Section 1.4

Rules for Exponents

If $a > 0$ and $b > 0$, the following hold for all real numbers x and y.

1. $a^x \cdot a^y = a^{x+y}$ 2. $\frac{a^x}{a^y} = a^{x-y}$ 3. $(a^x)^y = (a^y)^x = a^{xy}$
4. $a^x \cdot b^x = (ab)^x$ 5. $(\frac{a}{b})^x = \frac{a^x}{b^x}$

Exercise 1. Simplify: (a) $8^{4/3}$ (#2(a)) (b) $x(3x^2)^3$ (#2(b))

Class Exercise 1. Simplify: (a) $b^8(2b)^4$ (#3(a)) (b) $\frac{(6y^3)^4}{2y^5}$ (#3(b))

Class Exercise 2. Simplify: (a) $\frac{x^{2n}x^{3n-1}}{x^{n+2}}$ (#4(a)) (b) $\frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}$ (#4(b))

Definition: Let a be a positive real number other than 1. The function

$$f(x) = a^x$$

is the **exponential function with base a**.

Exercise 2. Graph $f(x) = 2^x$.

Class Exercise 3. Graph (a) $g(x) = 3^x$ (b) $h(x) = (\frac{1}{2})^x$.

Exercise 3. Find the exponential function $f(x) = Ca^x$ that passes though $(-1, 3)$ and $(1, \frac{4}{3})$. (#22)

Class Exercise 4. Find the exponential function $g(x) = Ca^x$ that passes through $(0, 3)$ and $(1, 6)$.

Class Exercise 5. Find the exponential function $g(x) = Ca^x$ that passes through $(1, 12)$ and $(2, 48)$.

Exercise 4. Suppose you are offered a job that lasts one month. Which of the following methods of payment do you prefer? (#24)
I. One million dollars at the end of the month.
II. One cent the first day of the month, two cents on the second day, four cents on the third day, and, in general, 2^{n-1} cents on the nth day.

Exercise 5. A bacterial culture starts with 500 bacteria and doubles in size every half hour. (#30)
(a) How many bacteria are there after 3 hours?
(b) How many bacteria are there after t hours?
(c) How many bacteria are there after 40 minutes?

The natural exponential function - Of all possible bases for an exponential function, the most useful in calculus as well as in real life, is the base 'e' exponential function called the natural exponential function. Observe, 'e' on your calculator and $e \approx 2.7183$ to 4 decimal places. The graph of $f(x) = e^x$ resembles the graph of $g(x) = 2^x$. However, $f(x) = e^x$ crosses the y-axis with a slope of 1! You will learn more about this in Chapter 3.

Class Exercise 6. Find the domain of $g(t) = \sin(e^{-t})$. (#20(a))

Class Exercise 7. Using a graph (graphing calculator!) to estimate the values of x such that $e^x > 1{,}000{,}000{,}000$. (#28)

Homework: 1, 3, 7, 11, 15, 19, 23