MATH 160 Homework 25 Name: ______________________

Due 11/10 MATH 160 section __________

1. Consider the function $f(x)$ given by

$$f(x) = \int_0^x \cos(y)dy.$$

   a. Find $f(\pi)$ and $f(3\pi/2)$.

   b. Find $\left.\frac{df}{dx}\right|_{x=2\pi}$.

2. Consider the function $G(x)$ given by

$$G(x) = \int_{-1}^{x-\sin(x)} \frac{t}{t+2}dt.$$

   Find $G'(\pi)$.

3. Consider the function $F(x)$ defined on $[-2, 2]$, where $F(x)$ is given by

$$F(x) = \int_{-2}^{x} tdt.$$

   a. Over what intervals is this function increasing? Decreasing? Explain how you know the function is increasing and decreasing over these intervals.

   b. Over what intervals is this function concave up? Concave down? Explain how you know the concavity of the function over these intervals.

   c. Find the global extrema of this function over the interval $[-2, 2]$.

4. The graph of the function $f(x) = 1-\sqrt{1-x^2}$ is the bottom half of a circle of radius 1 centered at $(0, 1)$. Define a function $F(x)$ over the interval $[0, 1]$ by

$$F(x) = \int_0^x (1-\sqrt{1-t^2})dt.$$

   a. Are there any values x values in $[0, 1]$ for which $F(x) < 0$? If so, explain how you know $F(x)$ is negative for these values. If not, explain how you know there are no such values.

   b. Explain how to see that $F(1) = 1 - \pi/4$.

   c. Find the exact value of $F(0)$ and explain how you know this is the exact value.

   d. Use derivatives to show that the function $F(x)$ is concave up on the interval $(0, 1)$.