Physics 7B Sections 103/107 (Bordel)

Date: March 14–16, 2022

Administrative Notes:

Wednesday is discussion again, no actual lab.
Held remotely again at this zoom link.

Lab grades posted.

MT2 covers material from eletric point charges up to and including DC circuits.

Potential

$V(x,y,z)$ — Scalar (SI Volt)

Suppose you have $V(x,y,z)$.

1) Electric Field

$V(x,y,z) \rightarrow \vec{E}(x,y,z)$

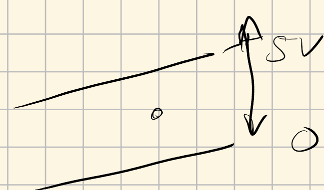
$$\vec{E} = -\vec{\nabla} V(x,y,z)$$

1D): $\vec{E}(x) = -\dfrac{dV}{dx}\,\hat{i}$

3D): $\vec{E}(x,y,z) = -\dfrac{\partial V}{\partial x}\,\hat{i} - \dfrac{\partial V}{\partial y}\,\hat{j} - \dfrac{\partial V}{\partial z}\,\hat{k}$

$V(x,y,z) \rightarrow \vec{E}(x,y,z)$

$V(x,y,z) + V_0$



1V = 6V

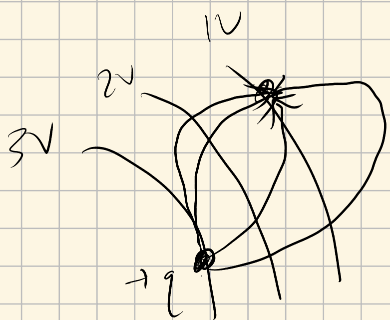
+

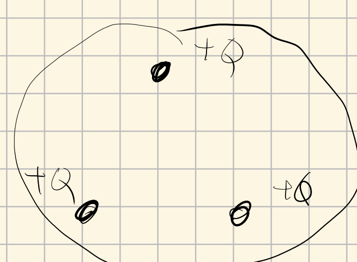
1V = 1V

2) $W = q\Delta V = q(V_f - V_i)$

$\dfrac{\Delta V}{\Delta x}$

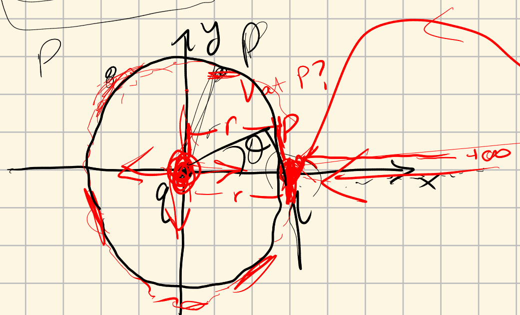
path-independent





Electric field $\longrightarrow$ potential

$$\boxed{\vec{E}(x,y,z)} \longrightarrow V(x,y,z)$$



What is the potential at a distance r from q relative to ∞?

∞ ↑ $V=0$

at ∞, $V=0$

$$\vec{E}\cdot d\vec{l}$$

$$\Delta V = -\int_{+\infty}^{r} k\frac{q}{x^2}\,\hat{i}\cdot dx\,\hat{i}$$

$V(r)$

$$= -\left[-k\frac{q}{x}\right]_{+\infty}^{r} = k\frac{q}{r} - \cancel{k\frac{q}{\infty}}^{\,0} = k\frac{q}{r} \quad \checkmark$$

$$V(r) - \underbrace{V(\infty)}_{=0} = k\frac{q}{r}$$

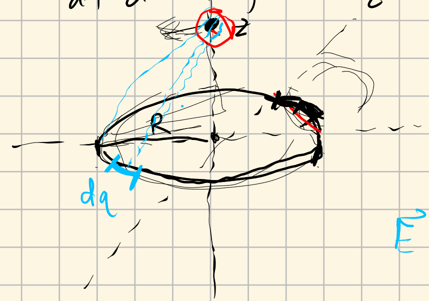
$$\boxed{V(r) = k\,q/r}$$ for a point charge

$$\vec{E} \rightarrow V$$

Problem # 1

Potential of a point charge is $k\frac{q}{r}$.

Suppose you have a circle of radius R carrying uniform linear charge density λ. What is the electric field at a height z above the center?



Approach #1:

$$\vec{E} = \int d\vec{E} = \cancel{\int d\vec{E}_x}^{\,0} + \cancel{\int d\vec{E}_y}^{\,0} + \boxed{\int d\vec{E}_z}$$

$|E| \cdot$ (trig function)

Approach #2:

Potential Also obeys superposition.

$$V_{total}(z) = \int dV$$

$$= \int k\frac{dq}{\sqrt{R^2+z^2}} = \frac{k}{\sqrt{R^2+z^2}}\underbrace{\int dq}_{Q_{total} = \lambda 2\pi R}$$

$$= \frac{k\,2\pi R\lambda}{\sqrt{R^2+z^2}}$$

$V(z) \longrightarrow \vec{E}(z)$?

