

Cheat Sheet - EE130

$q = 1.6 \cdot 10^{-19} \text{ C}$ $\epsilon_0 = 8.85 \cdot 10^{-14} \text{ F/cm}$
 $K_S = 11.8, K_O = 3.9$ $E_g(\text{Si}) = 1.12 \text{ eV}$
 Boltzman $k = 8.62 \cdot 10^{-5} \text{ eV/K}$
 Planck $h = 4.14 \cdot 10^{-15} \text{ eV}\cdot\text{s}$
 Free e Mass, $m_0 = 9.1 \cdot 10^{-31} \text{ kg}$
 Effective density of states $N_c = 3.22 \cdot 10^{19} \text{ cm}^{-3}$
 $dE_g = 3.5 \cdot 10^{-8} \cdot N^{(1/3)} \text{ eV}$

Basic Semiconductors Fundamentals

$$f(E) = \frac{1}{1 + e^{(E-E_f)/kT}} \text{ approx: } f(E) = e^{-(E-E_f)/kT}$$

Charge Neutrality: $p - n + N_D - N_A = 0$

$$n_i = \sqrt{N_C N_V} e^{-\frac{E_g}{2kT}} \quad g_c(E) = \frac{m_n \sqrt{2m_n(E-E_C)}}{\pi^2 \hbar^3}$$

$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{1/2} \quad M_n > M_p \text{ in Si}$$

$$p = n_i e^{\frac{E_i - E_f}{kT}} \quad E_i = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ln\left(\frac{m_p}{m_n}\right)$$

$$n = n_i e^{\frac{E_f - E_i}{kT}} \quad E_f - E_i = kT \ln\left(\frac{n}{n_i}\right) = -kT \ln\left(\frac{p}{n_i}\right)$$

Low T = freeze out; high T = intrinsic; else extrinsic

$$J_p = J_{drift} + J_{diff} = q \mu_p p E - q D_p \frac{dp}{dx}$$

$$J_n = J_{drift} + J_{diff} = q \mu_n n E + q D_n \frac{dn}{dx}$$

mobility units $\text{cm}^2/(\text{s}\cdot\text{V})$, diffusion = cm^2/s

$$\text{Diff len.: } L_N = \sqrt{D_N \tau_n} \quad v_{th} = \sqrt{\frac{3kT}{m_{eff}}} \quad v = \mu E$$

$$\frac{D}{\mu} = \frac{kT}{q} \quad \mu_p = \frac{q \tau_{mp}}{m_p}, \text{ units: } \frac{q \tau_{mp}}{m_p} \quad F = \frac{-qE}{m_p}$$

hi T = phonon scattering, low T = ion scattering

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)} \quad \sigma = q(\mu_n n + \mu_p p) \Omega^{-1} \text{ cm}^{-1}$$

Generation: band2band, R-G center, impact ion

Recomb: direct, R-G, Auger (2 collide, excite 1)

$$\text{rate of recombination} = \frac{dn}{dt} = \frac{\Delta n}{\tau} = \frac{\Delta p}{\tau}$$

Semiconductor Fabrication:

*oxidation = deposition of SiO2 layer

dry = thin, slow, precise, wet = thick, fast, imprecise

*lithography/etching = remove SiO2 with photoresist

*Dry/wet etch; dry=precise, wet = easy, cut sides

Antenna effect, charges left after etching, tunnel

*Ion Implantation = dopant atoms introduce into Si

low T vs diff. Dominant process now.

*Annealing/Diffusion = clean and spread

*Thin Film Deposition, spray metal, > clean sputter

*CVD – deposit ions/nitrides, etc.

