1. The first thing to note is that this region is anti-symmetric about the y-axis. As such, the volume of the solid generated over $-1 \leq x \leq 1$ will be twice that of the solid generated by rotating $0 \leq x \leq 1$. We find the volume of this region using cylindrical slabs. Each cross-section has a radius of $x - x^9$, so the volume is given by
$$2\int_0^1 \pi(x-x^9)^2 dx.$$

2. In this case we once again have a symmetry in the solid we are generating, so we can simply consider twice the volume of the solid generated by the top region. Here we will need to use cylindrical shells. The height of each shell will be given by $x - x^9$, and each shell will have a radius of x. Thus, we find the volume of the region to be given by
$$2 \cdot 2\pi \int_0^1 x(x-x^9)dx.$$

3. a. In this case the height of a cylindrical shell with radius y is given $12(y^2 - y^3)$. It follows the volume is given by
$$2\pi \int_0^1 y \cdot 12(y^2-y^3)dy = 24\pi \int_0^1 (y^3-y^4)dy = 24\pi\Big[\frac{y^4}{4} - \frac{y^5}{5}\Big|_0^1 = \frac{6\pi}{5}.$$
   b. Note that in this situation the axis of rotation is actually above the region we are rotating. We still find the height of a cylindrical shell to be given by $12(y^2-y^3)$. However, this time the radius of such a shell is given by $8/5 - y$. Thus, the volume is given by
$$2\pi \int_0^1 (8/5-y)12(y^2-y^3)dy = 24\pi \int_0^1 (\frac{8}{5}y^2 - \frac{13}{5}y^3 + y^4)dy = 24\pi\Big[\frac{8}{15}y^3 - \frac{13}{20}y^4 + \frac{y^5}{5}\Big|_0^1 = 2\pi.$$

4. a. Here there are two cases to consider, $0 < x \leq \pi/4$ and $x = 0$. In the first case
$$xg(x) = x \cdot (\tan(x))^2/x = (\tan(x))^2.$$
   In the second case, $x = 0$, we find that
$$xg(x) = x \cdot 0 = 0 = (\tan(0))^2 = (\tan(x))^2.$$
   b. We find the height of a cylindrical shell with radius x to be given by $g(x)$. Thus, the volume of the region is
$$\int_0^{\pi/4} 2\pi x g(x)dx = 2\pi \int_0^{\pi/4} (\tan(x))^2 dx$$
   by part (a). We find that the volume is given by
$$2\pi \int_0^{\pi/4} (\tan(x))^2 dx = 2\pi \int_0^{\pi/4} (\sec^2(x) - 1)dx = 2\pi[\tan(x) - x\Big|_0^{\pi/4} = 2\pi(1-\pi/4).$$

5. a. The height of a shell with radius y is given by $y^2 - 3y^2 + 2 = -2y^2 + 2$. Thus, the volume is given by
$$2\pi \int_0^1 y(-2y^2+2)dy = 4\pi \int_0^1 (y-y^3)dy.$$

b. Using slabs we first find the volume of the region with circular cross-sections of

$$y = \sqrt{\frac{2+x}{3}}.$$

Then we subtract the volume of the region with cross-sections of $y = \sqrt{x}$. We find the volume to be given by

$$\pi \int_{-2}^{1} \left(\sqrt{\frac{2+x}{3}}\right)^2 dx - \pi \int_0^1 \sqrt{x}^2 dx = \pi \int_{-2}^{1} \frac{2+x}{3} dx - \pi \int_0^1 x dx.$$

c. Both of the integrals above will give the same area, but the integral given by cylindrical shells is much simpler. Thus, we find the volume to be

$$4\pi \int_0^1 (y - y^3) dy = 4\pi \left[\frac{y^2}{2} - \frac{y^4}{4}\right|_0^1 = 4\pi(1/2 - 1/4) = \pi.$$