

1. The indefinite integral of any function includes an arbitrary constant of integration. What is the purpose of the arbitrary constant of integration, and why is it necessary?
2. The power rule for derivatives states that

$$\frac{d}{dx}x^n = nx^{n-1}.$$

Explain why the integral

$$\int \frac{1}{x} dx$$

cannot be evaluated using the power rule.

3. Evaluate the following integrals

a. $\int \frac{t\sqrt{t} + \sqrt{t}}{t^2} dt.$

b. $\int \sec(\pi x/2) \tan(\pi x/2) dx.$

c. $\int (2 \cos(2x) - 3 \sin(3x)) dx.$

4. a. Is it possible for a function to have two antiderivatives that differ by more than a constant? Explain why or why not.
b. Verify that $\int \frac{1}{(x+1)^2} dx = \frac{x}{x+1} + c.$
c. Verify that $\int \frac{1}{(x+1)^2} dx = -\frac{1}{x+1} + c.$
d. Are your answers to questions (a)-(c) contradictory? Explain.