$$\vec{E}(z) = -\vec{\nabla}V(z) = -\cancel{\frac{\partial V}{\partial x}}^{\,0}\hat{\imath} - \cancel{\frac{\partial V}{\partial y}}^{\,0}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}$$

$$\vec{E}(z) = -\frac{dV}{dz}\hat{k} = -\frac{dV}{dz}\hat{k}$$

$$V(z) = k\frac{2\pi R\lambda}{\sqrt{R^2+z^2}}$$

$$\frac{dV}{dz} = k\frac{2\pi R\lambda}{(R^2+z^2)^{3/2}} \cdot \left(-\frac{1}{2}\right)(2z) = -k\frac{2\pi R\lambda z}{(R^2+z^2)^{3/2}}$$

$$\boxed{\vec{E}(z) = k\frac{2\pi R\lambda z}{(R^2+z^2)^{3/2}}\hat{k}}$$

When there is symmetry, and enough of it, you can probably use Gauss's law to find the field

But otherwise, you might want to find the potential first and then compute its gradient to find the field.

Potential : SI Volts $= \frac{N\cdot m}{C} = \frac{J}{C}$

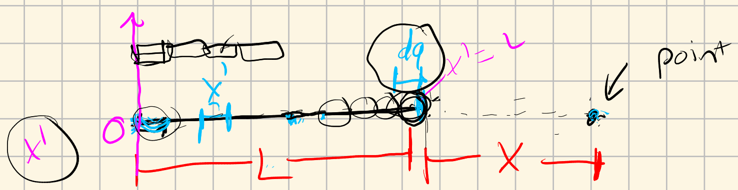
Electric Field : $N/C = V/m$

$(F = qE)$

$\left(E = -\frac{\partial V}{\partial x}\hat{\imath} \ldots\right)$

Problem

$$\sum_{i=0} \frac{q_i}{r_i^2}$$



A line of length L carries uniform linear charge density λ. What is the potential at a distance X from the end as shown above?

Assume $x > 0$.

$V(x)$?

1) $V(x) = \int dV$

$$= \int k \frac{dq}{x+L-x'} \qquad \int dq = \int_0^L \lambda \, dx' = \lambda L$$

$$= \int_0^L k \frac{\lambda \, dx'}{x+L-x'}$$

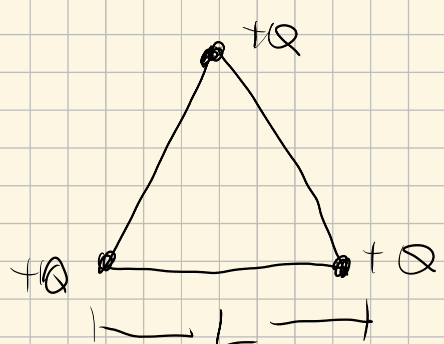
$$= k\lambda \left[-\ln(x+L-x') \right]_0^L$$

$$= k\lambda \left[-\ln(x) + \ln(x+L) \right]$$

$$= k\lambda \ln\left(\frac{x+L}{x} \right) \quad \checkmark$$

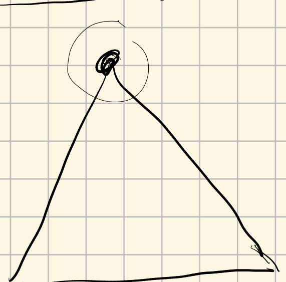
Problem

Consider 3 point charges all of value $+Q$. What is the work required to bring them from infinitely far apart to sit on the vertices of an equilateral triangle of side length L?



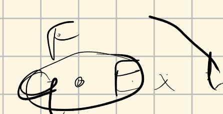
1st charge:

$W_1 = ?$



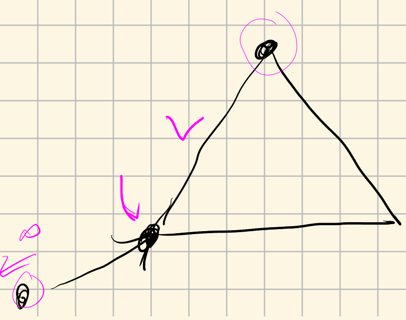
$$W = Q\Delta V = 0$$

2nd charge

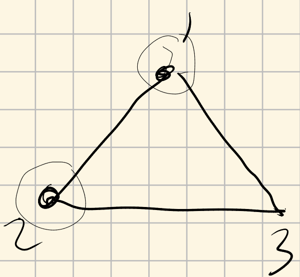


created by first charge

$$W_2 = Q\Delta V = Q(\underbrace{V_f}_{k\frac{Q}{L}} - \underbrace{V_i}_{0}) = k\frac{Q^2}{L}$$



