

implies that

$$W_{ki}^{(0)} = A_i \delta_{ki},$$

where usually we think of  $A_i$  depending only on  $x$ . Here  $A_i$  also depends on both  $\xi$  and  $t$  (for example, the time dependence determined via (1.2) could imply  $A_2 = \exp(i\xi^2 t)$ ). By considering the next order equation,  $O(1)$ , the  $x$  dependence of  $A_i$  may be obtained:

$$i(d_k - d_t)W_{ki}^{(1)} = \frac{\partial}{\partial x} W_{ki}^{(0)} - \sum_m N_{km} W_{mi}^{(0)} = \frac{\partial A_i}{\partial x} \delta_{ki} - A_i N_{ki}.$$

Since  $N_{ii} = 0$ ,  $\partial A_i / \partial x = 0$  and

$$W_{ki}^{(1)} = \begin{cases} 0, & k = i, \\ -N_{ki} A_i / i(d_k - d_t), & k \neq i, \end{cases}$$

where  $W_{ii}^{(1)} = 0$ , because  $W_{ii}^{(1)} \neq 0$  is equivalent to the introduction in  $A_i$  of  $O(1/\xi)$  term. In a similar way, the higher order terms may be directly calculated. Note that the equations for  $W_{ki}^{(p)}$  will be linear. From these results we see that

$$V_{ki} = A_i(\xi, t) \exp(i\xi d_t x) [\delta_{ki} + O(1/\xi)].$$

By considering the variables  $\Gamma_{ki} = V_{ki}/V_{ii}$ , the as yet unknown expression  $A_i(\xi, t)$  will be of no importance.

- <sup>1</sup>C.S. Gardner, J.M. Greene, M.D. Kruskal, and R.M. Miura, *Phys. Rev. Lett.* **19**, 1095 (1967).
- <sup>2</sup>C.S. Gardner, J.M. Greene, M.D. Kruskal, and R.M. Miura, *Comm. Pure Appl. Math.* **27**, 97 (1974).
- <sup>3</sup>V.E. Zakharov and A.B. Shabat, *Zh. Eksp. Teor. Fiz.* **61**, 118 (1971) [*Sov. Phys. JETP* **34**, 62 (1972)].
- <sup>4</sup>P.D. Lax, *Comm. Pure Appl. Math.* **21**, 467 (1968).
- <sup>5</sup>M.J. Ablowitz, D.J. Kaup, A.C. Newell, and H. Segur, *Phys. Rev. Lett.* **31**, 125 (1973).
- <sup>6</sup>M.J. Ablowitz, D.J. Kaup, A.C. Newell, and H. Segur, *Stud. Appl. Math.* **53**, 249 (1974).
- <sup>7</sup>V.E. Zakharov and S.V. Manakov, *Zh. Eksp. Teor. Fiz. Pis. Red.* **18**, 413 (1973) [*Sov. Phys. JETP Lett.* **18**, 243 (1973)].
- <sup>8</sup>M.J. Ablowitz and R. Haberman, *J. Math. Phys.* **16**, 2301 (1975).
- <sup>9</sup>R.M. Miura, C.S. Gardner, and M.D. Kruskal, *J. Math. Phys.* **9**, 1204 (1968).
- <sup>10</sup>K. Konno, H. Sanuki, and Y.H. Ichikawa, *Prog. Theor. Phys.* **52**, 886 (1974).
- <sup>11</sup>H. Sanuki and K. Konno, *Phys. Lett. A* **48**, 221 (1974).
- <sup>12</sup>M. Wadati, H. Sanuki, and K. Konno, *Prog. Theor. Phys.* **53**, 419 (1975).
- <sup>13</sup>E.A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations* (McGraw-Hill, New York, 1955).
- <sup>14</sup>A.H. Nayfeh, *Perturbation Methods* (Wiley, New York, 1973).

## Addendum: A classical perturbation theory [J. Math. Phys. 18, 110 (1977)]

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<sup>3</sup>J.K. Percus, *Comm. Pure Appl. Math.* **17**, 137 (1964).