Date: 01/19/2022

1) Scalars : examples

2) Vectors :
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   - addition again
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   - dot product
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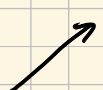
3) Taylor Series

Scalars

Just numbers : examples

energy	(SI unit Joules)		electric potential
length	(SI unit meter)		
time	(SI unit sec)		
mass	(. . . kg)		
temp	(. . . K)		

Vector



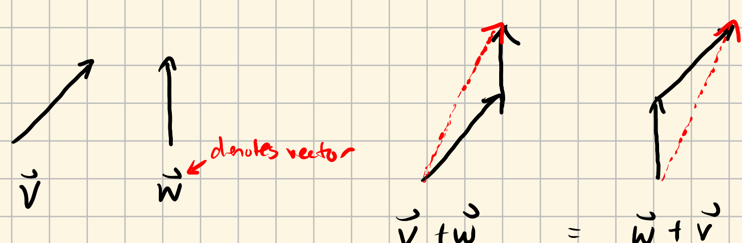
examples

velocity   momentum

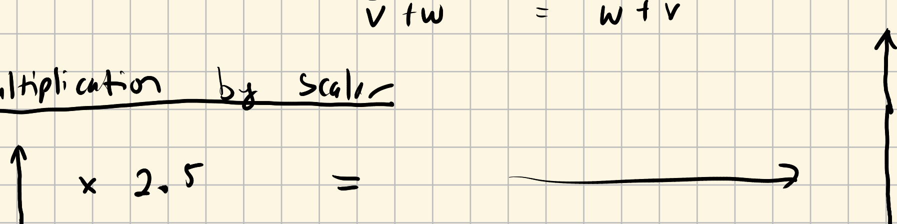
force   acceleration

electric field
magnetic field   magnetic potential

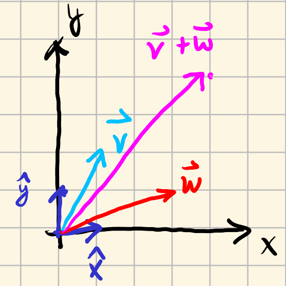
Addition



Multiplication by scalar



Unit vector: vector with magnitude 1



$$\vec{v} = 1\hat{x} + 2\hat{y}$$

$$\vec{w} = 3\hat{x} + 1\hat{y}$$

$$\vec{v} + \vec{w} = (1\hat{x} + 3\hat{y}) + (3\hat{x} + 1\hat{y})$$

$$= 4\hat{x} + 4\hat{y}$$

$$5 \cdot \vec{v} = 5 \cdot 1\hat{x} + 5 \cdot 2\hat{y}$$

$$= 5\hat{x} + 10\hat{y}$$

Dot Product: an operation on 2 vectors that returns a scalar

$$\vec{v} \cdot \vec{w} = \#$$

$$\vec{v} \cdot \vec{w} = v_x w_x + v_y w_y + v_z w_z$$

$$\vec{v} \cdot \vec{w} = (1\hat{x} + 3\hat{y}) \cdot (3\hat{x} + 1\hat{y})$$

$$= (1 \cdot 3) + (3 \cdot 1) + (0 \cdot 0)$$

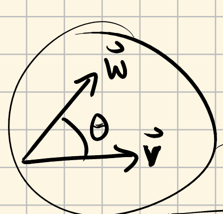
$$= 6$$

Length of a vector: $\sqrt{v_x^2 + v_y^2 + v_z^2}$ ← Pythagorean Thm

length of $\vec{v} = \sqrt{\vec{v} \cdot \vec{v}}$

Denote the length of a vector $\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$

→ $\vec{v} \cdot \vec{w} = v_x w_x + v_y w_y + v_z w_z = \underline{\|\vec{v}\| \|\vec{w}\| \cos(\theta)}$

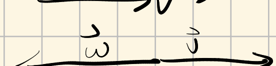


$$\cos(\theta) = \begin{cases} \theta = 0 & 1 \leftarrow \text{point in same direction} \\ \theta = \pi/2 & 0 \leftarrow \text{perpendicular} \\ \theta = \pi & -1 \leftarrow \text{opposite directions} \\ \theta = 3\pi/2 & 0 \leftarrow \text{perpendicular} \end{cases}$$

