Section 4.2

Rolle's Theorem: If f is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b) and if $f(a) = f(b)$, then $f'(c) = 0$ for at least one number c in (a, b).

Corollary: If f is continuous on a closed interval $[a, b]$ and if $f(a) = f(b)$, then f has at least one critical number in the open interval (a, b).

Exercise 1. Let $f(x) = 4x^2 - 20x + 29$. Show that f satisfies the hypotheses of Rolle's theorem on the interval $[1, 4]$, and find all real numbers c in the open interval $(1, 4)$ such that $f'(c) = 0$. Illustrate the results graphically. (Swok Sec 4.2 Ex 1)

Class Exercise 1. Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem. (#2,4)
(a) $f(x) = x^3 - x^2 - 6x + 2$, $[0, 3]$
(b) $f(x) = \cos 2x$, $[\pi/8, 7\pi/8]$

Mean Value Theorem: If f is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a},$$

or, equivalently,

$$f(b) - f(a) = f'(c)(b - a).$$

Exercise 2. If $f(x) = \frac{1}{4}x^2 + 1$, show that f satisfies the hypotheses of the mean value theorem on the interval $[-1, 4]$, and find a number c in (-1,4) that satisfies the conclusion of the theorem. Illustrate the results graphically. (Swok Sec 4.2 Ex 2)

Exercise 3. If $f(x) = x^3 - 8x - 5$, show that f satisfies the hypotheses of the mean value theorem on the interval $[1, 4]$, and find a number c in the open interval $(1, 4)$ that satisfies the conclusion of the theorem. (Swok Sec 4.2 Ex 3)

Class Exercise 2. (a) Show that the function f satisfies the hypotheses of the Mean Value Theorem on the given interval $[a, b]$.
(b) Find each value of c in (a, b) that satisfies the equation

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

(i) $f(x) = x^2 + 2x - 1$, $[0, 1]$
(ii) $f(x) = x^{2/3}$, $[0, 1]$
(iii) $f(x) = \sin^{-1}x$, $[-1, 1]$
(iv) $f(x) = \ln(x - 1)$, $[2, 4]$ (Waits Sec 4.2 #15-18)

Exercise 4. The speedometer of an automobile registers 50 mi/hr as it passes a mileage marker along a highway. Four minutes later, as the automobile passes a marker that is five miles away from the first, the speedometer registers 55 mi/hr. Use the mean value theorem to prove that the velocity exceeded 70 mi/hr at some time while the automobile was traveling between the two markers. (Swok Sec 4.2 Ex 4)

Class Exercise 3. A trucker handed in a ticket at a toll booth showing that in 2 hours he had covered 214 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why? (Waits Sec 4.2 #39)

Class Exercise 4. It took 20 sec for the temperature to rise from 0°F to 212°F when a thermometer was taken from a freezer and placed in boiling water. Explain why at some moment in that interval the mercury was rising at exactly 10.1°F/sec. (Waits Sec 4.2 #40)

Class Exercise 5. Classical accounts tell us that a 170-oar trireme once covered 184 sea miles in 24 hr. Explain why at some point during this feat the trireme's speed exceeded 7.5 knots (sea miles per hour). (Waits Sec 4.2 #41)

Class Exercise 6. A marathoner ran the 26.2-mi New York City Marathon in 2.2 h. Show that at least twice, the marathoner was running at exactly 11 mph. (Waits Sec 4.2 #42)

Homework: 1, 3, 9-17 ODD, 41