Section 2.3

Limit Laws

If L, M, c, and k are real numbers and $\lim_{x\to c} f(x) = \text{L}$ and $\lim_{x\to c} g(x) = M$, then

1. $\lim_{x\to c} (f(x) + g(x)) = L + M$ 2. $\lim_{x\to c} (f(x) - g(x)) = L$ - M
3. $\lim_{x\to c} (k \cdot f(x)) = k \cdot L$ 4. $\lim_{x\to c} (f(x) \cdot g(x)) = L \cdot M$
5. $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{M}$ 6. $\lim_{x\to c} [f(x)]^n = L^n$, n a positive integer
7. $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{L} = L^{1/n}$

Exercise 1. Use the observations $\lim_{x\to c} k = k$ and $\lim_{x\to c} x = c$ and the fundamental rules of limits to find the following limit: $\lim_{x\to c} (x^3 + 4x^2 - 3)$. (Hass Sec 2.2 Ex 5)

Class Exercise 1. Use the observations $\lim_{x\to c} k = k$ and $\lim_{x\to c} x = c$ and the fundamental rules of limits to find the following limit: $\lim_{x\to c} \frac{x^4+x^2-1}{x^2+5}$.

Class Exercise 2. Use the observations $\lim_{x\to c} k = k$ and $\lim_{x\to c} x = c$ and the fundamental rules of limits to find the following limit: $\lim_{x\to 2} \sqrt{4x^2 - 3}$.

<u>Theorem</u>: If $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots\ldots\ldots\ldots + a_0$, then

$$\lim_{x\to c} P(x) = P(c) = a_n c^n + a_{n-1}c^{n-1} + \ldots\ldots\ldots + a_0.$$

<u>Theorem</u>: If $P(x)$ and $Q(x)$ are polynomials and $Q(c) \neq 0$, then

$$\lim_{x\to c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}.$$

Exercise 2. Find $\lim_{x\to 3} \frac{5x^2-2x+1}{4x^3-7}$. (Swok Sec 2.3 Ex 5)

Class Exercise 3. Evaluate $\lim_{x\to 2} \frac{3x^2-4x}{5x^3-36}$. (Swok Sec 2.3 Ex 3)

Exercise 3. Find the following limit: $\lim_{x\to 2} \frac{x^2-6x+8}{x^2-4}$. (Briggs Sec 2.3 Ex 6a)

Exercise 4. Find the following limit: $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$. (Briggs Sec 2.3 Ex 6b)

Exercise 5. Evaluate:

$$\lim_{x\to 0} \frac{\sqrt{x^2+100}-10}{x^2}.$$ (Hass Sec 2.2 Ex 9)

Class Exercise 4. Evaluate the limit, if it exists. (#12-32 even)
(a) $\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4}$. (b) $\lim_{x\to -1} \frac{x^2-4x}{x^2-3x-4}$. (c) $\lim_{x\to -1} \frac{2x^2+3x+1}{x^2-2x-3}$.
(d) $\lim_{h\to 0} \frac{(2+h)^3-8}{h}$. (e) $\lim_{t\to 1} \frac{t^4-1}{t^3-1}$. (f) $\lim_{u\to 2} \frac{\sqrt{4u+1}-3}{u-2}$.
(g) $\lim_{x\to -1} \frac{x^2+2x+1}{x^4-1}$. (h) $\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$. (i) $\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h}$.
(j) $\lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4}$. (k) $\lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$.

<u>Squeeze Theorem</u>: If $g(x) \leq f(x) \leq h(x)$ for all $x \neq c$ for some interval about c, and

$$\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L,$$

then

$$\lim_{x\to c} f(x) = L.$$

Exercise 6. Use the Squeeze Theorem to find $\lim_{x\to 0} x^2 \sin \frac{1}{x^2}$. (Skow Sec 2.3 Ex 8)

Exercise 7. Given that $1 - \frac{x^2}{4} \leq u(x) \leq 1 + \frac{x^2}{4}$, find $\lim_{x\to 0} u(x)$. (Hass Sec 2.2 Ex 10)

Class Exercise 5. If $2x \leq g(x) \leq x^4 - x^2 + 2$ for all x, evaluate $\lim_{x\to 1} g(x)$. (#38)

Class Exercise 6. Prove that $\lim_{x\to 0^+} \sqrt{x}\, e^{\sin(\pi/x)}$. (#40)

Homework: 1, 3-19 (every 4th), 21-33 (every 4th), 39, 41