3^{rd} charge



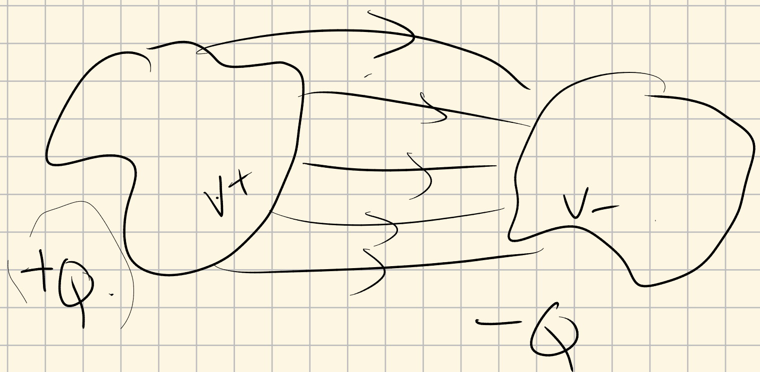
$$\left.\begin{array}{l} V_{13} = k\dfrac{Q}{L} \\ V_{23} = k\dfrac{Q}{L} \end{array}\right\} V_f = k\frac{2Q}{L}$$

$$V_i = 0$$

$$W_3 = Q\,(V_f - V_i) = k\frac{2Q^2}{L}$$

$$W_{total} = W_1 + W_2 + W_3 = 0 + k\frac{Q^2}{L} + k\frac{2Q^2}{2} = \boxed{3k\frac{Q^2}{L}}$$

