high; these factors are 0.64, 0.69, 0.82, 0.95, and 1.00 for cases 1 to 5 respectively. Now $\langle P(\phi)\rangle_{\mathrm{Av}}$ for the 20-Mev proton gas as a function of ϕ is given by:

$$\langle P(\phi)\rangle_{\mathrm{Av}}=\frac{\sum P_i(\phi,E_i)\frac{d\omega}{d\Omega}(\phi,E_i)f_i(E_i)v_i(E_i)}{\sum \frac{d\omega}{d\Omega}(\phi,E_i)v_i(E_i)}.$$

However, a large fraction of the nucleon gas in a nucleus has low-momentum components. A crude way to take into account its effects is to assume that 70 percent of the nucleons are at rest and 30 percent have 20-Mev kinetic energy in the plane of $\mathbf{P}_1$ and $\mathbf{P}_2$. But in reality the counters used in the coincidence measurement extend above and below such a plane, so that the effective momentum distribution is something between that in a plane and that in three dimensions. Consequently one guesses that a more proper ratio is about 2:1 and with this assumption obtains the calculated inelastic $p-p$ polarizations shown in Fig. 4.

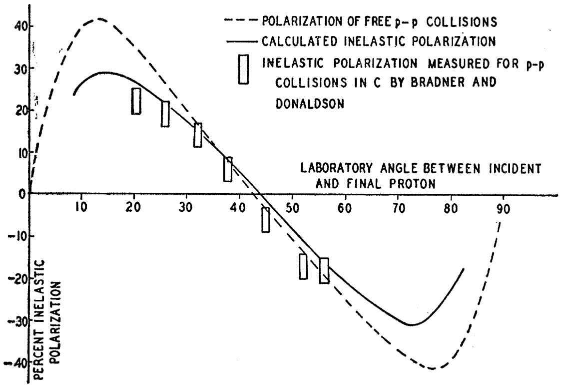

FIG. 4. $\langle P(\phi)\rangle_{\mathrm{Av}}$ for inelastic collisions of 310-Mev protons on a nucleus for which the probability to find a nucleon of energy 20 Mev is $\frac{1}{3}$ and of 0 Mev is $\frac{2}{3}$ in the plane defined by the incoming momenta, $\mathbf{P}_1$ and $\mathbf{P}_2$. The free $p-p$ polarization at 310 Mev is shown for comparison. The inelastic $p-p$ polarizations measured on carbon by coincidence methods at 285 Mev are indicated.

For comparison, the inelastic $p-p$ polarizations measured in coincidence by Bradner and Donaldson[1] for carbon at 285 Mev are shown plotted.

The experimental data can be fitted at least as well by more sophisticated choices of momentum distributions. The behavior of the inelastic polarization would seem very closely related to this kind of explanation.

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Hyperfine Structure Anomaly in Atomic *P*-States*

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LURIO and Prodell[1] have inferred a value for the hfs anomaly Δ for the two stable isotopes of gallium by comparing their measurements on the atomic $p_{\frac{1}{2}}$ states with the measurements of Daly and Holloway[2] on the $p_{\frac{3}{2}}$ states, assuming that there could be no anomaly in the latter. The purpose of this Letter is to point out that as a result of electronic configuration mixing there can in principle be an hfs anomaly in any atomic state, and to calculate this important effect for Ga. Furthermore, before one can use a measured value of Δ to study the nuclear magnetic structure through the Bohr-Weisskopf[3] (B-W) theory, the contribution to the anomaly of other mechanisms must be subtracted. We arrive at the conclusion that the net B-W effect in Ga appears rather smaller than expected, but that one cannot yet be certain of its precise value.

The hfs anomaly Δ in a particular atomic state is defined as follows:

$$\Delta=\frac{a_1}{\mu_1}\frac{\mu_2}{a_2}-1, \tag{1}$$

where a is the dipole hfs interaction constant measured in that state, μ is the nuclear magnetic moment, and the subscripts 1, 2 refer to two isotopes (2 heavier than 1). Theoretical explanations of Δ will usually depend on some coefficient b which represents the density at the nucleus of the electrons which are responsible for the hfs. Thus only valence electrons with $j=\frac{1}{2}$ are of interest, the value of b for a $p_{\frac{1}{2}}$ electron being smaller than that for an $s_{\frac{1}{2}}$ electron by roughly α^2Z^2.

Now it is well known[4] that in an atomic state nominally described as ^2P_j there is often a noticeable admixture of excited s-electron states contributing to the hfs. For such states we should use an effective value of b given by

$$b_{\mathrm{eff}}=(1-\beta_j)b_{pj}+\beta_j b_s, \tag{2}$$

where β_j, the fractional contribution of s-electrons to the hfs in the particular state ^2P_j, may be calculated from the dipole interaction constants $a_{\frac{1}{2}}$ and $a_{\frac{3}{2}}$ measured in both states of the doublet.[5]

$$\beta_{\frac{1}{2}}=(1-5\theta a_{\frac{3}{2}}/a_{\frac{1}{2}})/(1+5\theta) \tag{3}$$

$$\beta_{\frac{3}{2}}=-(a_{\frac{1}{2}}/a_{\frac{3}{2}})\beta_{\frac{1}{2}}. \tag{4}$$

For Ga, $a_{\frac{1}{2}}/a_{\frac{3}{2}}=7.02$, θ (a relativistic correction factor) $\cong 1.10_5$ so we have $\beta_{\frac{1}{2}}=0.033, \beta_{\frac{3}{2}}=-0.23$; the ratio $b_s/b_{p\frac{1}{2}}$ for the coefficients of the B-W theory is 23; so in the $p_{\frac{1}{2}}$ state we have $b_{\mathrm{eff}}=1.7b_{p\frac{1}{2}}$ and in the $p_{\frac{3}{2}}$ state $b_{\mathrm{eff}}=-5.3b_{p\frac{1}{2}}$. We thus have the result that there should already be a hfs anomaly in the $p_{\frac{3}{2}}$ state, about -3 times that in the $p_{\frac{1}{2}}$ state.

