Section 5.4

Recall that a function $F(x)$ is an antiderivative of a function $f(x)$ if $F'(x) = f(x)$.

Definition: The set of all antiderivatives of a function $f(x)$ is the **indefinite integral** of f with respect to x and is denoted by

$$\int f(x)\ dx.$$

The symbol $\int$ is an **integral sign**, the function f is the **integrand** of the integral, and x is the **variable of integration**.

Exercise 1. Find the general indefinite integral.
(a) $\int\ (\sqrt{x^3} + \sqrt[3]{x^2})\ dx$
(b) $\int\ (y^3 + 1.8y^2 - 2.4y)\ dy$

Class Exercise 1. Find the general indefinite integral.
(a) $\int\ v(v^2+2)^2\ dv$
(b) $\int\ (x^2 + 1 + \frac{1}{x^2+1})\ dx$
(c) $\int\ (\csc^2 t - 2e^t)\ dt$
(d) $\int\ \sec t\ (\sec t + \tan t)\ dt$
(e) $\int\ (\sin 2x)/(\sin x)\ dx$

The connection between the definite integral and the indefinite integral is given by Part 2 of the Fundamental Theorem. If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\ dx = \int f(x)\ dx\ ]_a^b.$$

Exercise 2. Evaluate the integral.
(a) $\int_1^2 (4x^3 - 3x^2 + 2x)\ dx$
(b) $\int_0^3 (1 + 6w^2 - 10w^4)\ dw$

Class Exercise 2. Evaluate the integral.
(a) $\int_{-1}^1 t(1-t)^2\ dt$
(b) $\int_1^2 (\frac{1}{x^2} - \frac{4}{x^3})\ dx$
(c) $\int_0^4 (3\sqrt{t} - 2e^t)\ dt$
(d) $\int_1^4 \frac{\sqrt{y}-y}{y^2}\ dy$
(e) $\int_0^1 (5x - 5^x)\ dx$
(f) $\int_{\pi/4}^{\pi/3} \csc^2\theta\ d\theta$
(g) $\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\ d\theta$
(h) $\int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x}\ dx$
(i) $\int_1^2 \frac{(x-1)^3}{x^2}\ dx$

Net Change Theorem: The net change in a differentiable function $F(x)$ over an interval $a \leq x \leq b$ is the integral of its rate of change:

$$F(b) - F(a) = \int_a^b F'(x)\ dx.$$

Example: Here are several interpretations of the Net Change Theorem.

(a) If $c(x)$ is the cost of producing x units of a certain commodity, then $c'(x)$ is the marginal cost. From the above theorem,

$$\int_{x_1}^{x_2} c'(x)\ dx = c(x_2) - c(x_1),$$

which is the cost of increasing production from x_1 units to x_2 units.

(b) If an object with position function $s(t)$ moves along a coordinate line, its velocity is $v(t) = s'(t)$. The above theorem says that

$$\int_{t_1}^{t_2} v(t)\ dt = s(t_2) - s(t_1),$$

so the integral of velocity is the **displacement** over the time interval $t_1 \le t \le t_2$. On the other hand, the integral of the speed $|v(t)|$ is the **total distance traveled** over the time interval.

Exercise 3. Consider again our analysis of a heavy rock blown straight up from the ground by a dynamite blast. The velocity of the rock at any time t during its motion was given by $v(t) = 160 - 32t$ ft/sec.
(a) Find the displacement of the rock during the time period $0 \le t \le 8$.
(b) Find the total distance traveled during this time period.

Class Exercise 3. The current in a wire is defined as the derivative of the charge: $I(t) = Q'(t)$. What does $\int_a^b I(t)\, dt$ represent?

Class Exercise 4. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)\, dt$ represent?

Class Exercise 5. If $f(x)$ is the slope of a trail at a distance of x miles from the start of the trail, what does $\int_3^5 f(x)\, dx$ represent?

Homework: 3, 9, 13, 19, 23, 27, 31, 35, 41, 45, 49, 53