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Scalars : just numbers

examples

mass	(SI unit kg)	electric potential	(SI unit Volts)
length	(SI unit m)	energy	(SI unit Joule)
volume	(SI unit L)		
temperature	(SI unit K)	charge	(SI unit Coulomb)
time	(SI unit sec)		$1e \leftrightarrow -1.602 \times 10^{-19}$ C
speed	(SI unit m/s)		

Vectors : direction, magnitude

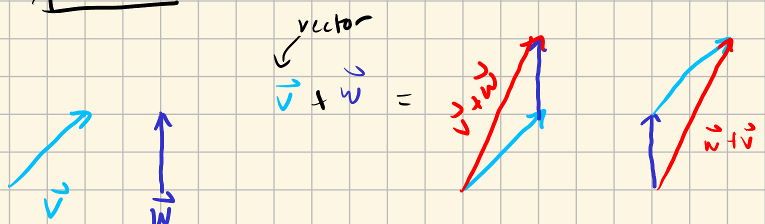
examples

velocity (SI unit m/s) → momentum (kg m/s)

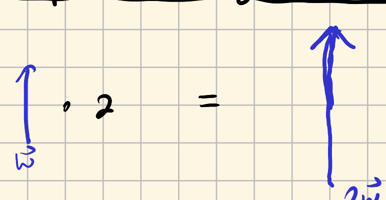
acceleration (SI unit m/s²) → force (N = kg m/s²

electric field, magnetic field

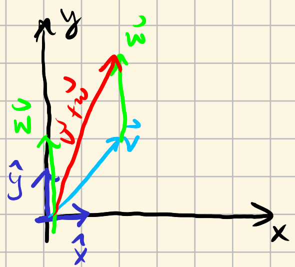
Addition



Multiplication by a scalar



Unit Vector: a vector with magnitude 1 $(\hat{x})$



$$\vec{v} = v_x \hat{x} + v_y \hat{y}$$
$$= 2\hat{x} + 2\hat{y}$$

Addition again

$$\vec{w} = 2\hat{y}$$
$$\vec{v} + \vec{w} = (2\hat{x} + 2\hat{y}) + 2\hat{y}$$
$$= 2\hat{x} + 4\hat{y}$$

Multiplication by scalar

$$2 \cdot \vec{v} = 2 \cdot (2\hat{x} + 2\hat{y}) = 4\hat{x} + 4\hat{y}$$

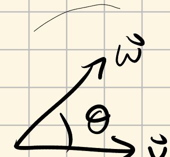
Dot Product: 2 vectors input $\longrightarrow$ 1 scalar output

$$\left.\begin{aligned} \vec{v} &= v_x \hat{x} + v_y \hat{y} + v_z \hat{z} \\ \vec{w} &= w_x \hat{x} + w_y \hat{y} + w_z \hat{z} \end{aligned}\right\} \quad \vec{v} \cdot \vec{w} = v_x w_x + v_y w_y + v_z w_z$$

$$\left.\begin{aligned} \vec{v} &= 2\hat{x} + 2\hat{y} \\ \vec{w} &= 2\hat{y} \end{aligned}\right\} \quad \vec{v} \cdot \vec{w} = (2 \cdot 0) + (2 \cdot 2) = 4$$

$\|\vec{v}\|$ $\rightarrow$ Magnitude of a vector: $\sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{\vec{v} \cdot \vec{v}}$

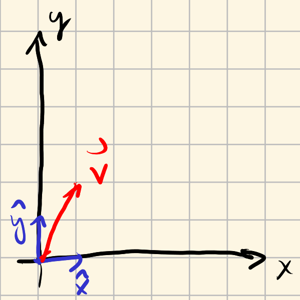
$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$
$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos\theta$$



$$\cos\theta = \begin{cases} \theta = 0 & \rightarrow 1 \quad \vec{v}, \vec{w} \text{ point in same direction} \\ \theta = \pi/2 & \rightarrow 0 \quad \vec{v}, \vec{w} \text{ perpendicular} \\ \theta = \pi & \rightarrow -1 \quad \vec{v}, \vec{w} \text{ point in opposite direction} \\ \theta = 3\pi/2 & \rightarrow 0 \quad \vec{v}, \vec{w} \text{ perpendicular} \end{cases}$$

$$\boxed{\frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} = \cos\theta}$$

