Section 1.2

Definition: A **mathematical model** is a mathematical description (often by means of a function or an equation) of a real world phenomenon such as the size of a population, the demand for a product, the speed of a falling object, the concentration of a product in a chemical reaction, the life expectancy of a person at birth, or the cost of emissions reductions.

In this section, we discuss different mathematical models used to formulate real-life problems.

Linear Models

Definition: A function of the form $f(x) = mx + b$, for constants m and b, is called a **linear function**.

Exercise 1. Recent studies indicate that the average surface temperature of the earth has been rising steadily. Some scientists have modeled the temperature by the linear function $T = 0.02t + 8.50$, where T is temperature in $^{\circ}$C and t represents years since 1900.
(a) What do the slope and T-intercept represent?
(b) Use the equation to predict the average global surface temperature in 2100. (#10)

Class Exercise 1. The manager of a weekend flea market knows from past experience that if he charges x dollars for a rental space at the market, then the number y spaces he can rent is given by the equation $y = 200$ - $4x$.
(a) Sketch a graph of this linear function. (Remember that the rental charge per space and the number of spaces rented can't be negative quantities.)
(b) What do the slope, the y-intercept, and the x-intercept of the graph represent? (#12)

Class Exercise 2. Jason leaves Detroit at 2:00 PM and drives at a constant speed west along I-96. He passes Ann Arbor, 40 miles from Detroit, at 2:50 PM.
(a) Express the distance traveled in terms of the time elapsed.
(b) Draw the graph of the equation in part (a).
(c) What is the slope of this line? What does it represent? (#14)

Polynomial Models

Definition: Let n be a nonnegative integer and let a_n, a_{n-1},........., a_2, a_1, a_0 be real numbers with $a_n \neq 0$. The function given by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_2 x^2 + a_1 x + a_0$$

is called a **polynomial function of x with degree n**.

Definition: Let a, b, and c be real numbers with $a \neq 0$. The function given by

$$f(x) = ax^2 + bx + c$$

is called a **quadratic function**.

Exercise 2. The polynomial function

$$f(x) = -49x^3 + 806x^2 + 3776x + 2503$$

models the number of AIDS cases diagnosed in the United States, $f(x)$, x years after 1983, where $0 \leq x \leq 8$. Find $f(6)$ and describe what this means in practical terms.

Class Exercise 3. The polynomial function

$$f(x) = -2212x^2 + 57,575x + 107,896$$

models the cumulative number of deaths from AIDS in the United States, $f(x)$, x years after 1900. Find and interpret $f(10)$.

Exponential Models

The exponential functions $y = e^{kx}$, where k is a nonzero constant, are frequently used for modeling exponential growth or decay. The function $y = y_0 e^{kx}$ is a model for **exponential growth** if $k > 0$ and a model for **exponential decay** if $k < 0$. Here y_0 represents a constant.

Exercise 3. Investment companies often use the model $y = Pe^{rt}$ in calculating the growth of an investment. Use this model to track the growth of \$100 invested in 2014 at an annual interest rate of 5.5%. (Hass Sec 1.5 Ex 3)

Logarithmic Models

Definition: For $x > 0$, $a > 0$, and $a \neq 1$,

$$y = \log_a x \text{ if and only if } x = a^y.$$

The function given by

$$f(x) = \log_a x$$

is called the logarithmic function with base a.

Exercise 4. Students in a mathematics class were given an exam and then retested monthly with an equivalent exam. The average scores for the class are given by the human memory model $f(t)$ = 80 - 17 log $(t + 1)$, $0 \leq t \leq 12$, where t is the time in months.
(a) What was the average score on the original exam $(t = 0)$?
(b) What was the average score after 4 months?
(c) What was the average score after 10 months?

Other Functions

Definition: A function of the form $f(x) = x^a$, where a is a constant, is called a **power function**. If $a = 1/n$, where n is a positive integer, then $f(x)$ is a root function.

Definition: **Rational functions** are of the form $f(x) = p(x)/q(x)$, where p and q are polynomials.

Definition: **Algebraic functions** are constructed using the operations of algebra: addition, subtraction, multiplication, division, and roots.

Definition: The **trigonometric functions** are sin x, cos x, tan x, cot x, sec x, and csc x; they are fundamental to mathematics and many areas of application.

Exercise 5. Classify each function as a power function, root function, polynomial function, rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.
(a) $f(x) = \log_2 x$ and (b) $g(x) = \sqrt[3]{x}$ (#2)

Class Exercise 4. Classify each function as a power function, root function, polynomial function, rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.
(a) $h(x) = x^2(2 - x^3)$ (b) $u(t) = 1 - 1.1t + 2.54t^2$
(c) $v(t) = 5^t$ (d) $w(\theta) = \sin \theta \cos^2 \theta$ (#2)

Homework: 3, 7, 13, 17, 21, 29, 33, 35