The result of Lurio and Prodell's measurement is

$$a_{\frac{1}{2}}^{69}/a_{\frac{1}{2}}^{71}=0.7870196_5\pm0.0000006,$$

and the earlier measurements of Daly and Holloway on the $p_{\frac{3}{2}}$ state give

$$a_{\frac{3}{2}}^{69}/a_{\frac{3}{2}}^{71}=0.7869949\pm0.0000013,$$

while the new value for the moment ratio given by Rice and Pound[6] is

$$\mu^{69}/\mu^{71}=0.7870147_6\pm0.0000012.$$

From these data, we have the two anomalies

$$\Delta_{\frac{1}{2}}=(6.2\pm2.3)\times10^{-6},$$
$$\Delta_{\frac{3}{2}}=(-25.2\pm3.2)\times10^{-6},$$

and the number given by Lurio and Prodell is

$$\Delta_{\frac{1}{2}}-\Delta_{\frac{3}{2}}=(31.5\pm2.4)\times10^{-6},$$

which is independent of the exact value of the moment ratio. The best check on our interpretation of this data is obtained from the ratio $\Delta_{\frac{3}{2}}/(\Delta_{\frac{1}{2}}-\Delta_{\frac{3}{2}})$ which is (-8.0 ± 1.6), while we predict $-5.3/(1.7+5.3)=-7.6$.

For the purpose of further analysis, the value of $\Delta_{\frac{1}{2}}-\Delta_{\frac{3}{2}}$ is still the most accurately known of these numbers, but we now see that as a result of the mixing s-electrons, this number represents seven times the anomaly that would be produced by a $p_{\frac{1}{2}}$ electron alone. Still, before we can interpret this as the B-W effect in Ga, we must first allow for other effects which can give an hfs anomaly. Crawford and Schawlow[7] have calculated the effect of the changing size of the nuclear charge distribution on the hfs of two isotopes. In Ga, the admixture of s-electrons again magnifies this effect in both states of the doublet, and we have estimated that this contributes about 20×10^{-6} to the difference $\Delta_{\frac{1}{2}}-\Delta_{\frac{3}{2}}$. If this is a correct and complete description of the problem, we are left with a B-W effect for a single $p_{\frac{1}{2}}$ electron of $1/7(31-20)10^{-6}=1.6\times10^{-6}$, while the simple theory predicts $\sim7\times10^{-6}$. (Our calculations use a nuclear charge radius of $1.2A^{\frac{1}{3}}\times10^{-13}$ cm.)

However, this number 1.6×10^{-6} may in fact be off by even a factor of 2 since the Crawford-Schawlow term was calculated using the doubtful assumption that the nuclear charge radius varies as $A^{\frac{1}{3}}$ as one adds two neutrons to Ga^{69}. The quantity $\gamma=(dR/R)(3A/dA)$ (which is usually <1) may, in principle, be deduced from measurements of the spectroscopic isotope shift; but the interpretation of such data in medium-weight atoms depends on the difficult calculation of the specific mass effect.[8] It may be that other observations, such as measurements of x-rays from μ-mesonic atoms of the separated isotopes, could be carried out with sufficient accuracy to give γ.

Still, the question of what other effects—such as second-order hfs proportional to μ^2, nuclear polarizability, etc.—might show up as part of an hfs anomaly greater than about 1×10^{-6} is as yet unanswered and should be the subject of further study.

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Ratio of the Magnetic Moments of the Stable Gallium Isotopes*

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INTERPRETATIONS by Schwartz of recent measurements of the ratios of the hyperfine structure splittings of the gallium isotopes[1] made an improved measurement of the ratio of their magnetic moments desirable. The most accurate value available from published results is derived as the ratio of the ratios of resonant frequencies to that of Na^{23} and is $\mu(Ga^{71})/\mu(Ga^{69})=1.27059\pm0.000023$.[2]

We have made a new determination using the recording oscillating rf spectrometer[3] and a permanent magnet. The sample was a solution of $GaCl_3$ in about 6N HCl. The line widths between points of maximum slope were 440 and 520 cps for Ga^{71} and Ga^{69}, respectively, of which width part appears to be attributable to inhomogeneity of the 6350-gauss static magnetic field. Trial experiments revealed considerably greater line width, and reduced intensity, in solutions of less acidity. The points of zero slope in the lines were determined by recording their derivatives with the spectrometer steadily scanning in frequency, making frequency marks every 20 cps as determined by standard procedures from our quartz crystal standard. The signal-to-noise amplitude ratios, (100:1 and 40:1) allowed determination of the zeroes of the recorded derivative curves to ±2 and ±4 cps.

A slow but reasonably steady drift of the static field, of thermal origin, necessitated alternate observations of each line and interpolation between pairs of measurements of one line to the time of observation of the other. For each direction of frequency scan four observations were made of one line and three of the