Physics 7B Sections 103/107 (Bordel)

Date: March 14–16, 2022

Administrative Notes:

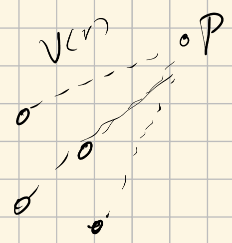
Wednesday is discussion again, no actual lab.
Held remotely again at this zoom link.

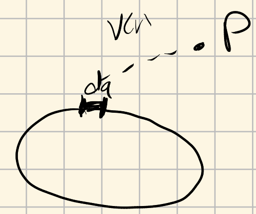
Lab grades posted.

MT2 covers material from eletric point charges up to and including DC circuits.

Potential

Scalar $\rightarrow$ obeys superposition





Adding scalars > adding fields/vectors

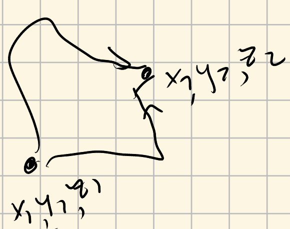
$\boxed{V(x,y,z)}$

$V(x,y,z) + 5V$

1) $V(x,y,z) \longrightarrow \vec{E}(x,y,z)$?

1D) $\vec{E}(x) = -\dfrac{dV}{dx}\hat{\imath}$

3D) $\vec{E}(x,y,z) = -\dfrac{\partial V}{\partial x}\hat{\imath} - \dfrac{\partial V}{\partial y}\hat{\jmath} - \dfrac{\partial V}{\partial z}\hat{k}$



2) Work required to move a point charge q from (x_1, y_1, z_1) to (x_2, y_2, z_2)

$$W = q\,\Delta V = q\left[V(x_2, y_2, z_2) - V(x_1, y_1, z_1)\right]$$

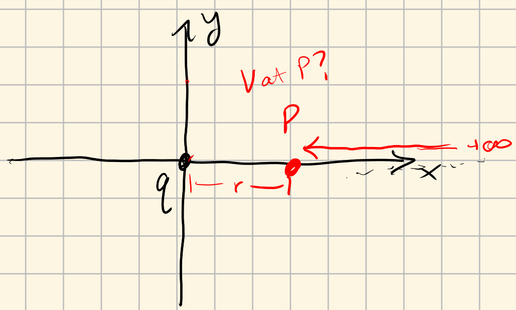
$\vec{\nabla} \times \vec{E} = 0 \Longleftrightarrow$ Electric field/force is conservative

1) $\vec{F} = q\vec{E}$ units of electric field: N/C

$\Delta V = -\int \vec{E} \cdot d\vec{\ell}$ units of potential: $\dfrac{N \cdot m}{C} = \dfrac{J}{C} = V$

Electric field $\longrightarrow$ potential

$\vec{E}(x,y,z) \longrightarrow V(x,y,z)$



What is the potential at a distance r from q relative to ∞?

at ∞, $V = 0$

$$\Delta V = -\int_{+\infty}^{r} k\frac{q}{x^2}\,\hat{\imath}\cdot d\vec{x}$$

$$= -\left[-k\frac{q}{x}\right]_{+\infty}^{r} = k\frac{q}{r} - \cancel{k\frac{q}{\infty}}^{\,0} = k\frac{q}{r}$$

$$V(r) - \underbrace{V(\infty)}_{=0} = k\frac{q}{r}$$

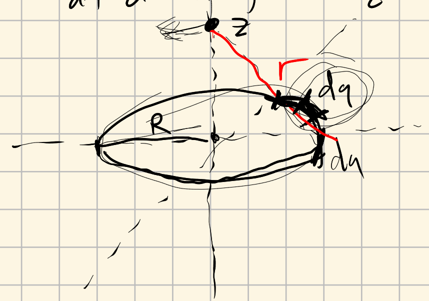
$$\boxed{V(r) = k\,q/r}$$ for a point charge

$\vec{E} \rightarrow V$

Problem # 1

Potential of a point charge is $k\frac{q}{r}$.

Suppose you have a circle of radius R carrying uniform linear charge density λ. What is the electric field at a height z above the center?



Approach #1

$$\vec{E} = \int d\vec{E} = \cancel{\int dE_x}^{0} + \cancel{\int dE_y}^{0} + \int dE_z$$

Approach #2

$$V(z) \rightarrow -\frac{dV}{dz}\hat{k}$$

1) Find potential at point z above center due to dq

2) Integrate over dq

$$V(z) = \int dV = \int k\frac{dq}{\sqrt{R^2+z^2}} = \int_0^R k\frac{\lambda 2\pi}{\sqrt{R^2+\cancel{z}}}\,dl$$

$$\int dq = Q_{total} = 2\pi R \cdot \lambda$$

$\lambda 2\pi dr$

$$= k\frac{2\pi\lambda}{\sqrt{R^2+z^2}}\int_0^R dl = k\frac{2\pi R\lambda}{\sqrt{R^2+z^2}}$$