Advanced lithography – EUV photo, ebeam, dip-pen

Positive = light, softens, negative = light hardens

Antenna effect – e- flow tunnel beneath oxide

Dop gasphase, solid source, in situ (deposit on surface)

PN Junctions

Forward Bias = Current flows P->N

Dep approx: assume carrier inside dep. region = 0

charge density out dep reg = 0 and q(Nd-Na) inside

$$\frac{d^2 V}{dx^2} = \frac{-dE}{dx} = \frac{-\rho}{\epsilon_s} \quad n: \quad V(x) = V_{bi} - \frac{qN_D}{2\epsilon_s}(x_n - x)^2$$

$$W = \sqrt{\frac{2\epsilon_s(V_{bi} - V_A)}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right)}$$

$$v_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) \quad p: \quad V(x) = \frac{qN_A}{2\epsilon_s}(x + x_p)^2$$

$$n\text{-side: } V(x) = V_{bi} - \frac{qN_D}{2\epsilon_s}(x_n - x)^2$$

$N_A x_p = N_D x_n$ common field in depletion region

one that reaches first is first to depleted

Dep reg. Widens under reverse bias

$$E\text{-field: } E(x) = \frac{-qN_A}{\epsilon_s}(x_p + x)$$

$$\text{Peak E-field: } E(0) = \frac{2qN}{\epsilon_s} (V_{bi} + |V_r|)^{1/2}$$

$$\text{Brkd Voltage: } V_{BR} = \frac{\epsilon_s E_{crit}^2}{2qN} - V_{bi}$$

$$\text{Cap diagram slope} = 2/qN \epsilon_s A^2$$

$$\text{cap/volt characteristics: } \frac{1}{C_{dep}^2} = \frac{W_{dep}^2}{A^2 \epsilon_s^2} = \frac{2(V_{bi} - V_A)}{qN \epsilon_s A^2}$$

Vbr decreases with increasing N or decreasing Eg

$$p_{p0}(-x_p) = N_A \quad n_{n0}(-x_n) = N_D \quad \text{majority}$$

$$n_{p0}(-x_p) = \frac{n_i^2}{N_A} \quad p_{n0}(-x_n) = \frac{n_i^2}{N_D} \quad \text{minority}$$

$$pn = n_i^2 e^{qV_A/kT} \quad \text{@ edge of dep region; maj/min}$$

$$n_p = n_{p0} + \Delta n_p(x) \quad p_n = p_{n0} + \Delta p_n(x)$$

Drift doesn't change with V b/c low numbers

Forward bias = more minority @ dep edge

Reverse bias = black hole @ dep edge

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L \quad \text{assume } E = 0$$

$$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L \quad L_p = \sqrt{D_p \tau_p}$$

$$\text{Steady State: } \frac{\partial \Delta p_n}{\partial t} \rightarrow 0$$

$$\text{No gradient/diff current: } D_P \frac{\partial^2 \Delta p_n}{\partial x^2} \rightarrow 0$$

$$\text{No thermal R-G: } \frac{\Delta p_n}{\tau_p} \rightarrow 0 \quad \text{No light: } G_L \rightarrow 0$$

Diode Saturation Currents:

$$I_0 = A q n_i^2 \left(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a} \right) \quad I = I_0 (e^{qV/kT} - 1)$$

IV curve shifts left @ high T b/c more diffusion

Charge storage: $I = Q/\tau_s$ charge/carrier lifetime

$$\text{Capacitance: } C = \tau_s G \quad \text{Conductance: } G = \frac{I_{DC} q}{kT}$$

PIN Junctions

Only e/h generated in dep reg contribute to current

Only light absorbed in dep reg is useful. Avalanches.

MS Junctions

lightly doped rectifying, heavily doped ohmic

Ideal assumptions: intimate contact, no oxide/charge

Not ideal: interface pinned Ef 0.4-0.9 eV below Ec

$$\Phi_{BN} = \Phi_M - X \quad \text{Barrier height; work(metal) - EA}$$

$$\Phi_{BN} = qV_{bi} + \left[\frac{1}{2} E_G - (E_{FS} - E_i)_{FB} \right] \quad \text{barrier height}$$

$$\Phi_{BP} = X + E_G - \Phi_M \quad \text{holes' barrier}$$

$$V_{bi} = \Phi_M - \Phi_S = \Phi_B - (E_C - E_F) \quad \text{built-in potential}$$

$$W = \sqrt{\frac{2\epsilon_s(V_{bi} - V_A)}{qN_D}} = \sqrt{\frac{2\epsilon_s(V_{bi} + V_A)}{qN_A}}$$

