Section 2.6

Definition: Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x\to\infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

Exercise 1. Find $\lim_{x\to\infty} \frac{1}{x}$.

Defintion: Let f be a function defined on some interval $(-\infty, a)$. Then

$$\lim_{x\to-\infty} f(x) = L$$

means that the value of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large negative.

Exercise 2. Find $\lim_{x\to-\infty} \frac{1}{x}$.

Exercise 3. Sketch the graph of an example of a function f that satisfies all of the given conditions: $\lim_{x\to2} f(x) = \infty$, $\lim_{x\to-2^+} f(x) = \infty$, $\lim_{x\to-2^-} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = 0$, $\lim_{x\to\infty} f(x) = 0$, $f(0) = 0$. (#6)

Class Exercise 1. Sketch the graph of an example of a function f that satisfies all of the given conditions: $\lim_{x\to\infty} f(x) = 3$, $\lim_{x\to2^-} f(x) = \infty$, $\lim_{x\to2^+} f(x) = -\infty$, f is odd. (#8)

Class Exercise 2. Sketch the graph of an example of a function f that satisfies all of the given conditions: $\lim_{x\to3} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = 2$, $f(0) = 0$, f is even. (#10)

Exercise 4. Find $\lim_{x\to\infty} (5 + \frac{1}{x})$. (Hass Sec 2.6 Ex 2a)

Exercise 5. Find $\lim_{x\to-\infty} \frac{\pi\sqrt{3}}{x^2}$. (Hass Sec 2.6 Ex 2b)

Exercise 6. Find the limit or show that it does not exist. (#16, 22, 26)
(a) $\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1}$ (b) $\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+1}}$ (c) $\lim_{x\to-\infty} x + \sqrt{x^2+2x}$

Class Exercise 3. Find the limit or show that it does not exist. (#18,20,24,28,30,32,34)
(a) $\lim_{x\to-\infty} \frac{4x^3+6x^2-2}{2x^3-4x+5}$ (b) $\lim_{t\to\infty} \frac{t-t\sqrt{t}}{2t^{3/2}+3t-5}$ (c) $\lim_{x\to-\infty} \frac{\sqrt{9x^6-x}}{x^3+1}$ (d) $\lim_{x\to\infty} \sqrt{x^2+1}$
(e) $\lim_{x\to\infty} (e^{-x} + 2 \cdot \cos 3x)$ (f) $\lim_{x\to-\infty} \frac{1+x^6}{x^4+1}$ (g) $\lim_{x\to\infty} \frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$

Definition: A line $y = b$ is a **horizontal asymptote** of the graph of a function $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = b \text{ or } \lim_{x\to-\infty} f(x) = b.$$

Exercise 7. Find the horizontal asymptotes of the graph of $f(x) = \frac{x^3-2}{|x|^3+1}$. (Hass Sec 2.6 Ex 4)

Class Exercise 4. Find the horizontal and vertical asymptotes of each curve. (#42, 44, 46)
(a) $y = \frac{x^2+1}{2x^2-3x-3}$ (b) $y = \frac{1+x^4}{x^2-x^4}$ (c) $y = \frac{2e^x}{e^x-5}$

Homework: 1-9 ODD, 13-41 (every 4th), 47, 51, 55, 59