Section 5.1

Exercise 1. Let A be the area of the region that lies under the graph of $f(x) = 1 - x^2$ between $x = 0$ and $x = 1$. Use two rectangles to approximate A.

Class Exercise 1. Estimate the area under the graph of $f(x) = \sqrt{x}$ from $x = 0$ to $x = 4$ using four approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate? (Section 5.1 #4)

Class Exercise 2. Evaluate the upper and lower sums for $f(x) = 1 + x^2$, -1 $\leq x \leq$ 1, with $n =$ 3 and 4. (Section 5.1 #8)

Exercise 2. The velocity function of a projectile fired straight into the air is $f(t) = 160$ - $9.8t$ m/sec. Use the summation technique just described to estimate how far the projectile rises during the first 3 sec. How close do the sums come to the exact value of 435.9 m?

Class Exercise 3. Speedometer readings for a motorcycle at 12-second intervals are given in the table.
(a) Estimate the distance traveled by the motorcycle during the time period using the velocities at the beginning of the time intervals.
(b) Give another estimate using the velocities at the end of the time periods.

t (s)	0	12	24	36	48	60
v (ft/s)	30	28	25	22	24	27

(Section 5.1 #14)

Class Exercise 4. When we estimate distances from velocity data, it is sometimes necessary to use times t_0, t_1, t_2, t_3,..... that are not equally spaced. We can still estimate distances using the time periods $\triangle\, t_i = t_i - t_{i-1}$. For example, on May 7, 1992, the space shuttle *Endeavour* was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters. Use these data to estimate the height above the earth's surface of the *Endeavour*, 62 seconds after liftoff.

Event	Time (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

(Section 5.1 #16)

Definition: The **area** A of the region S lies under the graph of the continuous function f is the limit of the sum of the areas of approximating rectangles:

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\ \triangle x + f(x_2)\ \triangle x + \cdots f(x_n)\ \triangle x\ ]$$

Exercise 3. Use the above definition to find an expression for the area under the graph of $f(x) = \frac{2x}{x^2+1}$ over the interval $[1, 3]$ as a limit. (#19)

Class Exercise 5. Use the above definition to find an expression for the area under the graph of $f(x) = x^2 + \sqrt{1 + 2x}$ over the interval $[4, 7]$. (#20)

Class Exercise 6. Use the above definition to find an expression for the area under the graph of $f(x) = \sqrt{\sin x}$ over the interval $[0, \pi]$. (#21)

Homework: 1, 5, 9, 13, 17, 23