$$\underbrace{\|\hat{x}\|^2}_{1} = \hat{x} \cdot \hat{x}$$



$$\vec{v} = v_x \hat{x} + v_y \hat{y}$$

$$\vec{v} \cdot \hat{x} = v_x \underbrace{(\hat{x} \cdot \hat{x})}_{1} + v_y \underbrace{(\hat{y} \cdot \hat{x})}_{0}$$

$$\boxed{\vec{v} \cdot \hat{x} = v_x} \quad + \cancel{v_y (0)}$$

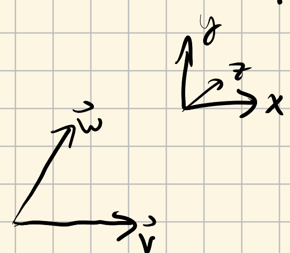
$$\boxed{\vec{v} \cdot \hat{y} = v_y}$$

ex:

$W = \vec{F} \cdot \vec{d}$ → electric potential is in terms of dot product

(X) Cross Product : 2 vectors input ⟶ 1 vector output

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix} = (v_y w_z - v_z w_y)\hat{x} - (v_x w_z - v_z w_x)\hat{y} + (v_x w_y - v_y w_x)\hat{z}$$



$\vec{v} \times \vec{w} = \vec{u}$

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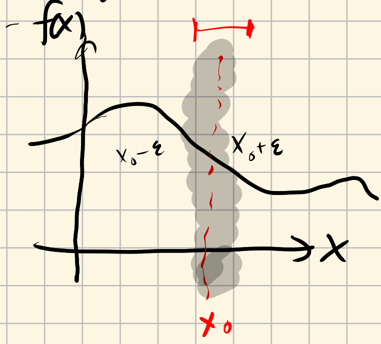
$$\boxed{\vec{l} = m\vec{v} \times \vec{r}}$$

Force on a charged particle moving through a magnetic field.

Taylor Series



find a simple approximation for $f(x)$ in $(x_0-\varepsilon,\ x_0+\varepsilon)$

Polynomial

$$f(x) \approx c_0 + c_1\underbrace{(x-x_0)}_{0} + c_2\underbrace{(x-x_0)^2}_{0} + c_3\underbrace{(x-x_0)^3}_{0} + \dots$$

$$f(x_0) = c_0$$

$$\rightarrow \frac{df}{dx} = c_1 + 2c_2\underbrace{(x-x_0)}_{0} + 3c_3\underbrace{(x-x_0)^2}_{0} + \dots$$

$$\left.\frac{df}{dx}\right|_{x=x_0} = c_1$$

$$\frac{d^2f}{dx^2} = 2c_2 + \underbrace{6c_3(x-x_0)}_{0} + \dots$$

$$\left.\frac{d^2f}{dx^2}\right|_{x=x_0} = 2c_2$$

$$\boxed{\begin{aligned} c_0 &= f(x_0) \\ c_1 &= f'(x_0) = \left.\frac{df}{dx}\right|_{x=x_0} \\ c_2 &= \frac{1}{2} f''(x_0) = \frac{1}{2}\left.\frac{d^2f}{dx^2}\right|_{x=x_0} \\ &\vdots \\ c_n &= \frac{1}{n!}\left.\frac{d^nf}{dx^n}\right|_{x=x_0} \end{aligned}}$$

$f(x) = e^x$

$x_0 = 0$

$c_0 = e^0 = 1$

$c_1 = f'(x_0) = e^{x_0} = e^0 = 1$

$c_2 = \frac{1}{2}(1) = \frac{1}{2}$

$c_3 = \frac{1}{6} \cdot 1 \quad \dots$

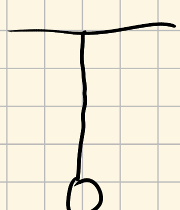
$e^x \rightarrow x = 5$

$$\sum_{n=0}^{\infty} \frac{e^5}{n!}(x-5)^n$$

$x = 3 \qquad n = 2$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

(Taylor series around $x=0$)



$F \sim mg\sin\theta$

$$m\frac{d^2x}{dt^2} = -kx$$

$$I\frac{d^2\theta}{dt^2} \sim -k\theta$$

$$\sin\theta = \boxed{\theta} - \frac{1}{6}\theta^3 + \dots$$