Section 4.9

Definition: A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for every x in I.

Definition: The process of finding the antiderivative of f is called **antidifferentiation**.

Exercise 1. Find an antiderivative for each of the following functions: (a) $f(x) = 2x$, (b) $g(x) = \cos x$, and (c) $h(x) = \frac{1}{x} + 2e^{2x}$. (Hass Sec 4.8 Ex 1)

Exercise 2. Find an antiderivative of $f(x) = 3x^2$ that satisfies $F(1) =$ -1. (Hass Sec 4.8 Ex 2)

Theorem: If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Exercise 3. Find the general antiderivative of $f(x) = \frac{3}{\sqrt{x}} + \sin 2x$. (Hass Sec 4.8 Ex 4)

Class Exercise 1. Find the general antiderivative of each of the following functions.
(a) $f(x) = x^5$ (b) $g(x) = \frac{1}{\sqrt{x}}$ (c) $h(x) = \sin 2x$
(d) $i(x) = \cos \frac{x}{2}$ (e) $j(x) = e^{-3x}$ (f) $k(x) = 2^x$ (Hass Sec 4.8 Ex 3)

Class Exercise 2. Find the general antiderivative of each of the following function. (#2-22 even)
(a) $f(x) = \frac{1}{2}x^2 - 2x + 6$ (b) $f(x) = 8x^9 - 3x^6 + 12x^3$ (c) $f(x) = x(2-x)^2$
(d) $f(x) = x^{3.4} - 2x^{\sqrt{2}-1}$ (e) $f(x) = e^2$ (f) $f(x) = \sqrt[3]{x^2} + x\sqrt{x}$ (g) $f(t) = \frac{3t^4 - t^3 + 6t^2}{t^4}$
(h) $r(\theta) = \sec\theta \tan\theta - 2e^\theta$ (i) $f(t) = \sin t + 2 \sinh t$
(j) $f(x) = 2\sqrt{x} + 6\cos x$ (k) $f(x) = \frac{2+x^2}{1+x^2}$

Definition: A **differential equation** is an equation that involves derivatives or differentials of an unknown function.

Definition: A function f is a **solution** of a differential equation if it satisfies the equation.

Definition: To **solve** a differential equation means to find all solutions.

Exercise 4. Solve the differential equation:

$$f'(x) = 6x^2 + x - 5$$

subject to the initial condition $f(0) = 2$. (Swok Sec 5.1 Ex 6)

Exercise 5. Solve the differential equation:

$$f''(x) = 5\cos x + 2\sin x$$

subject to the initial conditions $f(0) = 3$ and $f'(0) = 4$. (Swok Sec 5.1 Ex 7)

Class Exercise 3. Find f. (#26-46 even)
(a) $f''(x) = x^6 - 4x^4 + x + 1$ (b) $f''(x) = 6x + \sin x$ (c) $f'''(t) = e^t + t^{-4}$
(d) $f'(x) = 5x^4 - 3x^2 + 4$, $f(-1) = 2$ (e) $f'(t) = t + 1/t^3$, $t > 0$, $f(1) = 6$
(f) $f'(x) = (x^2-1)/x$, $f(1) = \frac{1}{2}$, $f(-1) = 0$ (g) $f'(x) = 4/\sqrt{1-x^2}$, $f(\frac{1}{2}) = 1$
(h) $f''(x) = 8x^3 + 5$, $f(1) = 0$, $f'(1) = 8$ (i) $f''(t) = 3/\sqrt{t}$, $f(4) = 20$, $f'(4) = 7$
(j) $f''(x) = x^3 + \sinh x$, $f(0) = 1$, $f(2) = 2.6$ (k) $f''(t) = 2e^t + 3\sin t$, $f(0) = 0$, $f(\pi) = 0$

Suppose a point P is moving on a coordinate line with an acceleration $a(t)$ at time t, and the corresponding velocity is $v(t)$. We learned earlier that $a(t) = v'(t)$. We also know that $v(t) = s'(t)$, where s is the position function of P.

Exercise 6. A stone is thrown vertically upward from a position 144 feet above the ground with an initial velocity of 96 ft/sec. Disregarding air resistance, find:
(a) the stone's distance above the ground after t seconds
(b) the length of time that the stone rises
(c) when and with what velocity the stone strikes the ground (Swok Sec 5.1 Ex 8)

Exercise 7. A hot-air balloon is ascending at the rate of 12 ft/sec is at a height of 80 ft above the ground when a package is dropped. How long does it take the package to reach the ground? (Hass Sec 4.8 Ex 5))

Class Exercise 4. A particle is moving with the given data. Find the position of the particle (#60, 62, 64) (a) $v(t) = 1.5\sqrt{t}$, $s(4) = 10$ (b) $a(t) = 3\cos t - 2\sin t$, $s(0) = 0$, $v(0) = 4$ (c) $a(t) = t^2 - 4t + 6$, $s(0) = 0$, $s(1) = 20$

Homework: 7, 13, 17, 21, 23, 27, 33-85 (every 4th)