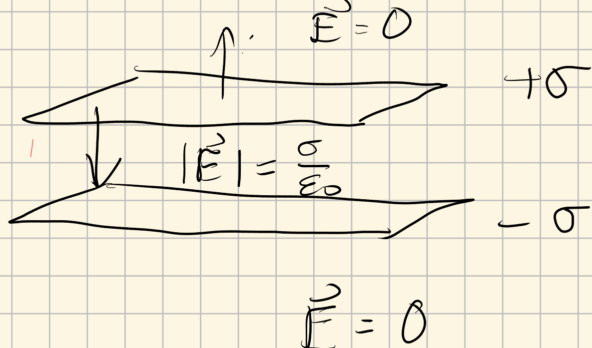
Capacitors

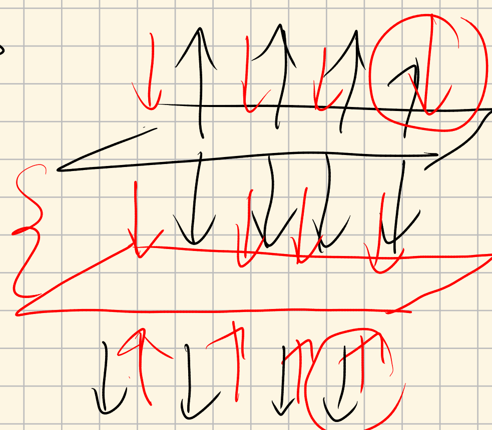


$$\frac{Q}{\Delta V} = C \qquad (F)$$

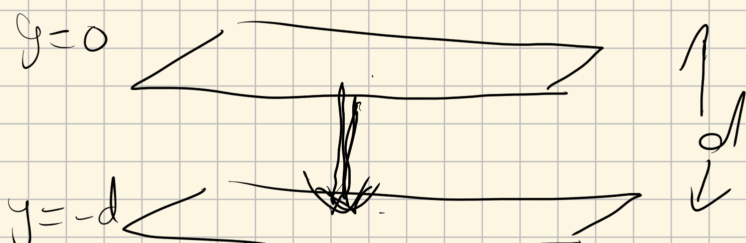
$\sigma > 0$

$$|\vec{E}| = \frac{\sigma}{2\varepsilon_0}$$



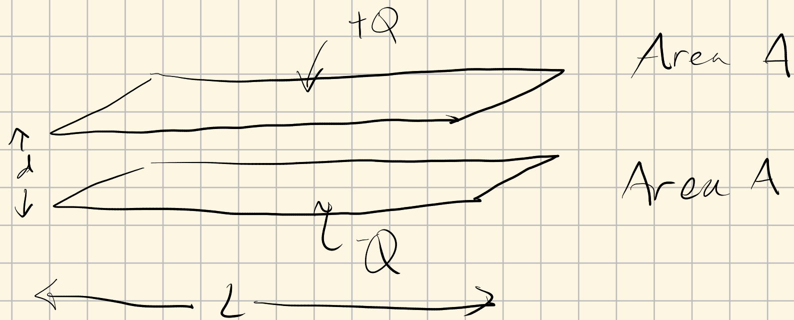


$$\frac{\sigma}{2\varepsilon_0}$$



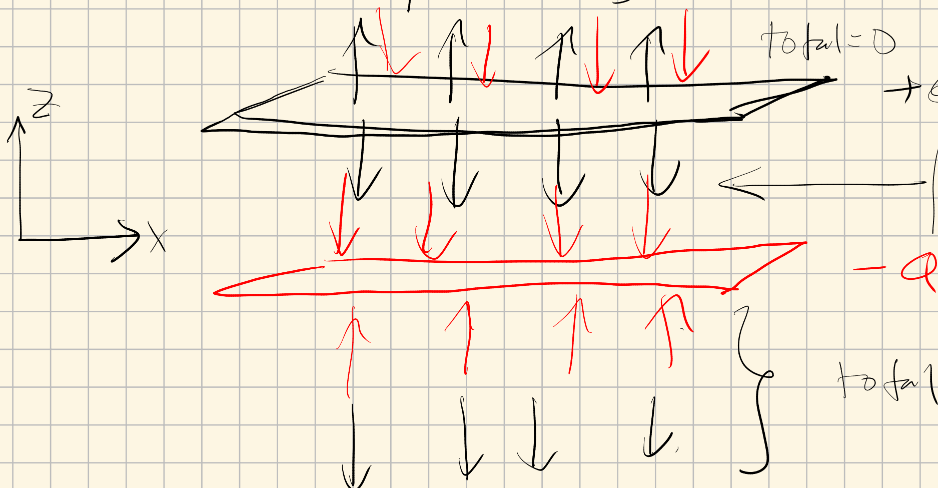
$$\vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{j} \qquad \Delta V = -\int \vec{E}\cdot d\vec{l} = -\int_0^{-d}\left(-\frac{\sigma}{\varepsilon_0}\hat{j}\right)(-dy\,\hat{j})$$

$$= \frac{\sigma}{\varepsilon_0} d$$



1) what does $\vec{E}$ look like?

when $d \ll L$, approximate this as 2 infinite sheets



total = 0

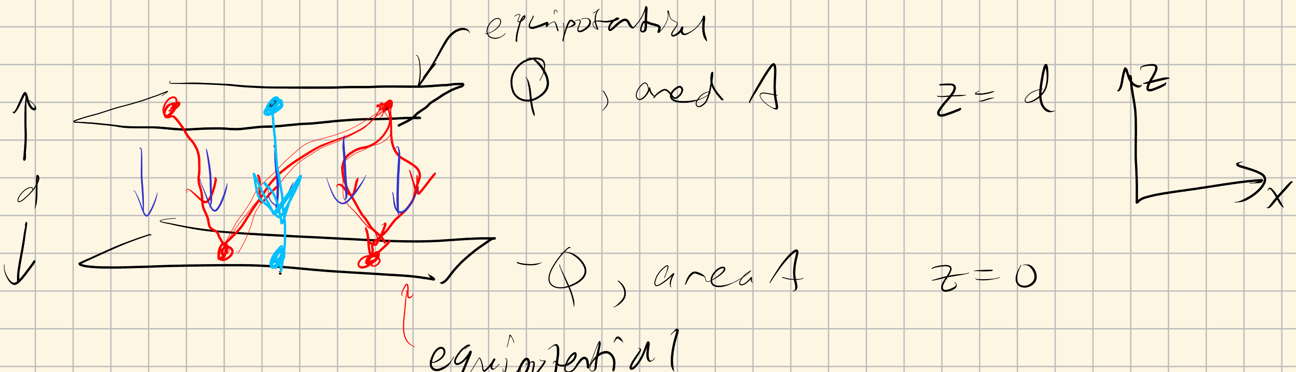
assume $+Q > 0$

$$\vec{E}_{total} = -\frac{\sigma}{2\varepsilon_0}\hat{k} - \frac{\sigma}{2\varepsilon_0}\hat{k}$$

$$= -\frac{\sigma}{\varepsilon_0}\hat{k}$$

electric field of ∞ sheet has constant $|E|$ everywhere

$\vec{E}$ field in between two oppositely charged infinite sheets will be $-\frac{\sigma}{\varepsilon_0}\hat{k}$



1) Imagine putting charge $+Q$ on one and $-Q$ on the other

2) Determine $\vec{E}$ between them

3) ΔV between the two conductors.

$$\vec{E} = -\vec{\nabla} V \longrightarrow \Delta V = -\int_a^b \vec{E} \cdot d\vec{l}$$

finite plate $\rightarrow$ infinite one

charge

Approx: finite plate of area A.

assume the charge is uniform ;

if I put $+Q$ on one plate and

$-Q$ on the other $\rightarrow$ infinite sheet.

$$\vec{E} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

$$\Delta V = -\int_d^0 \vec{E} \cdot d\vec{l} = -\frac{Q}{A\varepsilon_0}\int_d^0 dz = -\frac{Q}{A\varepsilon_0}(0-d)$$

$$= \frac{Q}{A\varepsilon_0} d$$

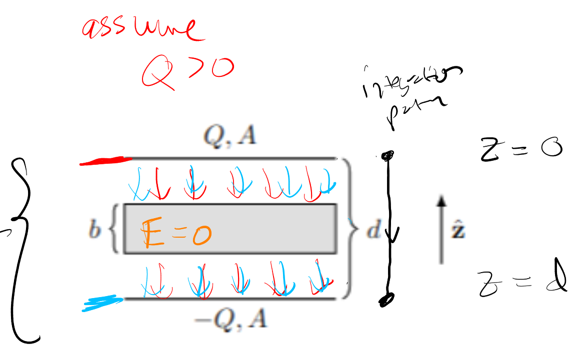
$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Q}{A\varepsilon_0} d} = \frac{A}{d}\varepsilon_0 \quad (F)$$