PN higher V for same I. MS more reverse current.
 MS = Ideal for rectifying high I, low V. $I_0 > PN$'s
 Contact both directions, dope heavily to tunnel
 Actual, MS is rectifying. Ohmic needs high N(thin)
 $P = e^{-H(\Phi_b - V_A)/\sqrt{N_D}}$ Ohmic when small barrier

Reduce height/reduce width. 2nd works. 1st too rare
Carrier injection @ contact: 3 modes (parameter)
 thermionic emission (work fcn, T, Forward bias),
 tunneling (high doping)

thermally activated tunneling (high T, high doping)
MOSC

Ideal assumption: no charge in oxide, same Work Fcn

$$\Phi_M = E_0 - E_{FM} \quad \Phi(x) = \frac{1}{q} [E_{i(bulk)} - E_{i(x)}]$$

$$\Phi_S = \frac{1}{q} [E_{i(bulk)} - E_{i(surface)}] \quad \Phi_F = \frac{1}{q} [E_{i(bulk)} - E_F]$$

$$W = \sqrt{\frac{2\epsilon_{si}|\Phi_s|}{qN}} \quad E_{ox} = (\epsilon_{si}/\epsilon_{ox})E_{si} \quad \phi_s = 2\phi_f$$

$$E_{max} = -\sqrt{\frac{2qN_D|\Phi_s|}{2\epsilon_{si}}} = \sqrt{\frac{2qN_A|\Phi_s|}{2\epsilon_{si}}} \quad \text{n/p type}$$

$$V_G = \phi_s + x_{ox} \frac{K_{si}}{K_{ox}} \sqrt{\left(\frac{2qN}{K_{si}\epsilon_0}\phi_s\right)} \quad \text{neg for N silicon}$$

Decrease t_{ox} decreases V_T and C_{min}

$$C_G = \frac{C_{ox}}{1 + \frac{\epsilon_{ox}W}{\epsilon_{si}x_{ox}}} \quad \text{Poly gate: } C = \frac{\epsilon_{ox}}{T_{ox} + W_{dpoly}/3}$$

$$C_{ox} = \epsilon_{ox}A/t_{ox} \quad C_{si} = \epsilon_{si}A/W \quad \text{Hi N = little effect}$$

Nonideal: nonmetal gate, charge traps, FB voltage
 Threshold voltage: larger doping requires small t_{ox} .
 Nonmetal gate is problematic; small oxide is good,
 but hard to get smaller. Want high dope in body.

Poly gate: $W_{dpoly} = \epsilon_{ox}V_{ox}/T_{ox}qN_{poly}$

$$W_{dpoly} = \sqrt{\frac{2\epsilon_s V_{poly}}{qN_{poly}}} \quad V_{fb} = \frac{E_g}{q} - \frac{kT}{q} \ln\left(\frac{N_{gate}}{N_{body}}\right)$$

$$V_{th} = V_{fb} + \phi_s + t_{ox} \frac{\epsilon_{si}}{\epsilon_{ox}} \sqrt{\frac{2qN_A\phi_s}{\epsilon_{si}}}$$

Effective t_{ox} increase: $\frac{\epsilon_{ox}}{T_{ox} + W_{dpoly}/3}$

Fixed charge $\Delta V_T = \frac{-1}{\epsilon_{ox}} \int_0^{t_{ox}} x \rho_{ox}(x) dx$ alter mobility

$$V_{FB} = \phi_{MS} - \frac{t_{ox}}{\epsilon_{SiO2}} Q_F \quad \Delta V_{G(i)} = \frac{-Q_F}{C_{ox}}$$

