Section 2.8

Definition: The **derivative** of the function $f(x)$ with respect to the variable x is the function f' whose value at x is

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h},$$

provided the limit exists.

There are many ways to denote the derivative of a function $y = f(x)$, where the independent variable is x and the dependent variable is y. Some common alternative notations for the derivative are

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = D(f)(x) = D_x f(x).$$

The symbols d/dx and D indicate the operation of differentiation. We read dy/dx as "the derivative of y with respect to x," and df/dx and $(d/dx)f(x)$ as "the derivative of f with respect to x."

Exercise 1. Differentiate $f(x) = \frac{x}{x-1}$. (Hass Sec 3.2 Ex 1)

Class Exercise 1. Find the derivative of the function using the definition of the derivative. State the domain of the function and the domain of its derivative. (#22-30 even)
(a) $f(x) = mx + b$ (b) $f(x) = 1.5x^2 - x + 3.7$ (c) $g(t) = \frac{1}{\sqrt{t}}$ (d) $f(x) = \frac{x^2-1}{2x-3}$ (e) $f(x) = x^{3/2}$.

Exercise 2. Graph the derivative of the functions $y = f(x)$ on the link posted in Canvas.

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Definition: A function f is **differentiable at a** if $f'(a)$ exists. It is **differentiable on an open interval** (a, b) [or (a,∞) or (a, b) or (a,∞) or $(-\infty, a)$ or $(-\infty,\infty)$] if it is differentiable at every number in the interval.

There are four different instances when a function fails to be differentiable.

1) a *corner*, where the one-sided derivatives differ

An example of a function would be $f(x) = |x|$.

2) a *cusp*, where the secants lines approach ∞ on one side and $-\infty$ from the other (an extreme case of a corner)

An example of a function would be $f(x) = x^{2/3}$.

3) a *vertical tangent*, where the slopes of the secant lines approach either ∞ or $-\infty$ from both sides (in this example, ∞)

An example of a function would be $f(x) = \sqrt[3]{x}$.

4) a *discontinuity* (which will cause one or both of the one-sided derivatives to be nonexistent).

An example of a function would be $f(x) = x/|x|$.

Exercise 3. For the graphs on the link posted in Canvas, compare the right-hand and left-hand derivatives to show that the function is not differentiable at the point P.

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Exercise 4. The graph of a function over a closed interval D is given on the link posted in Canvas. At what domain points does the function appear to be
(a) differentiable? (b) continuous but not differentiable?
(c) neither continuous nor differentiable

Class Exercise 4. The graph of a function over a closed interval D is given on the link posted in Canvas. At what domain points does the function appear to be
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Exercise 5. The function fails to be differentiable at $x = 0$. Tell whether the problem is a corner, cusp, a vertical tangent, or a discontinuity. (Waits Sec 3.2 #11, 12)
(a) $y = \begin{cases} \tan^{-1} x & x \neq 0 \\ 1 & x = 0 \end{cases}$ (b) $y = x^{4/5}$

Class Exercise 5. The function fails to be differentiable at $x = 0$. Tell whether the problem is a corner, cusp, a vertical tangent, or a discontinuity. (Waits Sec 3.2 #13-16)
(a) $y = x + \sqrt{x^2} + 2$ (b) $y = 3 - \sqrt[3]{x}$ (c) $y = 3x - 2|x| - 1$ (d) $y = \sqrt[3]{x}$

Exercise 6. Find all values of x for which the function is differentiable. (Waits Sec 3.2 #17,18)
(a) $f(x) = \frac{x^3-8}{x^2-4x-5}$ (b) $f(x) = \sqrt[3]{3x-6} + 5$

Class Exercise 6. Find all values of x for which the function is differentiable. (Waits Sec 3.2 #19-22)
(a) $P(x) = \sin(|x|)$-1 (b) $Q(x) = 3\cos(|x|)$
(c)

$$g(x) = \begin{cases} (x+1)^2, & x \leq 0 \\ 2x+1, & 0 < x < 3 \\ (4-x)^2, & x \geq 3 \end{cases}$$

(d) $C(x) = x|x|$

Definition: If f is a differentiable function, then its derivative f' is also a function, so f' may have a derivative of its own, denoted by $(f')' = f''$. This new function is called the **second derivative** of f because it is the derivative of the derivative of f.

Exercise 7. For $f(x) = x^2$, find $f''(x)$.

Class Exercise 7. For $f(x) = x^3$, find $f''(x)$.

Homework: 1-33 (every 4th)