1. a. From the Fundamental Theorem of Calculus we find that

$$f(x) = \int_0^x \cos(y)dy = \sin(y)\Big|_0^x = \sin(x) - \sin(0) = \sin(x).$$

It follows that $f(\pi) = \sin(\pi) = 0$ and $f(3\pi/2) = \sin(3\pi/2) = -1$.

b. From the Fundamental Theorem of Calculus we find that

$$\frac{df}{dx} = \frac{d}{dx}\int_0^x \cos(y)dy = \cos(x).$$

Thus,

$$\frac{df}{dx}\Big|_{2\pi} = \cos(2\pi) = 1.$$

2. The Fundamental Theorem of Calculus tells us that

$$\frac{d}{du}\int_{-1}^u \frac{t}{t+2}dt = \frac{u}{u+2}.$$

In this situation we can see that $G(x)$ is a composition of the above function of u and $u = x - \sin(x)$. Thus, in order to differentiate with respect to x we must use the chain rule. We find that

$$G'(x) = \frac{d}{dx}G(x) = \frac{dG}{du}\cdot\frac{du}{dx} = \frac{u}{u+2}\cdot(1-\cos(x)) = \frac{x-\sin(x)}{x-\sin(x)+2}\cdot(1-\cos(x)).$$

It follows that

$$G'(\pi) = \frac{\pi-\sin(\pi)}{\pi-\sin(\pi)+2}\cdot(1-\cos(\pi)) = \frac{\pi}{\pi+2}\cdot(1+1) = \frac{2\pi}{\pi+2}.$$

3. a. From the Fundamental Theorem of Calculus we find that

$$F'(x) = \frac{d}{dx}\int_{-2}^x tdt = x.$$

Thus, $F'(x) < 0$ on $[-2, 0)$, and $F'(x) > 0$ on $(0, 2]$. It follows that F is decreasing on $[-2, 0)$ and increasing on $(0, 2]$, as given by the sign of the derivative over these intervals.

b. We find that

$$F''(x) = 1 > 0$$

so the function is concave up on $[-2, 2]$.

c. To find the global extrema we must consider the value of the function at the critical point $x = 0$ and the end points $x = \pm 2$. Just considering the sign of the derivative we already know that $x = -2$ is a local minimum, $x = 0$ is a local maximum, and that $x = 2$ is a local minimum. Evaluating the value of the function at these points we find that

$$F(-2) = \int_{-2}^{-2} tdt = 0,$$

because the area accumulated underneath a function over an interval of 0 length must be 0. For an arbitrary x value we find

$$F(x) = \int_{-2}^x tdt = \frac{t^2}{2}\Big|_{-2}^x = \frac{x^2}{2} - \frac{(-2)^2}{2} = \frac{x^2}{2} - 2.$$

Thus, it follows that

$$F(0) = \frac{0^2}{2} - 2 = -2.$$

We could also see this by noting that the area accumulated over $[-2, 0]$ is the area between the line of slope 1 passing through the origin and then x-axis. This forms a triangle of base 2 and height 2, which has a negative signed area because it is below the x-axis. Finally, we can evaluate

$$F(2) = \frac{2^2}{2} - 2 = 0.$$

We could also see that $F(2) = 0$ by noting that x is an odd function, meaning it is antisymmetric about the y-axis. It follows that the area accumulated from $[-2, 0]$ will be exactly the opposite of the area accumulated over $[0, 2]$, so the overall area accumulated is 0.

4. a. There are no such x values in $[0, 1]$. The function $F(x)$ gives us the area between the bottom half of the given circle over the interval $[0, x]$. Since the circle is always above the x-axis, it follows that all of the area we accumulate (in moving from left to right) is positive.

   b. $F(1)$ is the area between the bottom of the circle and the x-axis over the interval $[0, 1]$. We can view this as the area of a square with sides of length one minus the area of one fourth of a circle of radius one. This is exactly $1 - \pi/4$.

   c. Over an interval of 0 length we accumulate no area, so we know that $F(0) = 0$.

   d. The Fundamental Theorem of Calculus tells us that

$$F'(x) = 1 - \sqrt{1 - x^2}.$$

   Differentiating again we find

$$F''(x) = -\frac{1}{2}(1 - x^2)^{-1/2} \cdot (-2x) = \frac{x}{\sqrt{1 - x^2}}.$$

   Over the interval $(0, 1)$ both the numerator and denominator of the above quotient are positive, so $F''(x) > 0$. It follows that the function F is concave up on $(0, 1)$.