Class 1

Abstract Algebra.

- group < symmetric group S_n. / linear group GL_n.
- ring: $R[x]$, $\mathbb{Z}$, division ✗
- field: $\mathbb{R}$ & $\mathbb{C}$. division ✓

D. Linear Transformation < D. scalar multiplication. / D addition

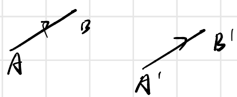
(beginning?)

D: identical vector <
- D vector on a plane R^2: directed segment — ordered pair of points.
- D translation: move the end without changing length & direction

? how why

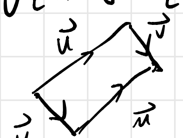
⇒ (formal)
- D vector is an equivalence class of directed segment modulo translation

$\overrightarrow{AB}$ and $\overrightarrow{A'B'}$ are < different as dir. seg $\overrightarrow{AB}$ / but same as vector $[\overrightarrow{AB}]$



- D vector addition: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$
- commutativity $[\overrightarrow{AB}] + [\overrightarrow{CD}] = [\overrightarrow{CD}] + [\overrightarrow{AB}]$

proof



{ D addition / parallelogram

- associativity

- distributive laws.

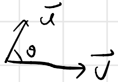
proof: proportional segment + similar triangle.

• D **inner product** imposed structure on vector space

: given 2 vectors $\vec{v}$, $\vec{u}$

we define $\langle \vec{v}, \vec{u} \rangle \in \mathbb{R}$

$\langle \vec{v}, \vec{u} \rangle = |\vec{u}| \cdot |\vec{v}| \cdot \cos\theta$



P: • inner product is defined to be a ~~linear~~ bilinear operation

structure of scalar multiplication & addition $\begin{cases} P_1 \text{ additivity} \\ P_2 \text{ homogeneity} \end{cases}$

$\Rightarrow \langle \vec{v_1} + \vec{v_2}, \vec{w} \rangle = \langle \vec{v_1}, \vec{w} \rangle + \langle \vec{v_2}, \vec{w} \rangle$

$\langle \lambda \vec{v}, \vec{w} \rangle = \lambda \langle \vec{v}, \vec{w} \rangle$

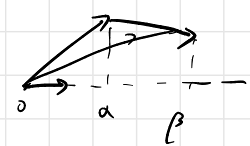
$\langle \vec{u}, \vec{u} \rangle = |u|^2$

$\cos\theta_{\vec{u},\vec{v}} = \dfrac{\langle u, v \rangle}{\langle u, u \rangle^{\frac{1}{2}} \cdot \langle v, v \rangle^{\frac{1}{2}}}$

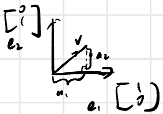
$\langle u_1 + u_2, v_1 + v_2 \rangle$

$= \langle u_1, v_1 \rangle + \langle u_2, v_2 \rangle + \langle u_1, v_2 \rangle + \langle u_2, v_1 \rangle$

Proof of additivity $\begin{cases} \langle \vec{v_1} + \vec{v_2}, \vec{i} \rangle = \langle \vec{v_1}, \vec{i} \rangle + \langle \vec{v_2}, \vec{i} \rangle \\ \qquad\qquad\qquad\quad \alpha \qquad\quad \beta \\ \text{homogeneity} \end{cases}$



- D. Basis vector



any other vector in $\mathbb{R}^2$ can be written as linear combination of BV

$v = \underline{a_1} e_1 + \underline{a_2} e_2$.

- e_1, e_2 forms an orthonormal basis.

orthogonal $\langle e_1, e_2 \rangle = 0$; $|e_1| = 1$ $|e_2| = 2$

$\Rightarrow$ claim: $v = \langle v, e_1 \rangle e_1 + \langle v_2, e_2 \rangle e_2$.

proof:

suppose $v = a_1 e_1 + a_2 e_2$, then we apply $\langle \cdot , e_1 \rangle$ to both sides

$$\begin{aligned}\langle v, e_1 \rangle &= \langle a_1 e_1 + a_2 e_2, e_1 \rangle \\ &= a_1 \langle e_1, e_1 \rangle + a_2 \langle e_2, e_1 \rangle \\ &= a_1 \cdot 1 + a_2 \cdot 0 = a_1\end{aligned}$$

- D coordinates

Given a basis E_1, E_2, we say a vector v has coordinates (a_1, a_2), if $v = a_1 E_1 + a_2 E_2$.

Note: change basis $\longrightarrow$ change coordinates.

- "Skewed" basis on $\mathbb{R}^2$

$\downarrow$

not orthonormal

- In general, if two v E_1, E_2 in $\mathbb{R}^2$ satisfy the property that, any vector $v \in \mathbb{R}^2$ can be written as

$$v = a_1 E_1 + a_2 E_2$$

then we say (E_1, E_2) is a basis.

$\Rightarrow$ 2D: non parallel vector definition.