Section 4.1

Definition: Let a function f be defined on a set S of real numbers, and let c be a number in S.

(i) $f(c)$ is the **absolute maximum value** of f on S if $f(x) \leq f(c)$ for every x in S.

(ii) $f(c)$ is the **absolute minimum value** of f on S if $f(x) \geq f(c)$ for every x in S.

Definition: Maximum and minimum values are sometimes called **extreme values**, or **extrema** of f.

Exercise 1. Let $f(x) = 4 - x^2$. Find the extrema of f on the following intervals: (a) $[-2, 1]$, (b) $(-2, 1)$, (c) $(1, 2]$, and (d) $(1, 2)$. (Swok Sec 4.1 Ex 1)

Exercise 2. Let $f(x) = \frac{1}{x^2}$. Find the extrema of f on (a) [-1,2] and (b) $[-1, 2)$. (Swok Sec 4.1 Ex 2)

Extreme Value Theorem: If a function f is continuous on a closed interval $[a, b]$, then f takes on a minimum value and a maximum value at least once on $[a, b]$.

Exercise 3. Identify each x-value at which any absolute extreme value occurs. Explain how your answer is consistent with the Extreme Value Theorem. (Waits Sec 4.1 #1-6)

Class Exercise 1. Identify each x-value at which any absolute extreme value occurs. Explain how your answer is consistent with the Extreme Value Theorem. (Waits Sec 4.1 #7-10)

Definition: Let c be a number in the domain of a function f.

(i) $f(c)$ is a **local maximum** of f if there exists an open interval (a, b) containing c such that $f(x) \leq f(c)$ for every x in (a, b).

(ii) $f(c)$ is a **local minimum** of f if there exists an open interval (a, b) containing c such that $f(x) \geq f(c)$ for every x in (a, b). (Swok Page 170)

Definition: Each local maximum or minimum is called a local extremum of f, and the totality of such numbers are the **local extrema** of f.

Exercise 4. Using a graphing calculator, find the extreme values and where they occur (Waits Sec 4.2 #19-23)
(a) $y = 2x^2 - 8x + 9$ (b) $y = x^3 - 2x + 4$ (c) $y = x^3 + x^2 - 8x + 5$
(d) $y = x^3 - 3x^2 + 3x - 2$ (e) $y = \sqrt{x^2 - 1}$

Class Exercise 2. Using a graphing calculator, find the extreme values and where they occur. (Waits Sec 4.2 #24-29)
(a) $y = \frac{1}{x^2-1}$ (b) $y = \frac{1}{\sqrt{1-x^2}}$ (c) $y = \frac{1}{\sqrt[3]{1-x^2}}$
(d) $y = \sqrt{3 + 2x - x^2}$ (e) $y = \frac{3}{2}x^4 + 4x^3 - 9x^2 + 10$ (f) $y = \frac{x}{x^2+1}$

Theorem: If a function f has a local extremum at a number c in an open interval, then either $f'(c) = 0$ or $f'(c)$ does not exist.

Corollary: If $f'(c)$ exists and $f'(c) \neq 0$, then $f(c)$ is not a local extremum of the function f.

Theorem: If a function f is continuous on a closed interval $[a, b]$ and has its maximum or minimum value at a number c in the open interval (a, b), then either $f'(c) = 0$ or $f'(c)$ does not exist.

Definition: A number c in the domain of a function f is a **critical number** of f if either $f'(c) = 0$ or $f'(c)$ does not exist.

Exercise 5. Find the critical numbers of f if $f(x) = (x + 5)^2 \sqrt[3]{x - 4}$. (Swok Sec 4.1 Ex 6)

Exercise 6. If $f(x) = 2 \sin x + \cos 2x$, find the critical numbers of f that are in the interval $[0, 2\pi]$. (Swok Sec 4.1 Ex 7)

Guidelines for finding the extrema of a continuous function f on $[a, b]$:

1. Find all the critical numbers of f in (a,b).
2. Calculate $f(c)$ for each critical number c found in guideline 1.
3. Calculate the endpoint values $f(a)$ and $f(b)$.
4. The absolute maximum and minimum values of f on $[a, b]$ are the largest and smallest function values calculated in guidelines 2 and 3.

Exercise 7. If $f(x) = x^3 - 12x$, find the absolute maximum and minimum values of f on the closed interval $[-3, 5]$ and sketch the graph of f. (Swok Sec 4.1 Ex 3)

Exercise 8. If $f(x) = (x-1)^{2/3} + 2$, find the absolute maximum and minimum values of f on $[0, 9]$, and sketch the graph of f (Swok Sec 4.1 Ex 4)

Exercise 9. If $f(x) = x^3$, prove that f has no local extremum. (Swok Sec 4.1 Ex 5)

Class Exercise 3. Use analytic methods to find the extreme values on the interval and where they occur. (Waits Sec 4.1 #11-18)

(a) $f(x) = \frac{1}{x} + \ln x$, $0.5 \leq x \leq 4$ (b) $g(x) = e^{-x}$, $-1 \leq x \leq 1$

(c) $h(x) = \ln(x+1)$, $0 \leq x \leq 3$ (d) $k(x) = e^{-x^2}$, $-\infty < x < \infty$

(e) $f(x) = \sin(x + \pi/4)$, $0 \leq x \leq 7\pi/4$ (f) $g(x) = \sec x$, $-\pi/2 < x < 3\pi/2$

(g) $f(x) = x^{2/5}$, $-3 \leq x < 1$ (h) $f(x) = x^{3/5}$, $-2 < x \leq 3$

Homework: 3-35 (every 4th), 41, 47-63 (every 4th)