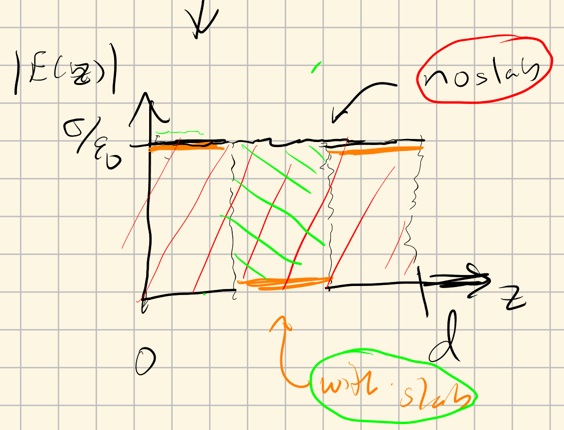
• Problem 3: A conductor inside a capacitor [3]

This is problem 7 of WS E6 in your course workbook.

Consider a parallel-plate capacitor with plates of area A separated by some small distance d. It is initially charged up to some charge Q, and then disconnected from the battery. A neutral conducting metal slab, also of area A and with width $b < d$, is then inserted between the plates as shown below. What is the capacitance of this configuration? (Hint: it may help to start by thinking about what the electric field looks like inside the capacitor.)

[1] We haven't specifically talked about conductors in any worksheet yet, but it's worth noting that conductors have some special properties. Most importantly, $\mathbf{E} = 0$ in the bulk of any conductor; any net charge on a conductor resides on the surface; and the surface of a conductor is always an equipotential. Review Section 21.9 in your textbook for more details [2].





$\Delta V \sim \frac{\sigma}{\varepsilon_0} d - \frac{\sigma}{\varepsilon_0} b$

ΔV

1) Imagine charge +Q on one conductor, −Q on the other
2) $\vec{E}$-field
3) ΔV between the 2 conductors
4) $C = Q/\Delta V$

$$\Delta V = -\int \vec{E}\cdot d\vec{l} = -\underbrace{\int_0^d |E(z)|\,dz}_{\text{area under the curve } |E(z)|} = -\left(\frac{\sigma}{\varepsilon_0}(d-b)\right)$$

$$\sigma = Q/A$$

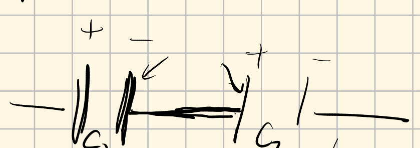
$$\downarrow$$

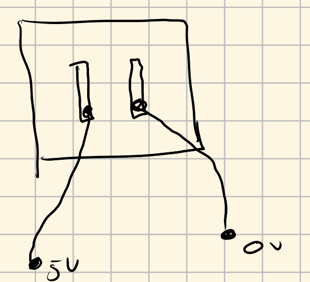
$$-\left(\frac{Q}{A\varepsilon_0}(d-b)\right)$$

$$\Delta V = -\left(\frac{Q}{A\varepsilon_0}(d-b)\right) \qquad b < d$$

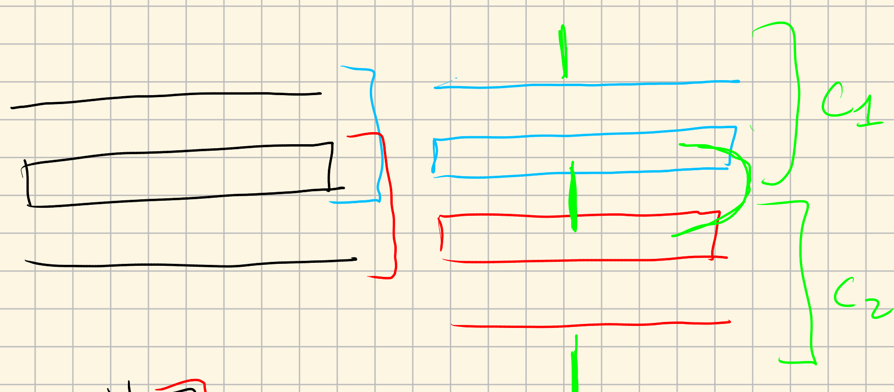
$$C = \frac{Q}{|\Delta V|} = \frac{Q}{\left|-\left(\frac{Q}{A\varepsilon_0}(d-b)\right)\right|} = \frac{A\varepsilon_0}{d-b}$$

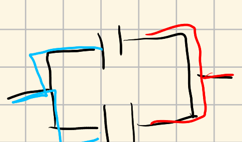
Capacitors in series





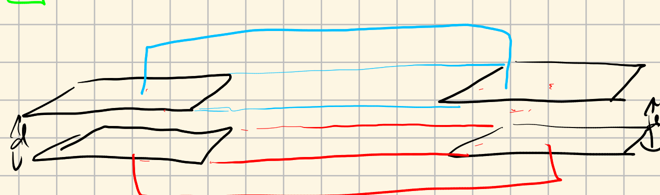
total cap? $\qquad \frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2}$



