1. The area of a typical cross-section is given by $\pi d^2/4$, where d is the diameter of a given circle. The diameter of a given circle is given by the distance between the two parabolas, so we find it to be
$$2 - x^2 - x^2 = 2(1 - x^2).$$
It follows that the area of a cross-section is given by
$$A(x) = \frac{\pi(2(1-x^2))^2}{4} = \pi(1 - 2x^2 + x^4).$$
In order to find the volume of the solid we integrate these cross-sections between the limits $x = -1$ and $x = 1$ over which they run. Thus,
$$V = \int_{-1}^{1} \pi(1 - 2x^2 + x^4)dx = \pi\Big[x - \frac{2}{3}x^3 + \frac{x^5}{5}\Big|_{-1}^{1} = \frac{16\pi}{15}.$$

2. The radius of a typical cross-section is given by $\sqrt{\cos(x)}$, so we find the area to be
$$A(x) = \pi\sqrt{\cos(x)}^2 = \pi\cos(x).$$
To find the region of integration we find the intersections of $\sqrt{\cos(x)}$ with the x-axis and the y-axis. Doing so we find the volume to be
$$\int_0^{\pi/2} \pi\cos(x)dx = \pi\sin(x)\Big|_0^{\pi/2} = \pi(1-0) = \pi.$$

3. a. In order to calculate the volume as we rotate around the y-axis it helps to first rewrite the line $y = 2x/3$ as a function of y. ie,
$$x = \frac{3}{2}y.$$
The radius of a cross-section is simply given by its x-coordinate, so the area of a cross-section is given by
$$A(y) = \pi\Big(\frac{3}{2}y\Big)^2 = \pi\frac{9}{4}y^2.$$
Since the line $y = 2x/3$ intersects the x-axis at the origin, we find our region bounded by the three lines to span $y = 0$ to $y = 2$. It follows that the volume of the region is given by
$$V = \int_0^2 \pi\frac{9}{4}y^2 = \pi\frac{3}{4}y^3\Big|_0^2 = 6\pi.$$

   b. When we rotate around the x-axis we generate a solid that has a hollow inside. Thus, we must subtract the area of this region from the volume of the solid swept out from the outside in order to find the volume of our solid. A typical cross-section from the outside is just a circle of radius 2. We then subtract the inside which is a circle of radius $y = 2x/3$. It follows that the area of a typical cross-section is
$$A(x) = \pi(2^2 - (2x/3)^2) = 4\pi(1 - x^2/9).$$
Now we need to find the limits of integration over which we rotate this region. We find the line $2 = 2y/3$ intersects the line $y = 2$ at an x-coordinate of 3, and we already know that it intersects the x-axis at the origin. It follows that
$$V = \int_0^3 4\pi(1 - x^2/9)dx = 4\pi(x - x^3/27)\Big|_0^3 = 4\pi(3-1) = 8\pi.$$

4. Rotating this region generates a solid with a hollow inside. The radius of the outside is 1, and the radius of the inside is given by $x = \tan(y)$. Since the curves $x = \tan(y)$ and $x = 1$ intersect at $y = \pi/4$, our limits of integration are from 0 to $\pi/4$. It follows the volume of the solid is given by

$$\int_0^{\pi/4} \pi(1 - \tan^2(y))dy.$$

Now we note that $\tan^2(y) = \sec^(y) - 1$, so we can transform the above integral into a manageable form. Doing so we find

$$V = \int_0^{\pi/4} \pi(2 - \sec^2(y))dy = \pi(2y - \tan(y))\Big|_0^{\pi/4} = \pi(\pi/2 - 1).$$

5. For each given pair of points there is a unique line that passes through both of them. These lines are $x = 1$, $y = 1$, and $y = x - 1$ or $x = y + 1$. The outer radius of a cross-section is given by $x = 1 + y$, whereas the inner radius is given by $x = 1$. Thus, the area of a cross-section is given by

$$A(y) = \pi((1 + y)^2 - 1^2) = \pi(1 + 2y + y^2 - 1) = \pi(2y + y^2).$$

The region of interest is revolved over the limits $y = 0$ to $y = 1$. It follows that the volume is given by

$$V = \int_0^1 \pi(2y + y^2)dy = \pi(y + \frac{y^3}{3})\Big|_0^1 = \frac{4\pi}{3}.$$

6. a. If we let $y_1 = 5/7 + x/7$ and $y_2 = \sqrt{1 - x^2}$ then

$$\begin{aligned} y_1(-4/5) &= 5/7 + (-4/5)/7 = 25/35 - 4/35 = 21/35 = 3/5 \\ y_2(-4/5) &= \sqrt{1 - (-4/5)^2} = \sqrt{1 - 16/25} = \sqrt{9/25} = 3/5 \end{aligned}$$

which shows that the point $(-4/5, 3/5)$ lies on both curves, or in other words, the curves intersect at this point. Similarly,

$$\begin{aligned} y_1(3/5) &= 5/7 + (3/5)/7 = 25/35 + 3/35 = 28/35 = 4/5 \\ y_2(3/5) &= \sqrt{1 - (3/5)^2} = \sqrt{1 - 9/25} = \sqrt{16/25} = 4/5. \end{aligned}$$

b. To find the area between these two curves we must see which is larger over the region of interest. Sampling a convenient point of $x = 0$ we see that $y_1(0) = 5/7$ and $y_2(0) = 1$, so y_2 is our top function. It follows that the area is given by

$$\int_{-4/5}^{4/5} (\sqrt{1 - x^2} - 5/7 - x/7)dx.$$

c. We know that the outer radius of a typical cross-section is given by y_2 (the larger of the two functions), and the inner radius is given by y_1. As a result, the area of a cross-section is given by

$$A(x) = \pi(1 - x^2 - (5/7 + x/7)^2).$$

To find the volume we integrate along x, and find that

$$V = \int_{-4/5}^{4/5} \pi(1 - x^2 - (5/7 + x/7)^2)dx.$$

d. When we revolve around the line $y = -1$ we need to rewrite our functions in terms of y. Thus, we have both of the functions

$$x_1 = 7y - 5$$

and

$$x_2 = \sqrt{1 - y^2}.$$

Since we found y_2 to be the top function before, we know it will be the leftmost function this time, defining the inner radius. The outer radius is given by the distance between x_2 and the line $x = -1$, which is

$$x_2 - (-1) = 1 + \sqrt{1 - y^2}.$$

The inner radius is given by the distance between x_1 and the line $x = -1$, which is

$$x_1 - (-1) = 7y - 4.$$

It follows that the volume is given by

$$\int_{3/5}^{4/5} \pi((1 + \sqrt{1 - y^2})^2 - (7y - 4)^2)dy.$$