Section 3.10

Definition: If f is differentiable at $x = a$, then the approximating function

$$L(x) = f(a) + f'(a)(x - a)$$

is the **linearization** of f at a. The approximation

$$f(x) \approx L(x)$$

of f by L is the **standard linear approximation** of f at a. The point $x = a$ is the **center** of the approximation.

Exercise 1. Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$. (Hass Sec 3.11 Ex 1)

Exercise 2. Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 3$. (Hass Sec 3.11 Ex 2)

Class Exercise 1. Find the linearization $L(x)$ of $f(x)$ at $x = a$. (Waits Sec 4.5 #1-6)
(a) $f(x) = x^3 - 2x + 3$, $a = 2$ (b) $f(x) = \sqrt{x^2+9}$, $a = -4$
(c) $f(x) = x + \frac{1}{x}$, $a = 1$ (d) $f(x) = \ln(x+1)$, $a = 0$
(e) $f(x) = \tan x$, $a = \pi$ (f) $f(x) = \cos^{-1}x$, $a = 0$

Definition: If $y = f(x)$, where f is a differentiable function, then the **differential** dx is an independent variable; that is dx can be given the value of any real number. The differential dy is then defined in terms of dx by the equation

$$dy = f'(x)\ dx$$

Definition: Let $y = f(x)$ and let $\triangle x$ be an increment of x. The **increment $\triangle y$ of y** is

$$\triangle y = f(x + \triangle x) - f(x).$$

Exercise 3. Let $y = 3x^2$ - 5 and let $\triangle x$ be an increment of x. (Swok Sec 3.5 Ex 1)
(a) Find general formulas for $\triangle y$ and dy.
(b) If x changes from 2 to 2.1, find the values of $\triangle y$ and dy.

Exercise 4. If $y = x^3$ and $\triangle x$ is an increment of x, find the following (Swok Sec 3.5 Ex 2)
(a) $\triangle y$ (b) dy
(c) $\triangle y$ - dy (d) the value of $\triangle y$ - dy if $x = 1$ and $\triangle x = 0.02$

Exercise 5. (a) Use differentials to approximate the change in $\sin\theta$ if θ changes from 60° to 61°.
(b) Find a linear approximation to $\sin 61°$. (Swok Sec 3.5 Ex 3)

Class Exercise 2. Find dy, and evaluate dy for the given value of x and dx. (Waits Sec 4.5 #19-26)
(a) $y = x^3 - 3x$, $x = 2$, $dx = 0.05$ (b) $y = \frac{2x}{1+x^2}$, $x = 2$, $dx = 0.1$
(c) $y = x^2 \ln x$, $x = 1$, $dx = 0.01$ (d) $y = x\sqrt{1-x^2}$, $x = 0$, $dx = -0.2$
(e) $y = e^{\sin x}$, $x = \pi$, $dx = -0.1$ (f) $y = 3\csc(1 - \frac{x}{3})$, $x = 1$, $dx = 0.1$
(g) $y + xy - x = 0$, $x = 0$, $dx = 0.01$ (h) $y = \sec(x^2 - 1)$, $x = 1.5$, $dx = 0.05$

Homework: 1, 3, 11-31 ODD, 35