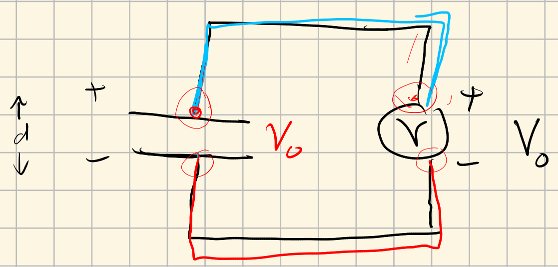


$$C_{tot} = C_1 + C_2$$

$$C = 2\frac{A}{d}\varepsilon_0$$



Suppose the capacitor has initial separation d, and is connected to voltage source (e.g. battery) and we pull the plates apart to distance $2d$.



$$Q = CV$$

What changes and by how much?

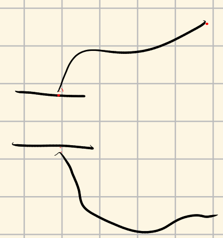
$$C = \varepsilon_0 \frac{A}{d}$$

$$Q \longrightarrow \frac{1}{2} Q$$

$$C \longrightarrow \frac{1}{2} C$$

$$V \longrightarrow V$$

Suppose we disconnect the battery <u>before</u> separating the plates to $2d$:



