Section 4.3

Theorem: Let f be continuous on $[a, b]$ and differentiable on (a, b).
(i) If $f'(x) > 0$ for every x in (a, b), then f is increasing on $[a, b]$.
(ii) If $f'(x) < 0$ for every x in (a, b), then f is decreasing on $[a, b]$.

Exercise 1. If $f(x) = x^3 + x^2 - 5x - 5$, (Swok Sec 4.3 Ex 1)
(a) find the intervals on which f is increasing and the intervals on which f is decreasing
(b) sketch the graph of f

Class Exercise 1. Find the intervals on which the function is increasing or decreasing. (Waits Sec 4.3 #13-19)
(a) $y = 4x^3 + 21x^2 + 36x - 20$ (b) $y = -x^4 + 4x^3 - 4x + 1$
(c) $y = 2x^{1/5} + 3$ (d) $y = 5 - x^{1/3}$
(e) $y = \frac{5e^x}{e^x + 3e^{0.8x}}$ (f) $y = \frac{8e^{-x}}{2e^{-x} + 5e^{-1.5x}}$
(g) $f(x) = \begin{cases} 2x & x < 1 \\ 2 - x^2 & x \geq 1 \end{cases}$

First Derivative Test: Suppose that c is a critical number of a continuous function f.
(a) If f' changes from positive to negative at c, then f has a local maximum at c.
(b) If f' changes from negative to positive at c, then f has a local minimum at c.
(c) If f' does not change sign at c (for example if f' is positive on both sides of c or negative on both sides), then f has no local maximum or minimum at c.

Exercise 2. If $f(x) = x^3 + x^2 - 5x - 5$, find the local extrema of f. (Swok Sec 4.3 Ex 2)

Exercise 3. If $f(x) = x^{1/3}(8 - x)$, find the local extrema of f. (Swok Sec 4.3 Ex 3)

Exercise 4. If $f(x) = x^{2/3}(x^2 - 8)$, find the local extrema of f. (Swok Sec 4.3 Ex 4)

Class Exercise 2. Find the local extreme values for the functions in the previous class exercise. (Waits Sec 4.3 #13-19)
(a) $y = 4x^3 + 21x^2 + 36x - 20$ (b) $y = -x^4 + 4x^3 - 4x + 1$
(c) $y = 2x^{1/5} + 3$ (d) $y = 5 - x^{1/3}$
(e) $y = \frac{5e^x}{e^x + 3e^{0.8x}}$ (f) $y = \frac{8e^{-x}}{2e^{-x} + 5e^{-1.5x}}$
(g) $f(x) = \begin{cases} 2x & x < 1 \\ 2 - x^2 & x \geq 1 \end{cases}$

Definition: Let f be differentiable on an open interval I. The graph of f is
(i) **concave upward** on I if f' is increasing on I
(ii) **concave downward** on I if f' is decreasing on I

Test for concavity: If the second derivative f'' of f exists on an open interval I, then the graph of f is
(i) **concave upward** on I if $f''(x) > 0$ on I
(ii) **concave downward** on I if $f''(x) < 0$ on I

Exercise 5. If $f(x) = x^3 + x^2 - 5x - 5$, determine intervals on which the graph of f is concave upward or is concave downward. (Swok Sec 4.4 Ex 1)

Exercise 6. If $f(x) = \sin x$, determine where the graph of f is concave upward and where it is concave downward. (Swok Sec Ex 2)

Class Exercise 3. Find the intervals on which the function is concave upward or concave downward. (Waits Sec 4.3 #13-19)
(a) $y = 4x^3 + 21x^2 + 36x - 20$ (b) $y = -x^4 + 4x^3 - 4x + 1$
(c) $y = 2x^{1/5} + 3$ (d) $y = 5 - x^{1/3}$
(e) $y = \frac{5e^x}{e^x + 3e^{0.8x}}$ (f) $y = \frac{8e^{-x}}{2e^{-x} + 5e^{-1.5x}}$
(g) $f(x) = \begin{cases} 2x & x < 1 \\ 2 - x^2 & x \geq 1 \end{cases}$

Definition A point $(c, f(c))$ on the graph of f is a **point of inflection** if the following two conditions are satisfied:
(i) f is continuous at c.
(ii) There is an open interval (a, b) containing c such that the graph is concave downward on (a, c) and concave upward on (c, b), or vice versa.

Exercise 7. If $f(x) = 12 + 2x^2 - x^4$, find points of inflection of f. (Swok Sec 4.3 Ex 3)

Exercise 8. If $f(x) = x^5 - 5x^3$, find the points of inflection of f. (Swok Sec 4.3 Ex 4)

Class Exercise 4. Find the points of inflection for the following functions. (Waits Sec 4.3 #13-19)
(a) $y = 4x^3 + 21x^2 + 36x - 20$ (b) $y = -x^4 + 4x^3 - 4x + 1$
(c) $y = 2x^{1/5} + 3$ (d) $y = 5 - x^{1/3}$
(e) $y = \frac{5e^x}{e^x + 3e^{0.8x}}$ (f) $y = \frac{8e^{-x}}{2e^{-x} + 5e^{-1.5x}}$
(g) $f(x) = \begin{cases} 2x & x < 1 \\ 2 - x^2 & x \geq 1 \end{cases}$

Second Derivative Test: Suppose that f'' is continuous near c.
(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.
(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

Exercise 9. If $f(x) = x^5$ - $5x^3$, find the local extrema of f. (Swok Sec 4.4 Ex 4)

Exercise 10. If $f(x) = 1$ - $x^{1/3}$, find the local extrema of f, discuss concavity, and find the points of inflection. (Swok Sec 4.4 Ex 5)

Class Exercise 5. Find the intervals on which the function is (i) increasing, (ii) decreasing, (iii) concave up, (iv) concave down, (v) local extreme values, (vi) inflection points . (Waits Sec 4.3 #21-27)
(a) $y = xe^{1/x^2}$ (b) $y = x^2\sqrt{9 - x^2}$
(c) $y = \tan^{-1}x$ (d) $y = x^{3/4}(5 - x)$
(e) $y = x^{1/3}(x - 4)$ (f) $y = x^{1/4}(x + 3)$
(g) $y = \frac{x^3 - 2x^2 + x - 1}{x - 2}$

Homework: 9, 11, 13, 45-57 ODD