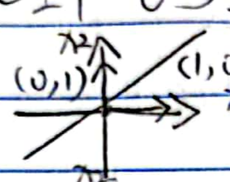
HW # 2 p.11 in [ODE]
Conic curve → Example
Matrices → Ex 1.3.1 (b, c, d, g) & Ex 1.3.2 (a-f)
p.15 Exercises 1.3.1

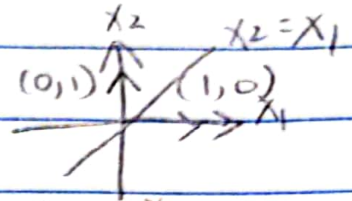
b) Describe geometrically the linear transformations defined by the 8 matrices

$$\begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix}, \begin{bmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{bmatrix}$$

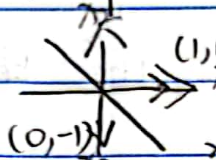
① $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$



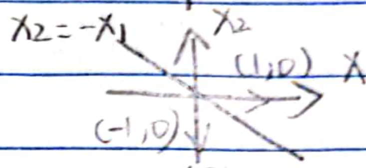
⑤ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$



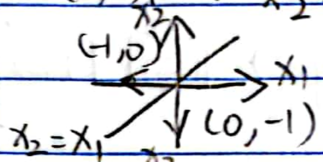
② $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$



⑥ $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

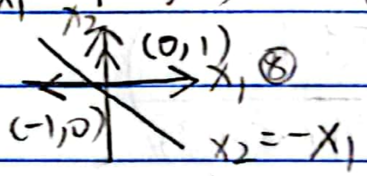


③ $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

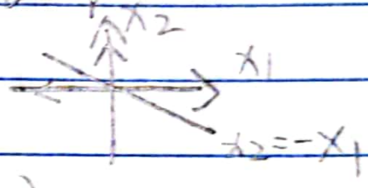


⑦ $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

④ $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$



⑧ $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$



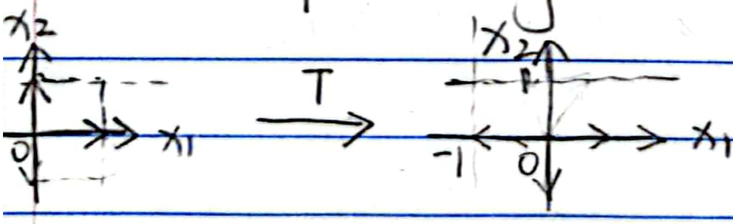
c) $x_1 x_2 = 1 \xrightarrow{T} x_1^2 - x_2^2 = 1 = (x_1 + x_2)(x_1 - x_2)$

$f_1 = \left(\frac{1}{2}, \frac{1}{2}\right), \; f_2 = \left(\frac{1}{2}, -\frac{1}{2}\right)$ or $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

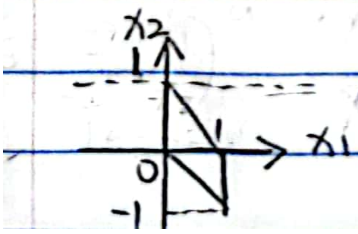
$X_1 = \dfrac{x_1 + x_2}{2}, \; X_2 = \dfrac{x_1 - x_2}{2}$

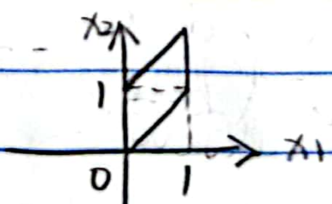
$$X_1^2 - X_2^2 = \frac{x_1^2 + 2x_1x_2 + x_2^2 - x_1^2 + 2x_1x_2 - x_2^2}{4} = x_1 x_2$$

d) transform unit square : $0 \le x_1, x_2 \le 1$ to the parallelogram : $0 \le x_1 + x_2, x_2 \le 1$



$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & \\ 1 & \end{bmatrix}$





9). $\vec{E_1} = \langle x_1, y_1 \rangle$, $T\vec{E_1} = T\langle x_1, y_1\rangle = \langle Tx_1, Ty_1\rangle$

$\vec{E_2} = \langle x_2, y_2\rangle$, $T\vec{E_2} = T\langle x_2, y_2\rangle = \langle Tx_2, Ty_2\rangle$

Inner product: $\vec{E_1} \cdot \vec{E_2} = |\vec{E_1}||\vec{E_2}| \cos\theta_1$

$$\theta_1 = \cos^{-1}\left(\frac{\vec{E_1}\cdot\vec{E_2}}{|\vec{E_1}||\vec{E_2}|}\right)$$

$T\vec{E_1} \cdot T\vec{E_2} = |T\vec{E_1}||T\vec{E_2}| \cos\theta_2$

$$\theta_2 = \cos^{-1}\left(\frac{T\vec{E_1}\cdot T\vec{E_2}}{|T\vec{E_1}||T\vec{E_2}|}\right)$$

$\vec{E_1}\cdot\vec{E_2} = x_1x_2 + y_1y_2$ $\qquad$ $T\vec{E_1}\cdot T\vec{E_2} = T^2(x_1x_2 + y_1y_2)$

$|\vec{E_1}||\vec{E_2}| = \sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}$ $\qquad$ $|T\vec{E_1}|\cdot|T\vec{E_2}| = \sqrt{T^4(x_1^2+y_1^2)(x_2^2+y_2^2)}$

$= T^2\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}$

$$\therefore \theta_1 = \cos^{-1}\left(\frac{x_1x_2+y_1y_2}{\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}}\right)$$

$$\theta_2 = \cos^{-1}\left[\frac{\cancel{T^2}(x_1x_2+y_1y_2)}{\cancel{T^2}\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}}\right] = \cos^{-1}\left(\frac{x_1x_2+y_1y_2}{\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}}\right)$$

$\theta_1 = \theta_2$

CS CamScanner