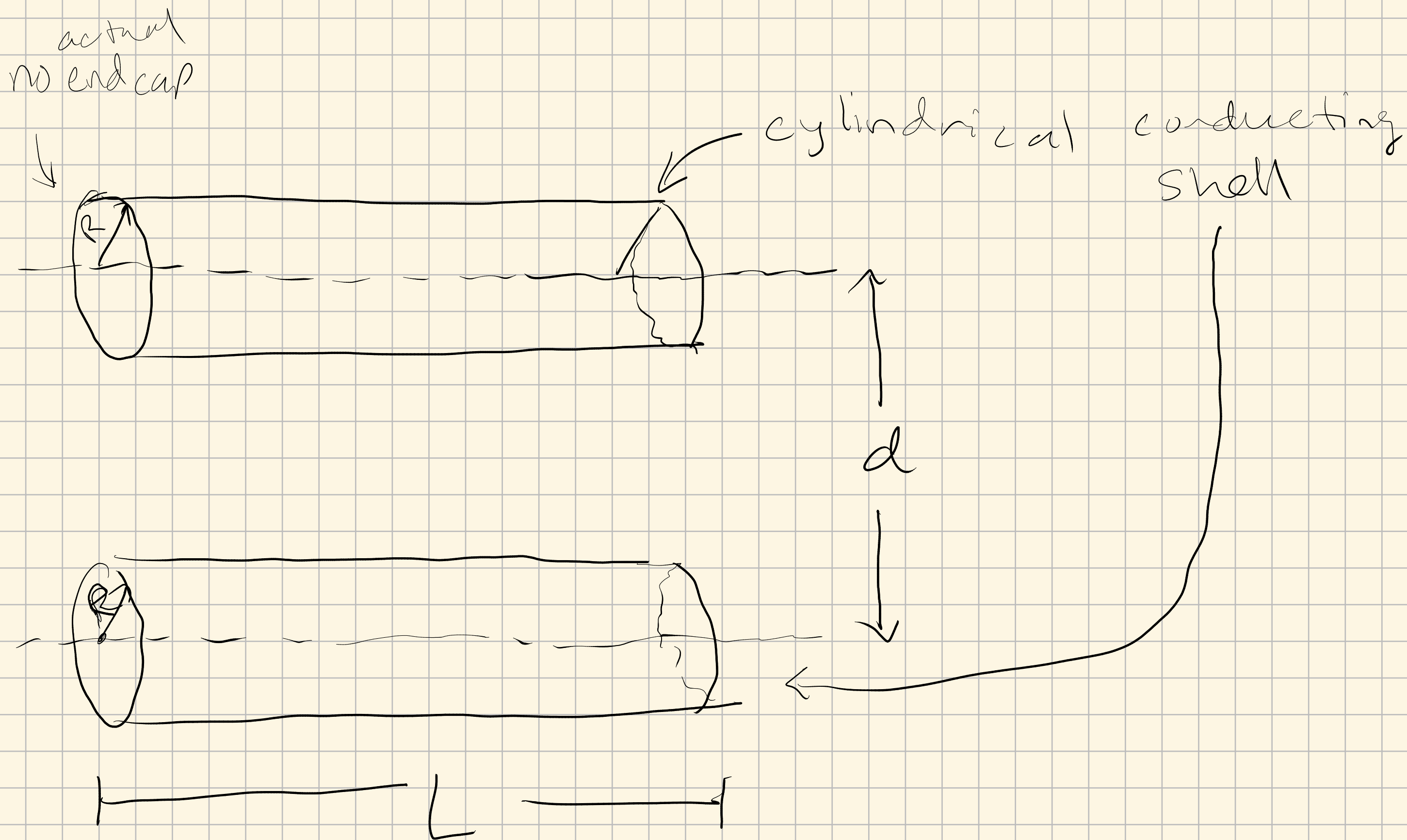
$$Q \longrightarrow Q$$

$$C \longrightarrow \frac{1}{2} C$$

$$V \longrightarrow 2V$$

$$C = \varepsilon_0 \frac{A}{d}$$

Problem



$R < d/2$

$d << L$

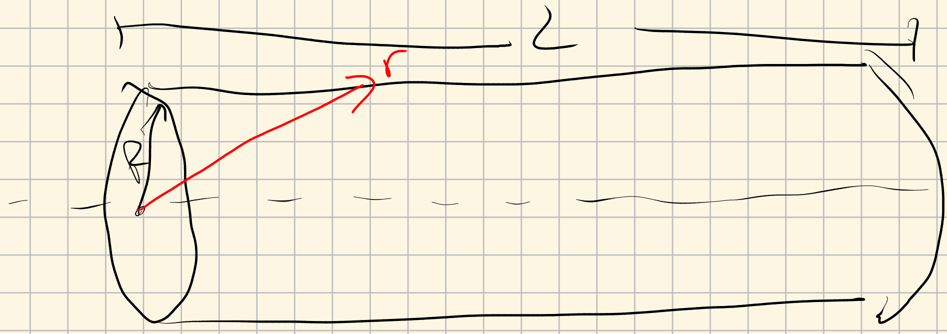
What is the capacitance of this configuration?

1. Imagine putting $+Q$ ....
2. $\vec{E}$-field

3. ΔV between surfaces of the conductors

4. C?

Electric field of one cylinder



top cylinder

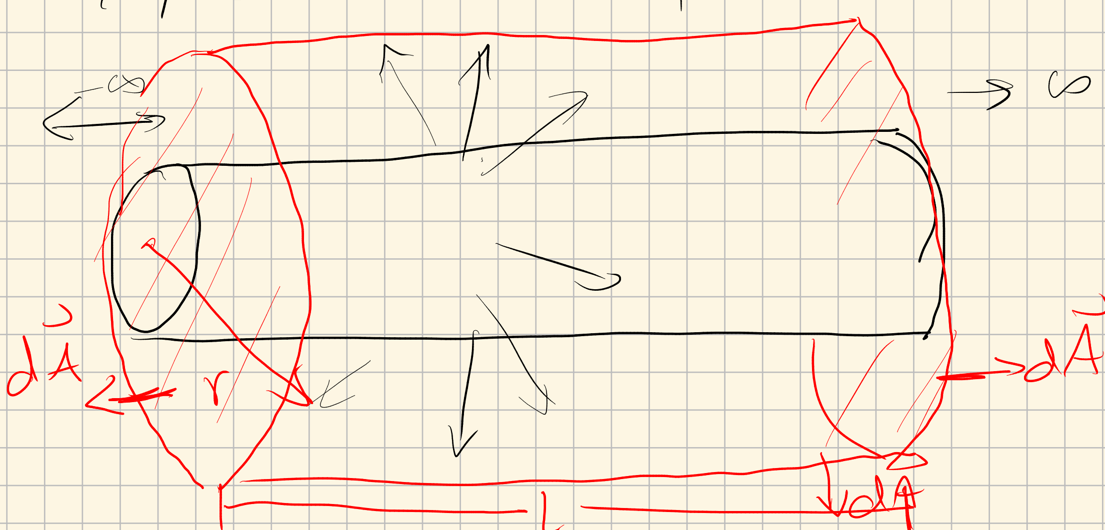
$L \gg d$

approx. ∞ - long

(for $\parallel$ plate, $L \gg d$)

$$\vec{E}_{total} = \vec{E}_{top} + \vec{E}_{bottom}$$

Gauss's Law for ∞ cylinder



$$\Phi = \int_{\substack{closed\\surface}} \vec{E} \cdot d\vec{A} = \int_{\substack{lateral\\surface}} \vec{E} \cdot d\vec{A} + \cancel{\int_{\substack{end\\caps}} \vec{E} \cdot d\vec{A}}$$

0

$$\Phi = \int_{\text{lateral surface}} \vec{E} \cdot d\vec{A} \simeq \int_{\text{lateral surface}} |E||dA|$$

