Section 1.3

Vertical and Horizontal Shifts

Class Exercise 1. Graph the following functions on the same set of axes
(a) $f_1(x) = x^2$. (b) $f_2(x) = x^2 + 1$. (c) $f_3(x) = x^2 - 1$.
(d) $f_4(x) = (x - 1)^2$. (e) $f_5(x) = (x + 1)^2$.

Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.
1. Vertical shift c units *upward*: $h(x) = f(x) + c$
2. Vertical shift c units *downward*: $h(x) = f(x) - c$
3. Horizontal shift c units to the *right*: $h(x) = f(x - c)$
4. Horizontal shift c units to the *left*: $h(x) = f(x + c)$

Class Exercise 2. Graph the following functions on the same set of axes using the rules above:
(a) $f_1(x) = \sin x$ (b) $f_2(x) = \sin(x - \pi/2)$ (c) $f_3(x) = \sin(x + \pi)$.

Nonrigid Transformations

Class Exercise 3. Graph the following functions on the same set of axes:
(a) $f_1(x) = \sin x$. (b) $f_2(x) = \sin 2x$. (c) $f_3(x) = \sin \frac{x}{2}$.
(d) $f_4(x) = 2 \sin x$. (e) $f_5(x) = \frac{1}{2} \sin x$.

Horizontal shifts, vertical shifts, and reflections are **rigid transformations** because the basic shape of the graph is unchanged. These transformations change only the *position* of the graph in the coordinate plane. **Nonrigid transformations** are those that cause a *distortion*- a change in the shape of the original graph. For instance, a nonrigid transformation of the graph of $y = f(x)$ is represented by $g(x) = cf(x)$, where the transformation is a **vertical stretch** if $c > 1$ and a **vertical shrink** if $0 < c < 1$. Another nonrigid transformation of the graph $y = f(x)$ is represented by $h(x) = f(cx)$, where the transformation is a **horizontal shrink** if $c > 1$ and a **horizontal stretch** if $0 < c < 1$.

Class Exercise 4. Graph the following functions on the same set of axes:
(a) $f_1(x) = \sqrt{x}$ (b) $f_2(x) = 2\sqrt{x}$ (c) $f_3(x) = \frac{1}{3}\sqrt{x}$

Reflections

Class Exercise 5. Graph $f_1(x) = \sqrt{x}$ and $f_2(x) = \sqrt{-x}$ on the same set of axes.

Class Exercise 6. Graph $g_1(x) = x^2$ and $g_2(x) = -x^2$ on the same set of axes.

Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.
1. Reflection in the x-axis: $h(x) = -f(x)$.
2. Reflection in the y-axis: $h(x) = f(-x)$.

Class Exercise 7. Graph $f_1(x) = x^3$ and $f_2(x) = -x^3$ on the same set of axes.

Class Exercise 8. Graph $f_1(x) = \tan x$ and $f_2(x) = \tan(-x)$ on the same set of axes.

Exercise 1. Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions and then applying the appropriate transformations. (#10, 12, 14)
(a) $y = (x - 1)^3$ (b) $y = x^2 + 6x + 4$ (c) $y = 4 \sin 3x$

Class Exercise 9. Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions and then applying the appropriate transformations. (#16, 18, 20, 22, 24)
(a) $y = \frac{2}{x} - 2$ (b) $y = 1 - 2\sqrt{x + 3}$ (c) $y = |x| - 2$ (d) $y = \frac{1}{4}\tan(x - \pi/4)$

Combination of Functions

If f and g are functions, we define the **sum** $f + g$, **difference** $f - g$, **product** fg, and quotient f/g as follows:
$(f + g)(x) = f(x) + g(x) \quad (f - g)(x) = f(x) - g(x)$
$(fg)(x) = f(x)g(x) \quad (\frac{f}{g})(x) = \frac{f(x)}{g(x)}$

The domain of $f + g$, $f - g$, and fg is the *intersection* of the domain of f and g - that is, the numbers that are *common* to both domains. The domain of f/g consists of all numbers x in the intersection such that $g(x) \neq 0$.

Exercise 2. For $f(x) = \sqrt{x+5}$ and $g(x) = \sqrt{x+5}$, find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $(f/g)(x)$ and find the domain of those functions.

Class Exercise 10. Find (a) $f + g$, (b) $f - g$, (c) fg, and (d) f/g and state their domains.
(i) $f(x) = \sqrt{3-x}$ and (ii) $g(x) = \sqrt{x^2-1}$ (#30)

Composite Functions

Definition: The **composite function** $f \circ g$ of f and g is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $(f \circ g)$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

Exercise 3. Find the domain of the composition $(f \circ g)(x)$ for the functions given by

$$f(x) = x^2 - 9 \text{ and } g(x) = \sqrt{9-x^2}. \text{ (Larson Sec 1.6 Ex 7)}$$

Exercise 4. Given $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - x - 6$, find (a) $g \circ f$ and (b) $g \circ g$, and their domains. (Briggs Sec 1.1 Ex 6)

Class Exercise 11. Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$ and their domains. (#32, 34, 36)
(i) $f(x) = x - 2$ and $g(x) = x^2 + 3x + 4$
(ii) $f(x) = \sqrt{x}$ and $g(x) = \sqrt[3]{1-x}$
(iii) $f(x) = \frac{x}{1+x}$ and $g(x) = \sin 2x$

Homework: 3, 7, 13, 19, 23, 29-49 (every 4th), 53, 59, 63, 67