Section 2.2

Exercise 1. How does $f(x) = \frac{x^2-1}{x-1}$ behave near $x = 1$? (Hass Sec 2.2 Ex 1)

Definition: Suppose $f(x)$ is defined on an open interval about c, *except possibly at c itself.* If $f(x)$ is arbitrarily close to L as we like for all x sufficiently close to c, we say that f approaches the **limit** L as x approaches c, and write

$$\lim_{x \to c} f(x) = \text{L},$$

which is read "the limit of $f(x)$ as x approaches c is L."

For any $\lim_{x \to c}$, we are not looking for what happens at $x = c$, but what happens as x approaches c.

Definition: Sometimes the values of a function f tend to different limits as x approaches a number c from opposite sides. When this happens, we call the limit of f as x approaches from the right the **right-hand limit** of f at c and the limit as x approaches c from the left the **left-hand limit** of f at c.

$\lim_{x \to c^+} f(x) =$ the limit of f as x approaches c from the right

$\lim_{x \to c^-} f(x) =$ the limit of f as x approaches c from the left

Fact: $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

Exercise 2. Find $\lim_{x \to 0} \frac{x}{|x|}$.

Definition: Let f be a function defined on both sides of a, except possibly at a itself.

(a) Then $\lim_{x \to a} f(x) = \infty$ means the values of $f(x)$ can be made arbitrarily large positive (as large as we please) by taking x sufficiently close to a, but not equal to a.

Exercise 3. Find $\lim_{x \to 0} \frac{1}{x^2}$.

(b) $\lim_{x \to a} f(x) = -\infty$ means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to a, but not equal to a.

Exercise 4. Find $\lim_{x \to 0} -\frac{1}{x^2}$.

Exercise 5. For the graph in my Canvas video for Exercise 5, find the following limits (SD Notes):
(a) $\lim_{x \to -2^-} f(x)$
(b) $\lim_{x \to -2^+} f(x)$ (c) $\lim_{x \to -2} f(x)$ (d) $\lim_{x \to -1^-} f(x)$ (e) $\lim_{x \to -1^+} f(x)$
(f) $\lim_{x \to -1} f(x)$ (g) $\lim_{x \to 0^-} f(x)$ (h) $\lim_{x \to 0^+} f(x)$ (i) $\lim_{x \to 0} f(x)$

Class Exercise 1. For the graph in my Canvas video for Exercise 5, find the following limits (SD Notes):
(a) $\lim_{x\to 1^-} f(x)$ (b) $\lim_{x\to 1^+} f(x)$ (c) $\lim_{x\to 1} f(x)$ (d) $\lim_{x\to 2^-} f(x)$ (e) $\lim_{x\to 2^+} f(x)$
(f) $\lim_{x\to 2} f(x)$ (g) $\lim_{x\to 3^-} f(x)$ (h) $\lim_{x\to 3^+} f(x)$ (i) $\lim_{x\to 3} f(x)$

Exercise 6. Sketch the graph of an example of a function f that satisfies all of the given conditions. (#16) (a) $\lim_{x\to 0} f(x) = 1$ (b) $\lim_{x\to 3^-} f(x) =$ -2
(c) $\lim_{x\to 3^+} f(x) = 2$ (d) f(0) = -2 (e) $f(3) = 1$

Class Exercise 2. Sketch the graph of an example of a function f that satisfies all of the given conditions. (#18) (a) $\lim_{x\to 0^-} f(x) = 2$ (b) $\lim_{x\to 0^+} f(x) = 0$ (c) $\lim_{x\to 4^-} f(x) = 3$
(d) $\lim_{x\to 4^+} f(x) = 0$ (e) $f(0) = 2$ (f) $f(4) = 1$

Exercise 7. Use a table to estimate the value of the limit: $\lim_{x\to 0}$ (tan $3x$)/(tan $5x$). (#24)

Class Exercise 3. Use a table of values to estimate the value of the limit: $\lim_{x\to 0} \frac{9^x - 5^x}{x}$. (#26)

Exercise 8. Show that the limits don't exist: $\lim_{x\to 0} \frac{1}{x}$ (Swok Sec 2.1 Ex 3)

Exercise 9. Find $\lim_{x\to -3^-} \frac{x+2}{x+3}$. (#30)

Class Exercise 4. Find: (a) $\lim_{x\to 5^-} \frac{e^x}{(x-5)^3}$ (b) $\lim_{x\to \pi^-} \cot x$ (c) $\lim_{x\to 2^-} \frac{x^2-2x}{x^2-4x+4}$ (#32, 34, 36)

__**Definition**__: The line $x = a$ is called a __**vertical asymptote**__ of $y = f(x)$ if at least <u>one</u> of the following is true: $\lim_{x\to a} f(x) = \infty$, $\lim_{x\to a^-} f(x) = \infty$, $\lim_{x\to a^+} f(x) = \infty$, $\lim_{x\to a} f(x) = -\infty$, $\lim_{x\to a^+} f(x) = -\infty$.

Exercise 10. Find the vertical asymptotes of the function: $y = \frac{x^2+1}{3x-2x^2}$. (#38)

Class Exercise 5. Find the vertical asymptotes of the function: $y = \frac{1}{x^2-5x+6}$.

Homework: 5-11 ODD, 17-21 ODD, 27-41 ODD