1. When we differentiate a function, we lose information. In a sense, the derivative of a function tells us how the function is changing, but we don't know where the function is starting from. The arbitrary constant represents this fact - that we we need additional information in order to reconstruct our original function. To solve a physical problem, where we should only have one solution, we can use some known measured data to match the general form of the solution, which includes the arbitrary constant.

2. To evaluate the integral we want to find a function f such that

$$\frac{df}{dx} = \frac{1}{x}.$$

This would correspond (in a sense) to using the power rule for derivatives for the case $n = 0$. However, the power rule for derivatives does not apply for $n = 0$ (which can be seen by looking at its hypotheses). If we were to blindly try and use the expression given we can see it does not help us, because we would have

$$\begin{aligned} \frac{d}{dx}x^0 &= 0 \cdot x^{0-1} \\ \frac{d}{dx}1 &= 0 \cdot x^{-1} \\ 0 &= 0 \end{aligned}$$

which tells us nothing.

3. a. $\displaystyle\int \frac{t\sqrt{t} + \sqrt{t}}{t^2} dt = \int (t^{1/2} + t^{-3/2}) dt = \frac{t^{3/2}}{3/2} + \frac{t^{-1/2}}{-1/2} + c.$

   b. $\displaystyle\int \sec(\pi x/2) \tan(\pi x/2) dx = \frac{2}{\pi} \cdot \sec(\pi x/2).$

   c. $\displaystyle\int (2\cos(2x) - 3\sin(3x)) dx = \sin(2x) + \cos(3x).$

4. a. No. If two functions have the same derivative, that means that they are changing identically at all times. Thus, if we consider two functions that are equal at a single point, and have the same derivative, they must remain equal. It follows that two antiderivatives of a single function can differ at most by a constant.

   b. We verify this through differentiation, as an antiderivative F satisfies

$$\frac{dF}{dx} = \frac{1}{(x+1)^2}.$$

   We find that

$$\frac{d}{dx}\left(\frac{x}{x+1} + c\right) = \frac{(x+1) - x}{(x+1)^2} = \frac{1}{(x+1)^2},$$

   so this is indeed an antiderivative.

   c. Analagously, we find that

$$\frac{d}{dx}\left(\frac{-1}{x+1} + c\right) = \frac{(x+1) \cdot 0 - (-1)}{(x+1)^2} = \frac{1}{(x+1)^2},$$

   so this also an antiderivative.

d. Given that antiderivatives can differ by at most a constant, we expect that the antiderivatives in part (b) and (c) must differ by some constant. In fact, we find that

$$\frac{x}{x+1} - \frac{x+1}{x+1} = \frac{x-x-1}{x+1} = \frac{-1}{x+1},$$

so indeed these two antiderivatives differ by a constant, namely 1. This is consistent with our results from (a)-(c).