Fixed charge due to ionized silicon not oxidized.
 Mobile ions shift CV curves. Positive shifts left
 Interface traps smooth out curve; degrade mobility
 More surface scattering with lower t_{ox}
 Lower t_{ox} shifts CV curve down, lowers V_T
MOSFET

$$V_G = 2\phi_F + x_{ox} \frac{\epsilon_{si}}{\epsilon_{ox}} \sqrt{\left(\frac{2qN}{\epsilon_{si}}\phi_s\right)} \quad \text{n chan (p-si)}$$

$$V_G = 2\phi_F - x_{ox} \frac{\epsilon_{si}}{\epsilon_{ox}} \sqrt{\left(\frac{2qN}{\epsilon_{si}}\phi_s\right)} \quad \text{p chan (n-si)}$$

$$I_D = \frac{W\mu_n}{L} C_{ox} (V_G - V_T V_{DS} - \frac{V_{DS}^2}{2}); 0 < V_{DS} < V_{DSsat}$$

$$I_D = \frac{W\mu_n}{2L} C_{ox} (V_G - V_T)^2; V_{DS} > V_{DSsat}; V_G > V_T$$

For above, $C_{ox} = e_{ox}/t_{ox}$, or cap per unit area
 Only apply to diffusive devices, today quasi-ballistic
 * Square law also ignores bulk charge effect, assumes
 gate charge balanced by inversion charge, not dep
 * Also ignores changes in dep width

Threshold and Subthreshold

Mobility degrades at high V_{GS} , minority carriers
 flow at low V_{GS} (subthreshold). Exponential decay.
 Small subthreshold swing is desirable, get sharper
 slope.

$$I_{ds} e^{qV_{gs}/\eta kT} \quad \eta = 1 + \frac{C_{dep}}{C_{ox}} \quad S = 60mV \left(1 + \frac{C_{dep}}{C_{ox}}\right)$$

Lower swing by Increase C_{ox} , problem – tunneling,
 scattering
 Decrease C_{dep} w/ lighter doping. Kills V_t
 Decrease Temp

Velocity saturates b/c of high energy collisions.

$$I_D = WQ_{inv} v_{drift} \quad \frac{1}{V_{Dsat}} = \frac{1}{V_{GS} - V_T} + \frac{1}{\epsilon_{sat}L}$$

$$Q_{inv} = C_{ox}(V_{GS} - V_T - V_D)$$

Ballistic devices often exceed v_{sat} .
 Series resistance shifts IdVg curve to right.

Lowers Id/Vd curve respectively. Degrades perf.
 Not all Vd drops across channel; some in contacts
 V_t rolloff – V_t decreases with decreasing length
 I_{off} becomes too large if V_t becomes too small.
 Consequence of reducing oxide thickness.

Reduce tox

- Larger C_{ox} raises Ion, better e-field
- Reduce subthreshold swing
- Control V_t rolloff
- Bad – breakdown due to high field; leakage

Define EOT = E_{siO2}/E_{gate} dielectric * t_{ox}
 Hi-k challenges: chemical reaction w/ gate
 lower surface mobility, too low V_t for PMOS
 Source/drain leakage in body is a problem. Drain
 controls this part. Reduce this w/ UTB, FINFET, SOI.

BJT

Good design: minority carriers don't recombine in B
 Emitter current almost all from carriers from B
 Control doping in base but balance w/ dep width
 $I_E = I_B + I_C$ current flows to C in pnp, E in npn

Bias Mode	E-B Interface	C-B Interface
Saturation	Forward	Forward
Active	Forward	Reverse
Inverted	Reverse	Forward
Cutoff	Reverse	Reverse

Pnp emitter efficiency $\gamma = \frac{I_{Ep}}{I_E} = \frac{I_{Ep}}{I_{Ep} + I_{En}}$

Base transport factor $\alpha_T = \frac{I_{Cp}}{I_{Ep}}$

Common base gain $\alpha_{dc} = \gamma \alpha_T \quad I_C = \alpha_{dc} I_E + I_{CB0}$

Common Emitter dc gain $B_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}} = \frac{I_C}{I_B}$

$$I_C = B_{dc} I_B + I_{CE0}$$

Base width modulation / punchthrough
 High CB bias causes early effect, sloped I_c vs V_{ec}
 Base gets shorter and thinner.