Section 3.6

In this section, we use implicit differentiation to find the derivative of the logarithmic functions.

Exercise 1. Find $\frac{d}{dx}$ (ln x). (Hass Page 179)

Exercise 2. Use implicit differentiation to find $\frac{d}{dx}$ ($\log_a x$). (Hass Page 180)

From the previous two exercises, we have established the following two formulas:

$$\frac{d}{dx}(\log_a x) = 1/(x \ln a)$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Exercise 3. If $f(x) = \ln(x^2 - 6)$, find $f'(x)$. (Swok Sec 3.6 Ex 1)

Exercise 4. If $f(x) = \ln\sqrt{x+1}$, find $\frac{dy}{dx}$. (Swok Sec 3.6 Ex 2)

Exercise 5. If $f(x) = \ln|4 + 5x - 2x^3|$, find $f'(x)$. (Swok Sec 3.6 Ex 3)

Class Exercise 1. Find dy/dx. (Waits Sec 3.9 #21-32)
(a) $y = \ln(x^2)$ (b) $y = (\ln x)^2$ (c) $y = \ln(1/x)$
(d) $y = \ln(10/x)$ (e) $y = \ln(x+2)$ (f) $y = \ln(2x+2)$
(g) $y = \ln(2 - \cos x)$ (h) $y = \ln(x^2 + 1)$ (i) $y = \ln(\ln x)$
(j) $y = x \ln x - x$ (k) $y = \log_4 x^2$ (l) $y = \log_5 \sqrt{x}$

Guidelines for logarithmic differentiation
1. $y = f(x)$
2. $\ln y = \ln f(x)$
3. $D_x[\ \ln y\ ] = D_x[\ \ln f(x)\ ]$
4. $\frac{1}{y}\ D_x y = D_x[\ \ln f(x)]$
5. $D_x y = f(x)\ D_x[\ \ln f(x)]$

Exercise 6. If $y = \frac{(5x-4)^3}{\sqrt{2x+1}}$, use logarithmic differentiation to find $D_x y$. (Swok Sec 3.6 Ex 6)

Exercise 7. If $y = (\sin x)^x$, find $\frac{dy}{dx}$. (Waits Sec 3.9 #43)

Class Exercise 2. Find dy/dx. (Waits Sec 3.9 #44-46)
(a) $y = (x)^{\tan x}$ (b) $y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$ (c) $y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$

Homework: 5, 11, 13, 15, 21, 29, 33, 37, 45, 49, 53, 55

Section 3.9

In this section we look at problems that ask for the rate at which some variable changes when it is known how the rate of some other variable (or perhaps several variables) changes. The problem of finding a rate of change from other known rates of change is called a *related rates problem.*

Exercise 8. Two variables x and y are functions of t and are related by the equation

$$x^3 - 2y^2 + 5x = 16.$$

If $dx/dt = 4$ when $x = 2$ and $y = -1$, find the corresponding value of dy/dt. (Swok Sec 3.8 Ex 1)

Exercise 9. A ladder 20 feet long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 12 feet above the ground? (Swok Sec 3.8 Ex 2)

Here are the guidelines for solving related rates problems:
1. Read the problem carefully several times, and think about the given facts and the unknown quantities that are to be found.
2. Sketch a picture or diagram and label it appropriately, introducing variables for unknown quantities.
3. Write down all the known facts, expressing the given and unknown rates of derivatives of the variables introduced in guideline 2.
4. Formulate a *general* equation that relates the variables.
5. Differentiate the equation formulate in guideline 4 implicitly with respect to t, obtaining a *general* relationship between the rates.
6. Substitute the *known* values and rates, and the find the unknown rate of change.

Exercise 10. At 1:00 P.M., ship A is 25 miles due south of ship B. If ship A is sailing west at a rate of 16 mi/hr and ship B is sailing south at a rate of 20 mi/hr, find the rate at which the distance between the ships is changing at 1:30 P.M. (Swok Sec 3.8 Ex 3)

Class Exercise 3. A 13 ft-ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec. (Waits Sec 4.6 #13)
(a) How fast is the top of the ladder sliding down the wall at that moment?
(b) At what rate is the area of the triangle formed by the ladder wall, and ground changing at that moment?
(c) At what rate is the angle θ between the ladder and the ground changing at the moment?

Class Exercise 4. Inge flies a kite at a height of 300 ft, the wind carrying the kite horizontally away at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her? (Waits Sec 4.6 #14)

Exercise 11. A water tank has the shape of an inverted right circular cone of altitude 12 feet and base radius 6 feet. If water is being pumped into the tank at a rate of 10 gal/min, approximate the rate at which the water level is rising when the water is 3 feet deep. (Swok Sec 3.8 Ex 4)

Class Exercise 5. A trough is 15 ft long and 4 ft across the top. Its ends are isosceles triangles with height 3 ft. Water runs into the trough at the rate of 2.5 ft^3/min. How fast is the water level rising when it is 2 ft deep? (Waits Sec 4.6 #12)

Class Exercise 6. The mechanics at Lincoln Automotive are reboring a 6-in. deep cylinder to fit a new piston. The machine they are using increases the cylinder's radius one-thousandth of an inch every min. How rapidly is the cylinder volume increasing when the bore (diameter) is 3.800 in? (Waits Sec 4.6 #15)

Class Exercise 7. Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the (a) height and (b) radius changing when the pile is 4 m high? Give your answer in cm/min. (Waits Sec 4.6 #16)

Class Exercise 8. Water is flowing at the rate of 50 m^3/min from a concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. (a) How fast is the water level falling when the water is 5 m deep? (b) How fast is the radius of the water's surface changing at that moment? Give your answer in cm/min. (Waits Sec 4.6 #17)

Homework: 3-21 ODD, 31, 43