1. Since cosine has an amplitude of 1, we know that it never exceeds 1, so $\cos^2(x)$ must be our bottom function. We find that

$$1 - \cos^2(x) = \sin^2(x) = \frac{1}{2} \cdot (1 - \cos(2x)).$$

Integrating we find

$$\text{Area} = \frac{1}{2}\int_0^{\pi} (1 - \cos(2x))dx = \frac{1}{2}\Big[x - \frac{\sin(2x)}{2}\Big|_0^{\pi} = \frac{1}{2}[(\pi - 0) - (0 - 0)] = \frac{\pi}{2}.$$

2. Our first task is to find the points of intersection of these two functions. We know that we cannot find any intersections for $|x| > 1$, because the sine function never exceeds 1. By plotting both of these functions we can see that they intersect at $-1, 0, 1$. Thus, there are two subintervals over which we must calculate the area $[-1, 0]$ and $[0, 1]$. By choosing convenient evaluation points of -0.5 and 0.5 we find that

$$0.5 < \sin(0.5 \cdot \pi/2) = \sin(\pi/4) = \sqrt{2}/2 \approx 0.707,$$

and

$$-0.5 > \sin(-0.5 \cdot \pi/2) = -\sqrt{2}/2 \approx 0.707.$$

Thus, we find the area is given by

$$\begin{aligned}
\text{Area} &= \int_{-1}^{0} (x - \sin(\pi x/2))dx + \int_0^1 (\sin(\pi x/2) - x)dx \\
&= \int_{-1}^{0} (x - \sin(\pi x/2))dx - \int_0^1 (x - \sin(\pi x/2))dx \\
&= \Big[\frac{x^2}{2} + \frac{2}{\pi}\cos(\pi x/2)\Big|_{-1}^{0} - \Big[\frac{x^2}{2} + \frac{2}{\pi}\cos(\pi x/2)\Big|_0^1 \\
&= \Big[0 + \frac{2}{\pi} - \frac{1}{2} - 0\Big] - \Big[\frac{1}{2} + 0 - 0 - \frac{2}{\pi}\Big] \\
&= \frac{4}{\pi} - 1.
\end{aligned}$$

3. We begin by looking for the points of intersection

$$\begin{aligned}
12y^2 - 12y^3 &= 2y^2 - 2y \\
12y^2(1 - y) &= -2y(1 - y)
\end{aligned}$$

This immediately yields the solutions $y = 0$ and $y = 1$. If we assume that $y \neq 1$ and $y \neq 0$ then we can divide both sides of the above equation by $y(1 - y)$, as it will be a nonzero product. We then find that

$$12y = -2$$

or $y = -1/6$. This gives us two subintervals to consider, $[-1/6, 0]$ and $[0, 1]$. By choosing evaluation points within these subintervals we find the cubic polynomial is the bottom function over the left subinterval and the top function over the right subinterval. Thus, the area is given by

$$\begin{aligned}\text{Area} &= \int_{-1/6}^{0} (2y^2 - 2y - 12y^2 + 12y^3)dy + \int_0^1 (12y^2 - 12y^3 - 2y^2 + 2y)dy \\ &= \int_{-1/6}^{0} (12y^3 - 10y^2 - 2y)dy - \int_0^1 (12y^3 - 10y^2 - 2y)dy \\ &= \left[3y^4 - \frac{10}{3}y^3 - y^2\right|_{-1/6}^{0} - \left[3y^4 - \frac{10}{3}y^3 - y^2\right|_0^1 \\ &= 0 - \left[3(-1/6)^4 - \frac{10}{3}(-1/6)^3 - (-1/6)^2\right] - \left[3 - \frac{10}{3} - 1\right] \\ &= \frac{13}{1296} + \frac{4}{3} = \frac{1741}{1296} \approx 1.3433.\end{aligned}$$

4. If we sketch the curves over the region of interest we see that we want the area between the line $y = 1$ and $y = x^2/4$ over $[0, 2]$ minus the area of the triangle (with base and height 1) formed between $y = x$ and $y = 1$. Thus we find the area to be given by

$$\text{Area} = \int_0^2 (1 - x^2/4)dx - \frac{1}{2}(1)(1) = \left[x - \frac{x^3}{12}\right|_0^2 - \frac{1}{2} = (2 - 8/12) - \frac{1}{2} = \frac{5}{6}.$$

5. We begin by finding the intersection points

$$\begin{aligned} x^4 - 4x^2 + 4 &= x^2 \\ x^4 - 5x^2 + 4 &= 0 \\ (x^2 - 4)(x^2 + 1) &= 0 \\ (x+2)(x-2)(x+1)(x-1) &= 0 \end{aligned}$$

Thus we have the subintervals $[-2, -1]$, $[-1, 1]$, and $[1, 2]$. Evaluating the the functions at the appropriate places to see which is higher over each subinterval we find

$$\begin{aligned}\text{Area} &= \int_{-2}^{-1} (x^2 - x^4 + 4x^2 - 4)dx + \int_{-1}^{1} (x^4 - 4x^2 + 4 - x^2)dx + \int_1^2 (x^2 - x^4 + 4x^2 - 4)dx \\ &= \int_{-2}^{-1} (-x^4 + 5x^2 - 4)dx - \int_{-1}^{1} (-x^4 + 5x^2 - 4)dx + \int_1^2 (-x^4 + 5x^2 - 4)dx \\ &= \left[-\frac{x^5}{5} + \frac{5x^3}{3} - 4x\right|_{-2}^{-1} - \left[-\frac{x^5}{5} + \frac{5x^3}{3} - 4x\right|_{-1}^{1} + \left[-\frac{x^5}{5} + \frac{5x^3}{3} - 4x\right|_1^2 \\ &= 8\end{aligned}$$