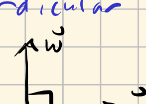
$$\boxed{\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|}}$$

$\theta = 0$

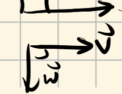
$\theta = \pi$

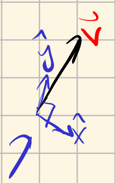


$\theta = \pi/2$



$\theta = 3\pi/2$





$\vec{v} = v_x \hat{x} + v_y \hat{y}$

$\hat{x} \cdot \hat{x} = \|\vec{x}\|^2$ ← 1 (unit vector)

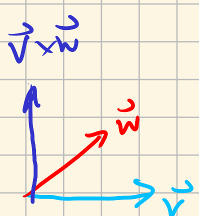
$\vec{v} \cdot \hat{x} = v_x \underbrace{\hat{x} \cdot \hat{x}}_{1} + v_y \underbrace{\hat{y} \cdot \hat{x}}_{0}$

$\vec{v} \cdot \hat{x} = v_x$

$\vec{v} \cdot \hat{y} = v_y$

(X) Cross Product : two vectors ⟶ another vector

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix} = (v_y w_z - v_z w_y)\hat{x} - (v_x w_z - v_z w_x)\hat{y} + (v_x w_y - v_y w_x)\hat{z}$$

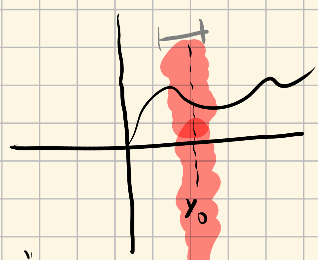


$\underbrace{\|\vec{v} \times \vec{w}\|}_{\geq 0} = \|\vec{v}\| \|\vec{w}\| \sin\theta$

$$\sin\theta = \begin{cases} \theta = 0 & 0 \\ \theta = \pi/2 & 1 \\ \theta = \pi & 0 \\ \sim\theta = 3\pi/2 \sim & \sim -1 \sim \end{cases}$$

Taylor Series

$f(x)$ ← approximate $f(x)$ in some neighborhood of x_0 $(x_0 - \varepsilon, x_0 + \varepsilon)$



Polynomials : is there a polynomial of x that approximates $f(x)$ in the interval $(x_0 - \varepsilon, x_0 + \varepsilon)$

"ansatz"

$$f(x) = c_0 + c_1(x - x_0) + c_2(x - x_0)^2 + c_3(x - x_0)^3 + \ldots$$

$$f(x_0) = c_0 \rightarrow \frac{d}{dx} f(x)\Big|_{x = x_0} = \overset{0}{0} + \boxed{c_1} + 2c_2 \overset{0}{(x - x_0)} + 3c_3 \overset{0}{(x - x_0)^2} + \ldots$$

$$\frac{d^2}{dx^2} f(x)\Big|_{x=x_0} = 2c_2 + 6c_3(x-x_0) + \dots$$

(→ 0)

$$c_0 = f(x_0)$$

$$c_1 = f'(x_0) = \frac{d}{dx} f(x)\Big|_{x=x_0}$$

$$c_2 = \frac{1}{2} f''(x_0) = \frac{1}{2}\frac{d^2}{dx^2} f(x)\Big|_{x=x_0}$$

$$c_n = \frac{1}{n!}\frac{d^n}{dx^n} f(x)\Big|_{x=x_0}$$

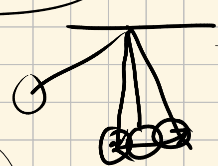
$$m\frac{d^2x}{dt^2} = -kx$$

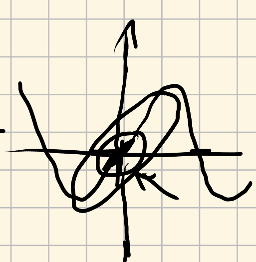
(hopefully)

$$f(x) \simeq c_0 + c_1(x-x_0) + c_2(x-x_0)^2 + \dots$$

$$mg\sin\theta \rightarrow \frac{d^2\theta}{dt^2} = -k\theta$$

$$\sin\theta = \theta + \dots$$





$$x = \frac{\ell_A}{\ell_B} < 1$$