Section 5.3

The Fundamental Theorem of Calculus, Part 1: If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\, dt \qquad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

Exercise 1. Use the Fundamental Theorem to find dy/dx if
(a) $y = \int_a^x (t^3 + 1)\, dt$

(b) $y = \int_x^5 3t \sin t\, dt$

(c) $y = \int_1^{x^2} \cos t\, dt$

(d) $y = \int_{1+3x^2}^4 \frac{1}{2+3^t}\, dt$

Class Exercise 1. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.
(a) $g(x) = \int_3^x e^{t^2-t}\, dt$
(b) $g(r) = \int_0^r \sqrt{x^2+4}\, dx$
(c) $G(x) = \int_x^1 \cos \sqrt{t}\, dt$
(d) $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\, dz$
(e) $y = \int_0^{x^4} \cos^2\theta\, d\theta$
(f) $y = \int_{\sin x}^1 \sqrt{1+t^2}\, dt$

The Fundamental Theorem of Calculus, Part 2: If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, a function such that $F' = f$.

Exercise 2. Evaluate $\int_{-2}^3 (6x^2 - 5)\, dx$.

Exercise 3. Find the area A of the region between the graph of $y = \sin x$ and the x-axis from $x = 0$ to $x = \pi$.

Exercise 4. Evaluate $\int_{-1}^2 (x^3 + 1)^2\, dx$.

Exercise 5. Evaluate $\int_1^4 (5x - 2\sqrt{x} + \frac{32}{x^3})\, dx$.

Class Exercise 2. Evaluate the integral.
(a) $\int_{-1}^1 x^{100}\, dx$
(b) $\int_0^1 (1 + \frac{1}{2}u^4 - \frac{2}{5}u^9)\, du$
(c) $\int_1^8 x^{-2/3}\, dx$
(d) $\int_{-5}^5 e\, dx$
(e) $\int_0^4 (4 - t)\sqrt{t}\, dt$
(f) $\int_0^2 (y - 1)(2y + 1)\, dy$
(g) $\int_0^{\pi/4} \sec\theta \tan\theta\, d\theta$
(h) $\int_0^3 (2 \sin x - e^x)\, dx$
(i) $\int_1^{18} \sqrt{\frac{3}{z}}\, dz$
(j) $\int_0^1 \cosh t\, dt$
(k) $\int_1^2 \frac{4+u^2}{u^3}\, du$
(l) $\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}}\, dx$

Homework: 9-53 (every 4th), 73, 75