Section 2.7

Definition: The **slope m_a of the tangent line** to the graph of a function f at $P(a, f(a))$ is

$$m_a = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h},$$

provided the limit exists.

Exercise 1. Let $f(x) = x^2$, and let a be any real number. (Swok Sec 3.1 Ex 1)
(a) Find the slope of the tangent line to the graph of f at $P(a, a^2)$.
(b) Find an equation of the tangent line at $R(-2, 4)$.

Class Exercise 1. Find the equation of the tangent line to the curve, $y = x^3 - 3x + 1$, at the point (2,3). (#6)

Class Exercise 2. Find the equation of the tangent line to the curve, $y = \frac{2x+1}{x+2}$ at the point (1,1). (#8)

Definition: The **average velocity** v_{av} of an object that travels a distance d in time t is $v_{av} = \frac{d}{t}$.

Definition: Suppose a point P moves on a coordinate line l such that its coordinate at time t is $s(t)$. The velocity v_a of P at time a is

$$v_a = \lim_{h\to 0} \frac{s(a+h)-s(a)}{h},$$

provided the limit exists.

The limit in the definition is also called the **instantaneous velocity** of P at time a.

Exercise 2. A sandbag is dropped from a hot-air balloon that is hovering at a height of 512 feet above the ground. If air resistance is disregarded, then the distance $s(t)$ from the ground to the sandbag after t seconds is given by

$$s(t) = -16t^2 + 512.$$

Find the velocity of the sandbag at: (a) $t = a$ sec (b) $t = 2$ sec
(c) the instant it strikes the ground (Swok Sec 3.1 Ex 2)

Class Exercise 3. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height (in meters) after t seconds is given by $H = 10t - 1.86t^2$. (#14)
(a) Find the velocity of the rock after one second.
(b) Find the velocity of the rock when $t = a$.
(c) When will the rock hit the surface?
(d) With what velocity will the rock hit the surface?

Class Exercise 4. The displacement (in meters) of a particle moving in a straight line is given by $s = t^2 - 8t + 18$, where t is measured in seconds. (#16)
(a) Find the average velocity over each time interval: (i) [3,4], (ii) [3.5,4], (iii) [4,5], (iv) [4,4.5].
(b) Find the instantaneous velocity when $t = 4$.
(c) Draw the graph of s as a function of t and draw the secant lines whose slopes are the average velocities in part (a) and the tangent line whose slope is the instantaneous velocity in part (b).

Definition: The **derivative of a function f at a number** a, denoted by $f'(a)$, is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

if this limit exists.

Exercise 3. Find the derivative of the function $f(x) = x^2 - 8x + 9$ at the number a. (Stew Sec 2.7 Ex 4)

Class Exercise 5. Find $f'(a)$ for the following functions. (#28, 30, 32)
(a) $f(t) = 2t^3 + t$ (b) $f(x) = x^{-2}$ (c) $f(x) = \frac{4}{\sqrt{1-x}}$.

Homework: 1-17 (every 4th), 23, 27, 39, 41, 43, 47