$$= |E| \boxed{\int_{\text{lateral surface}} |dA|}$$

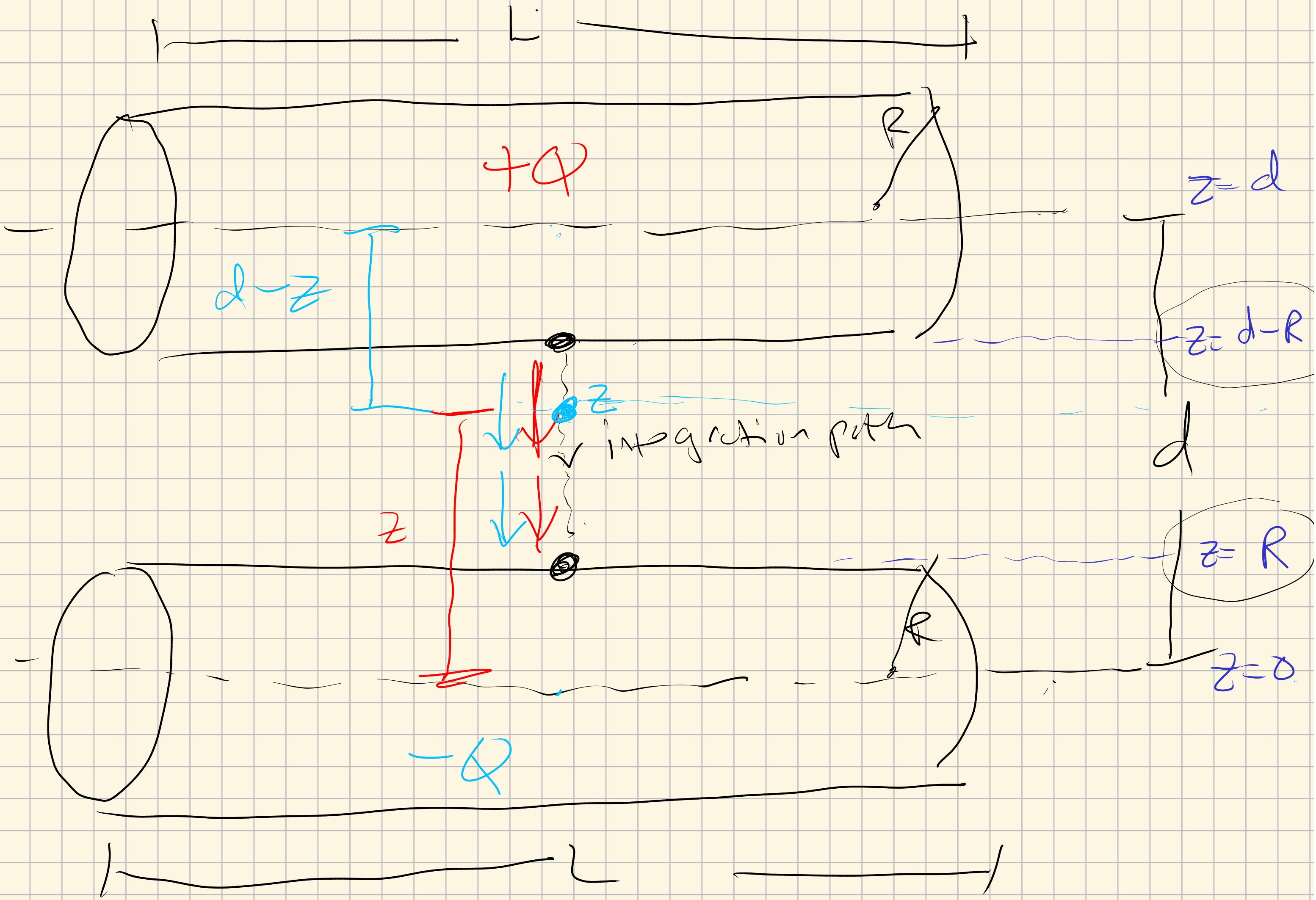
area of lateral surface

$$= |E| (2\pi r L)$$

$$\Phi = \frac{Q}{\varepsilon_0}$$

$$|E| 2\pi r L = \frac{Q}{\varepsilon_0}$$

$$|E| = \frac{Q}{\varepsilon_0} \frac{1}{2\pi \underbrace{r}_{\text{distance from the center of the cylinder}} L}$$



$$\vec{E}_{total} = \left(- \frac{Q}{2\pi (d-z) L \varepsilon_0} - \frac{Q}{2\pi z L \varepsilon_0} \right) \hat{k}$$

$$\Delta V = - \int_{z=d-R}^{z=R} \vec{E} \cdot d\vec{\ell} = - \int_{d-R}^{R} \left(- \frac{Q}{2\pi (d-z) L \varepsilon_0} - \frac{Q}{2\pi z L \varepsilon_0} \right) dz$$

$$= \frac{Q}{2\pi L \varepsilon_0} \int_{d-R}^{R} \left(\frac{1}{d-z} + \frac{1}{z} \right) dz$$

$$= \frac{Q}{2\pi L \varepsilon_0} \left[-\ln\left(\frac{d-R}{R}\right) + \ln\left(\frac{R}{d-R}\right) \right]$$

$$= \frac{Q}{2\pi L \varepsilon_0} \ln\left(\frac{R^2}{(d-R)^2}\right)$$

$$= \frac{Q}{\pi L \varepsilon_0} \ln\left(\frac{R}{d-R}\right)$$

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{\frac{Q}{\pi L \varepsilon_0} \ln\left(\frac{R}{d-R}\right)}$$

$$= \frac{\pi L \varepsilon_0}{-\ln\left(R/(d-R)\right)}$$

$R < d/2$

ex: $\dfrac{d/4}{d/2 - d/4}$

$$= \frac{\pi L \varepsilon_0}{\ln\left[\frac{d-R}{R}\right]}$$