

1. Evaluate the following integrals

a. Let $u = 7x - 2$, $du = 7dx$. We find that

$$\int 28(7x - 2)^{-5} dx = \int 4u^{-5} du = 4 \frac{u^{-4}}{-4} + c = -(7x - 2)^{-4} + c.$$

b. Let $u = 1 - r^3$, so $du = -3r^2 dr$. It follows $u(0) = 1$ and $u(1) = 0$. We find that

$$\int_0^1 \frac{9r^2 dr}{\sqrt{1 - r^3}} = - \int_1^0 3u^{-1/2} du = -6u^{3/2} \Big|_1^0 = 0 - (-6) = 6.$$

c. Let $u = \sin(2\theta)$ so $du = 2 \cos(2\theta) d\theta$ or $du/2 = \cos(2\theta) d\theta$. Then it follows

$$\int \csc^2(2\theta) \cot(2\theta) d\theta = \frac{1}{2} \int u^{-2} du = \frac{1}{2} \cdot (-u^{-1}) + c = -\frac{1}{2 \sin(2\theta)} + c.$$

d. Let $u = z^2 - 1$ so $du = 2z dz$. We find that

$$\int 8z(z^2 - 1)^{1/3} dz = \int 4u^{1/3} du = 3u^{4/3} + c = 3(z^2 - 1)^{4/3} + c.$$

e. Let $u = 1 + v^{3/2}$ so $du = (3/2) \cdot v^{1/2} dv$. It follows $u(1) = 2$ and $u(4) = 8$. We find that

$$\int_1^4 \frac{10\sqrt{v}}{(1 + v^{3/2})^2} dv = \int_2^8 \frac{20}{3} u^{-2} du = -\frac{20}{3u} \Big|_2^8 = -\frac{20}{24} - \left(-\frac{20}{6}\right) = \frac{5}{2}.$$

f. Let $u = \tan(a)$ so $du = \sec^2 a da$. We find that

$$\int \tan^2(a) \sec^2(a) da = \int u^2 du = \frac{u^3}{3} + c = \frac{\tan^3(a)}{3} + c.$$

g. Let $u = \sin(\sqrt{x})$ so that $du = \cos(\sqrt{x}) \cdot (1/2)x^{-1/2} dx$, so $2du = \cos(\sqrt{x})x^{-1/2}$. It follows that

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x} \sin^2(\sqrt{x})} dx = 2 \int u^{-2} du = -2u^{-1} + c = -\frac{2}{\sin(\sqrt{x})} + c.$$

h. In this case we choose $u = 1 - \sin(2t)$ so that $du = -2 \cos(2t) dt$, and $u(0) = 1$ while $u(\pi/4) = 0$. We find that

$$\int_0^{\pi/4} (1 - \sin(2t))^{3/2} \cos(2t) dt = -\frac{1}{2} \int_1^0 u^{3/2} du = -\frac{1}{5} u^{5/2} \Big|_1^0 = 0 - \left(-\frac{1}{5} \cdot 1^{5/2}\right) = \frac{1}{5}.$$

2. First we find the general solution using indefinite integration. We need to evaluate

$$s(t) = \int 6 \sin(2t) dt.$$

Letting $u = 2t$ we find $du = 2dt$, so

$$\int 6 \sin(2t) dt = \int 3 \sin(u) du = 3 \cos(u) + c = 3 \cos(2t) + c.$$

Using the initial condition we find

$$0 = s(0) = 3 \cos(0) + c = 3 + c.$$

Thus, $c = -3$, so we find

$$s(t) = 3 \cos(2t) - 3.$$

Evaluating this position function at $t = \pi/4$ we find

$$s(\pi/4) = 3 \cos(\pi/2) - 3 = 0 - 3 = -3.$$