Section 1.1

Definition: A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B.
Definition: The set A is called the **domain** of f, and the **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain.

A function can be though of as a machine. For example, a machine that makes pies (e.g. if you put in apples, you get an apple pie; you don't also get a peach pie or a cherry pie).

The four possible ways to represent a function are: verbally (by a description in words), numerically (by a table of values), visually (by a graph), and algebraically (by an explicit formula).

Here are some real-world examples of functions (Hass):
1) The temperature at which water boils depends on the elevation above sea level (the boiling pot drops as you ascend).
2) The interest paid on a cash investment depends on the length of time the investment is held.
3) The area of a circle depends on the radius of the circle.
4) The distance an object travels at constant speed along a straight-line path depends on the elapsed time.

Exercise 1. Temperature readings T (in °F) were recorded every two hours from midnight to 2:00 PM in Phoenix on September 10, 2008. The time t was measured in hours from midnight

t	0	2	4	6	8	10	12	14
T	82	75	74	75	84	90	93	94

(a) Use the readings to sketch a rough graph of T as a function of t.

(b) Use your graph to estimate the temperature at 9:00 A.M. (#24)

Exercise 2. Sketch a rough graph of the number of hours of daylight as a function of the time of year. (#16)

Class Exercise 1. Sketch a rough graph of the market value of a new car as function of time for a period of 20 years. Assume the car is well maintained. (#18)

Class Exercise 2. You place a frozen pie in an oven and bake it for an hour. Then you take it out and let it cool before eating it. Describe how the temperature of the pie changes as time passes. Then sketch a rough graph of the temperature of the pie as a function of time. (#20)

Definition: The expression

$$\frac{f(a+h)-f(a)}{h}$$

is called a **difference quotient** and it occurs frequently in calculus.

Exercise 3. Simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$, for $f(x) = 3x^2 - x$. (Briggs Sec 1 Ex 9)

Class Exercise 3. Evaluate the difference quotient

$$\frac{f(a+h)-f(a)}{h}$$

for the function $f(x) = x^3$. (#28)

Class Exercise 4. Evaluate the difference quotient

$$f(x) = \frac{f(x)-f(1)}{x-1}$$

for the function $f(x) = \frac{x+3}{x+1}$. (#30)

We learned from algebra that you cannot take the square root of a negative number and you are not allowed to divide by 0.

Exercise 4. Let $h(x) = \sqrt{4 - x^2}$. Find the domain and range and sketch the graph. (#38)

Exercise 5. Suppose $f(x) = \frac{2x^3-5}{x^2+x-6}$. Find the domain of the function. (#32)

Class Exercise 5. Suppose $g(t) = \sqrt{3-t} - \sqrt{2+t}$. Find the domain of the function. (#34)

Class Exercise 6. Suppose $f(u) = \frac{u+1}{1+\frac{1}{u+1}}$. Find the domain of the function. (#36)

<u>**Definition**</u>: (i) A function f is called <u>increasing</u> on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I (i.e. the graph of f <u>rises</u> from left to right on the interval I.)

(ii) It is called <u>decreasing</u> on I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I (i.e. the graph <u>falls</u> from left to right on the interval I.)

Exercise 6. Find the intervals where the function $f(x) = x^2$ is increasing and decreasing.

Class Exercise 7. Find the intervals where the function $f(x) = -x^2$ is increasing and decreasing.

Some functions are defined by different formulas in different parts of their domains. Such functions are called <u>piecewise defined functions</u>.

Exercise 7. An example of a piecewise defined function is $f(x) = |x|$. Graph the function.

Exercise 8. Graph the function: $g(x) = \begin{cases} x^2 - 2 & x < -2 \\ 8x - 5 & x \geq -2 \end{cases}$

Class Exercise 8. Graph the function: $g(x) = \begin{cases} 3 - \frac{1}{2}x & x \leq 2 \\ 2x - 5 & x > 2 \end{cases}$ (#48)

Class Exercise 9. Graph the function: $g(x) = \begin{cases} x + 9 & x < -3 \\ -2x & |x| \leq 3 \\ -6 & x > 3 \end{cases}$ (#50)

<u>**Definition**</u>: (a) If a function f satisfies $f(-x) = f(x)$ for every number x in its domain, then f is called an <u>even function</u>. A nice example of such a function is $f(x) = \cos x$.

Graphically, all even functions are symmetric with respect to the y-axis.

(b) If a function f satisfied $f(-x) = -f(x)$ for every number x in its domain, then f is called an <u>odd function</u>. A nice example of such a function is $f(x) = \sin x$.

Graphically, all odd functions are symmetric with respect to the origin.

Exercise 9. Determine whether $f(x) = \frac{x^2}{x^4+1}$ is even, odd, or neither. (#74)

Class Exercise 10. Determine whether $f(x) = x|x|$ is even, odd, or neither. (#76)

Class Exercise 11. Determine whether $f(x) = 1 + 3x^3$ - x^5 is even, odd, or neither. (#78)

Exercise 10. A rectangle has area 16 m^2. Express the perimeter of the rectangle as a function of the length of one of its sides. (#58)

Class Exercise 12. Express the surface area of a cube as a function of its volume. (#60)

Class Exercise 13. A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, express the area A of the window as a function of the width x of the window. (#62)

Homework: 3, 15-75 (every 4th)