Section 2.1

What is a tangent? A tangent to a curve is a line that touches the curve.

For a circle, a tangent line touches it at exactly one point. However, for complicated curves a tangent can touch at more than one point.

Definition: A **secant line** is a line that cuts (intersects) a curve more than once.

Exercise 1. Find the equation of the tangent line to the curve $y = x^3$ at the point $P(1, 1)$. We must find the slope of the line first.

(a) Fill in the following table:

P	Q	m_{PQ}
(1,1)	(2,8)	
(1,1)	(1.5,1.5^3)	
(1,1)	(1.1,1.1^3)	
(1,1)	(1.01,1.01^3)	
(1,1)	(1.001,1.001^3)	
(1,1)	(0,0)	
(1,1)	(0.5,0.5^3)	
(1,1)	(0.9,0.9^3)	
(1,1)	(0.99,0.99^3)	
(1,1)	(0.999,0.999^3)	

(b) What is your conjecture for the slope of the tangent line?
(c) Write an equation of the tangent line.
(d) Sketch the graph and the tangent line.

Velocity Problem

What is the average velocity? The average velocity = (change in position)/(time elapsed).

Exercise 2. The displacement (in meters) of an object moving in a straight line is given by: $s = 1 + 2t + \frac{1}{4}t^2$, where t is measured in seconds.

(a) Find the average velocity over each time period.

Interval	$[1, 3]$	$[1, 2]$	$[1, 1.5]$	$[1, 1.1]$
Average Velocity				

Definition: **Instantaneous velocity** is the velocity of an object in motion at a specific point of time.
(b) Estimate the instantaneous velocity when $t = 1$.
(c) Find the instantaneous velocity at $t = a$. Show algebraic work.
(d) Sketch the displacement function and interpret the average velocities from part (a) as the slopes of the secant lines.
(e) What is the interpretation of the instantaneous velocity from part (b)?

Class Exercise 1. A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after t minutes. When the data in the table are graphed, the slope of the tangent line represents the heart rate in beats per minute.

t	36	38	40	42	44
Heartbeats	2530	2661	2806	2948	3080

The monitor estimates this value by calculating the slope of a secant line. Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with the given values of f.
(a) $t = 36$ and $t = 42$

(b) t = 38 and t = 42
(c) t = 40 and t = 42
(d) t = 42 and t = 44

What are your conclusions? (#2)

Class Exercise 2. The point $P(0.5, 0)$ lies on the curve y = cos πx.
(a) If Q is the point (x, cosπx), use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x:

(i) 0 (ii) 0.4 (iii) 0.49 (iv) 0.499 (v) 1 (vi) 0.6 (vii) 0.51 (viii) 0.501

(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(0.5, 0)$.

(c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(0, 5, 0)$.

(d) Sketch the curve, two of the secant lines, and the tangent line. (#4)

Class Exercise 3. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, the height in meters t seconds later is given by y = 10t - 1.86t^2.

(a) Find the average velocity over the given time intervals:

(i) $[1, 2]$, (ii) $[1, 1.5]$, (iii) $[1, 1.1]$, (iv) $[1, 1.01]$, (v) $[1, 1.001]$.

(b) Estimate the instantaneous velocity when t = 1. (#6)

Class Exercise 4. The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion s = 2 sin πt + 3 cos πt, where t is measured in seconds.

(a) Find the average velocity during each time period: (i) $[1, 2]$, (ii) $[1, 1.1]$, (iii) $[1, 1.01]$, (iv) $[1, 1.001]$.
(b) Estimate the instantaneous velocity of the particle when t = 1. (#8)

Homework: 1-9 ODD