Section 4.5

Here are the guidelines for sketching the graph of $f(x)$:

1. **Domain of f**: Find the domain of f- that is, all real numbers x such that $f(x)$ is defined.

2. **Continuity of f**: Determine whether f is continuous on its domain, and, if not, find and classify the discontinuities.

3. **x- and y- intercepts**: The x-intercepts are solutions of the equation $f(x) = 0$; the y-intercept is the function value $f(0)$, if it exists.

4. **Symmetry**: If f is an even function, the graph is symmetric with respect to the y-axis. If f is an odd function, the graph is symmetric with respect to the origin.

5. **Critical numbers and local extrema**: Find $f'(x)$ and determine the critical numbers- that is, the values of x such that $f'(x) = 0$ or $f'(x)$ does not exist. Use the first derivative test to help find local extrema. Employ the sign of $f'(x)$ to find intervals on which f is increasing ($f'(x) > 0$) or is decreasing ($f'(x) < 0$). Determine whether there are corners or cusps on the graph.

6. **Concavity and points of inflection**. Find $f''(x)$, and use the second derivative test whenever appropriate. If $f''(x) > 0$ on an open interval I, the graph is concave upward. If $f''(x) < 0$, the graph is concave downward. If f is continuous at c and if $f''(x)$ changes sign at c, then $P(c, f(c))$ is a point of inflection.

7. **Asymptotes**

Horizontal: If $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$, then the line $y = L$ is a horizontal asymptote.

Vertical: If $\lim_{x\to a^+} f(x)$ or $\lim_{x\to a^-} f(x)$ is either ∞ or $-\infty$, then the line $x = a$ is a vertical asymptote.

Exercise 1. Graph $f(x) = \frac{2x^2}{9-x^2}$. (Swok Sec 4.5 Ex 1)

Exercise 2. Graph $f(x) = \frac{x^2}{x^2-x-2}$. (Swok Sec 4.5 Ex 2)

Exercise 3. Graph $f(x) = \frac{x^2-9}{2x-4}$. (Swok Sec 4.5 Ex 3)

Class Exercise 1. Graph the function. (#2-18 even)
(a) $y = 2 + 3x^2 - x^3$ (b) $y = x^4 - 8x^2 + 8$ (c) $y = x^5 - 5x$
(d) $y = (4 - x^2)^5$ (e) $y = \frac{x^2-4}{x^2-2x}$ (f) $y = \frac{x}{x^2-9}$
(g) $y = \frac{x^2}{x^2+9}$ (h) $y = 1 + \frac{1}{x} + \frac{1}{x^2}$ (i) $y = \frac{x}{x^3-1}$

Exercise 4. Graph the function $f(x) = \frac{2x}{\sqrt{x^2+1}}$. (Swok Sec 4.5 Ex 4)

Class Exercise 2. Graph the function. (#22-34 even)
(a) $y = 2\sqrt{x} - x$ (b) $y = \sqrt{x^2+x} - x$ (c) $y = x\sqrt{2-x^2}$
(d) $y = \frac{x}{\sqrt{x^2-1}}$ (e) $y = x^{5/3} - x^{2/3}$
(f) $y = \sqrt[3]{x^3+1}$ (g) $y = x + \cos x$

Homework: 1-15 ODD