–2225

DOI: 10.1142/S0217732311036681

TACHYONS AND GRAVITATIONAL CHERENKOV RADIATION

CHARLES SCHWARTZ
Department of Physics, University of California,
Berkeley, CA 94720, USA
schwartz@physics.berkeley.edu

Received 1 July 2011
Revised 23 August 2011

We calculate the rate at which a free tachyon (faster than light particle) would emit gravitational radiation. It is very small.

Keywords: Tachyons; gravitational waves; Cherenkov radiation.

In a recent work,[1] I have suggested considering that faster-than-light particles (tachyons) might exist in the universe and could be a significant element in cosmological models. That study treated tachyons as classical particles, within the Special and General Theories of Relativity, and found a number of new and interesting results. This present paper looks at a possible criticism of that theoretical approach: namely, whether faster-than-light particles can be considered as stable entities in view of Cherenkov-type radiative effects.

Traveling faster than c, the speed of light, tachyons would be expected to emit gravitational radiation — analogous to the electromagnetic radiation (Cherenkov radiation) observed when electrically charged particles travel at high speeds through a medium in which light waves propagate at speeds less than c.

In fact, earlier authors[2–4] have considered the case of electrically charged tachyons; and one would expect a very rapid emission of electromagnetic radiation.[5] In our present studies we focus on tachyons that are electrically neutral. Viewing them as classical particles, we now wish to ascertain whether they would be stable with regard to the emission of gravitational radiation.

We start this calculation with formulas given by Weinberg[6] for gravitational radiation from a given source.

$$\frac{dE}{d\Omega} = 2G \int d\omega\, \omega^2 \left[T^{\lambda\nu *}(\mathbf{k},\omega) T_{\lambda\nu}(\mathbf{k},\omega) - \frac{1}{2} |T^{\lambda}_{\lambda}(\mathbf{k},\omega)|^2 \right], \tag{1}$$

for the total energy emitted per solid angle in the direction $\hat{\mathbf{k}}$, with the Fourier transformed energy momentum tensor,

$$T_{\mu\nu}(\mathbf{k},\omega) \equiv \frac{1}{2\pi}\int d^4x\, e^{ik\cdot x}\, T_{\mu\nu}(\mathbf{x},t)\,. \tag{2}$$

For a free tachyon, we have

$$T_{\mu\nu}(x) = m\int d\tau\, \dot{\xi}_\mu \dot{\xi}_\nu \delta^4(x-\xi(\tau))\,, \tag{3}$$

$$\xi^\mu(\tau) = (\gamma\tau, \gamma\mathbf{v}\tau)\,, \qquad \gamma = 1/\sqrt{v^2-1}\,, \qquad \dot{\xi}^\mu\dot{\xi}_\mu = -1\,. \tag{4}$$

This gives us the Fourier transform,

$$T_{\mu\nu}(\mathbf{k},\omega) = m\dot{\xi}_\mu\dot{\xi}_\nu\delta(\gamma\omega(1-\hat{\mathbf{k}}\cdot\mathbf{v}))\,. \tag{5}$$

The argument of this delta-function is the Lorentz invariant quantity $\dot{\xi}_\mu k^\mu$. It tells us that the radiation goes out along a cone, just as with familiar Cherenkov radiation; and this can only happen for velocities v that are greater than $c = 1$.

Now we put (5) into (1) and get,

$$\frac{dE}{d\Omega} = 2G\int d\omega\, \omega^2 \frac{m^2}{2}\delta(\gamma\omega(1-\hat{\mathbf{k}}\cdot\mathbf{v}))\delta(0)\,. \tag{6}$$

We make sense out of this by noting that

$$\delta(0) = \frac{1}{2\pi}\int d\tau\, e^{i(0)\tau} = \frac{\Delta\tau}{2\pi} = \frac{\Delta t}{2\pi\gamma}\,, \tag{7}$$

where Δt is the time interval over which we observe this process of radiation.

Now we integrate over all angles and get the rate of energy emission,

$$\frac{\Delta E}{\Delta t} = G\frac{m^2}{\gamma^2 v}\int d\omega\,\omega\,. \tag{8}$$

We need to introduce some cutoff for the integral over ω; and this we take from the basic quantum relation, $E = \hbar\omega$, where this E of the emitted gravitational quantum cannot be more than the total energy E of the tachyon. Thus we end with,

$$\frac{\Delta E}{\Delta t} \sim \frac{G}{2}\frac{m^2}{\gamma^2 v}(E/\hbar)^2\,. \tag{9}$$

Noting that $E = m\gamma$, and putting in the factors of c, we get our final result,

$$\frac{\Delta E}{\Delta t} \sim \frac{G}{2}\frac{m^4c^4}{\hbar^2 v}\,. \tag{10}$$

Putting in numbers we have:

$$\frac{\Delta E}{\Delta t} \sim \left(\frac{mc^2}{eV}\right)^4 \left(\frac{c}{v}\right) 10^{-41}\ \text{eV/sec}\,. \tag{11}$$

Suppose we guess that the neutrino is a tachyon, with mass somewhat less than one electron Volt. Over the course of cosmic time, $\sim 10^{17}$ sec, the gravitational radiation it will have emitted will be incredibly small at around 10^{-24} eV.

This result affirms the idea that we can consider uncharged tachyons as effectively stable classical particles; and this gives additional support to the previous study[1] of their behavior under General Relativity.

Acknowledgment

I thank K. Bardakci for some helpful discussions on this topic.

References

1. C. Schwartz, *J. Math. Phys.* **52**, 052501 (2011).
2. O. M. P. Bilaniuk, V. K. Deshpande and E. C. G. Sudarshan, *Am. J. Phys.* **30**, 718 (1962).
3. O. M. Bilaniuk and E. C. G. Sudarshan, *Phys. Today* **22**, 43 (1969).
4. G. Feinberg, *Phys. Rev. D* **17**, 1651 (1978).
5. However, for a contrary view, see E. Recami, *Riv. Nuovo Cimento* **9**, 80 (1986).
6. S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (John Wiley & Sons, 1972), pp. 260–263.