Section 4.7

Exercise 1. A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity? (Swok Sec 4.6 Ex 1)

Here are the guidelines for solving optimization problems:
1. Read the problem carefully several times, and think about the given facts as well as the unknown quantities that are to be found.
2. If possible, sketch a picture or diagram and label it appropriately, introducing variables for unknown quantities. Words such as *what*, *find*, *how much*, *how far*, or *when* should alert you to the unknown quantities.
3. Write down the known facts together with any relationships involving the variables.
4. Determine which variable is to be maximized or minimized, and express this variable as a function of *one* of the other variables.
5. Find the critical numbers of the function obtained in guideline 4.
6. Determine the extrema by using Guidelines (4.9) or the first or second derivative test. Check for endpoint extrema whenever appropriate.

Exercise 2. An open box with a rectangular base is to be constructed from a rectangular piece of cardboard 16 inches wide and 21 inches long by cutting a square from each corner and then bending up the resulting sides. Find the size of the corner square that will produce a box having the largest possible volume. (Disregard the thickness of the cardboard.) (Swok Sec 4.6 Ex 2)

Class Exercise 1. What is the largest possible area for a right triangle whose hypotenuse is 5 cm long, and what are its dimensions? (Waits Sec 4.4 #2)

Class Exercise 2. What is the smallest possible perimeter possible for a rectangle whose area is 16 m^2, and what are its dimensions? (Waits Sec 4.4 #3)

Class Exercise 3. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions? (Waits Sec 4.4 #6)

Class Exercise 4. You are planning to make an open rectangular box from an 8- by 15-in. piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way, and what is its volume? (Waits Sec 4.4 #7)

Class Exercise 5. You are planning to close off a corner of the first quadrant with a line segment 20 units long running from $(a, 0)$ to $(0, b)$. Show that the area of the triangle enclosed by the segment is largest when $a = b$. (Waits Sec 4.4 #8)

Class Exercise 6. A 216-m^2 rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed? (Waits Sec 4.4 #10)

Exercise 3. A circular cylindrical metal container, open at the top, is to have a capacity of 24π in.^3 The cost of the material used for the bottom of the container is 15 cents per in.^2, and that of the material used for the curved part is 5 cents per in.^2 If there is no waste of material, find the dimensions that will minimize the cost of the material. (Swok Sec 4.6 Ex 3)

Exercise 4. Find the maximum volume of a right circular cylinder that can be inscribed in a cone of altitude 12 centimeters, and base radius 4 centimeters, if the axes of the cylinder and cone coincide. (Swok Sec 4.6 Ex 4)

Class Exercise 7. A 1125-ft^3 open-top rectangular tank with a square base x ft on a side and y ft deep is to built with its top flush with the ground to catch runoff water. The costs associated with tank involve not only the material from which the tank is made but also an excavation charge proportional to the product xy If the total cost is

$$c = 5(x^2 + 4xy) + 10xy,$$

what values of x and y will minimize it? (Waits Sec 4.4 #12)

Class Exercise 8. You are designing a rectangular poster to contain 50 in^2 of printing with a 4-in. margin at the top and bottom and a 2-in. margin at each side. What overall dimensions will minimize the amount of paper used? (Waits Sec 4.4 #13)

Class Exercise 9. Two sides of a triangle have lengths a and b, and the angle between them is θ. What value of θ will maximize the triangle's area? Hint: $A = (\frac{1}{2})\ ab \sin\theta$. (Waits Sec 4.4 #15)

Class Exercise 10. What are the dimensions of the lightest open-top right circular cylindrical can that will hold a volume of 1000 cm^3? (Waits Sec 4.4 #16)

Class Exercise 11. Jane is 2 mi offshore in a boat and wishes to reach a coastal village 6 mi down a straight shoreline from the point nearest the boat. She can row 2 mph and can walk 5 mph. Where should she land her boat to reach the village in the least amount of time? (Waits Sec 4.4 #20)

Homework: 3, 7, 11, 19, 27, 31, 37-57 (every 4th)

Section 4.8

<u>**Newton's Method**</u>: Let f be a differentiable function, and suppose r is a real zero of f. If x_n is an approximation to r, then the next approximation x_{n+1} is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

provided $f'(x_n) \neq 0$.

Exercise 5. Use Newton's method to approximate $\sqrt{7}$ to five decimal places. (Swok Sec 4.8 Ex 1)

Exercise 6. Find the largest positive real root of $x^3 - 3x + 1 = 0$ to four decimal places. (Swok Sec 4.8 Ex 2)

Exercise 7. Approximate the real root of $x - \cos x = 0$ to three decimal places. (Swok Sec 4.8 Ex 3)

Class Exercise 12. Use Newton's method with the specified initial approximation x_1 to find x_3, the third approximation to the root of the given equation: $\frac{1}{3}x^3 + \frac{1}{2}x^2 + 3 = 0$, $x_1 = -3$. (#6) (Give your answer to four decimal places.)

Class Exercise 13. Use Newton's method with the specified initial approximation x_1 to find x_3, the third approximation to the root of the given equation: $x^7 + 4 = 0$, $x_1 = -1$. (#8)

Class Exercise 14. Use Newton's method to approximate the given number correct to eight decimal places: $\sqrt[100]{100}$. (#12)

Class Exercise 15. Use Newton's method to approximate the root of $2.2x^5 - 4.4x^3 + 1.3x^2 - 0.9x - 4.0 = 0$ in the interval [-2,-1] correct to six decimal places. (#14)

Class Exercise 16. Use Newton's method to approximate the root of $3 \sin x = x$ correct to six decimal places. (#16)

Class Exercise 17. Use Newton's method to find all roots of the equation correct to six decimal places. (#18, 20, 22) (a) $\sqrt{x+1} = x^2 - x$ (b) $\frac{1}{x} = 1 + x^3$ (c) $\sin x = x^2 - 2$

Homework: 1-7 ODD, 11-21 ODD