$= R$

$$V(z) \longrightarrow \vec{E}(z) \ ?$$

$$\vec{E}(z) = -\cancel{\frac{\partial V}{\partial x}}^{\,0}\,\hat{\imath} - \cancel{\frac{\partial V}{\partial y}}^{\,0}\,\hat{\jmath} - \frac{\partial V}{\partial z}\,\hat{k}$$

$$\frac{dV}{dz} = k \frac{2\pi R \lambda}{(R^2+z^2)^{3/2}} \cdot \left(-\frac{1}{2}\right)(2z)$$

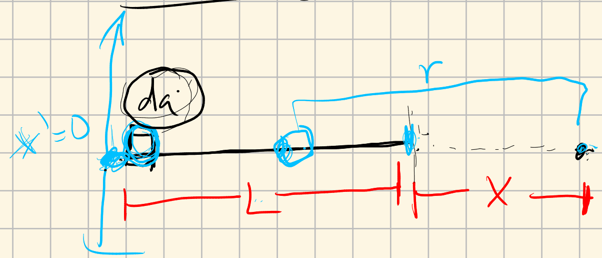
$$\vec{E}(z) = - \ -k \frac{2\pi R z \lambda}{(R^2+z^2)^{3/2}}\,\hat{k}$$

$$= k \frac{2\pi R z \lambda}{(R^2+z^2)^{3/2}}\,\hat{k} .$$

When there is symmetry, and enough of it, you can probably use Gauss's law to find the field

But otherwise, you might want to find the potential first and then compute its gradient to find the field.

Problem



A line of length L carries uniform linear charge density λ. What is the potential at a distance x from the end as shown above?

$V(x)$ Assume $x > 0$.

$$V(x) = \int dV = \int \frac{k\,dq}{(L-x')+x}$$

$$dq = \lambda\, dx'$$

$$V(x) = \int_0^L \frac{k\,\lambda\, dx'}{(L-x')+x} = k\lambda \int_0^L \frac{1}{(L-x')+x}\, dx'$$

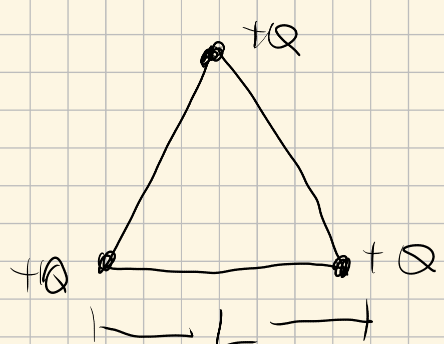
$$= k\lambda \Big[-\ln\left[(L-x')+x\right] \Big]_0^L$$

$$= k\lambda\left[-\ln x\right] - k\lambda\left[-\ln(L+x)\right]$$

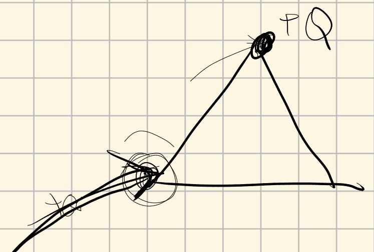
$$= k\lambda \ln\left(\frac{L+x}{x}\right)$$

Problem

Consider 3 point charges all of value $+Q$. What is the work required to bring them from infinitely far apart to sit on the vertices of an equilateral triangle of side length L?

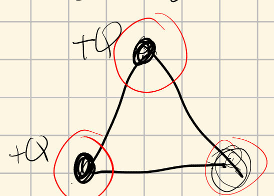


Work for first charge: $W = 0$



Second charge: $W = Q\,\Delta V = Q\left(k\frac{Q}{L} - 0\right)$

$$= k\frac{Q^2}{L}$$



Third charge: $W = Q\,\Delta V = Q\left(V_f - \overset{0}{V_i}\right) = Q\left(k\frac{Q}{L} + k\frac{Q}{L} - 0\right)$

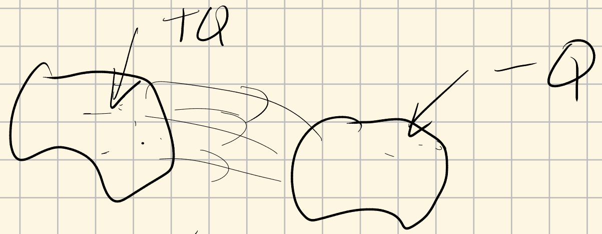
$$= 2k\frac{Q^2}{L}$$

$$W_{total} = \cancel{W_1}^{\,0} + W_2 + W_3$$

$$= 3K\frac{q^2}{L}$$

Capacitors

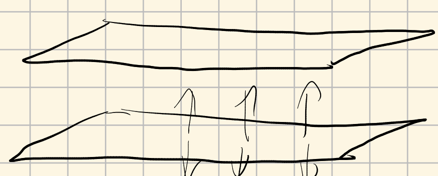
Two pieces of metal at different voltages



$\Delta V \neq 0$

$$\frac{Q}{\Delta V} = C \qquad (F)$$

$$Q = C\,\Delta V$$

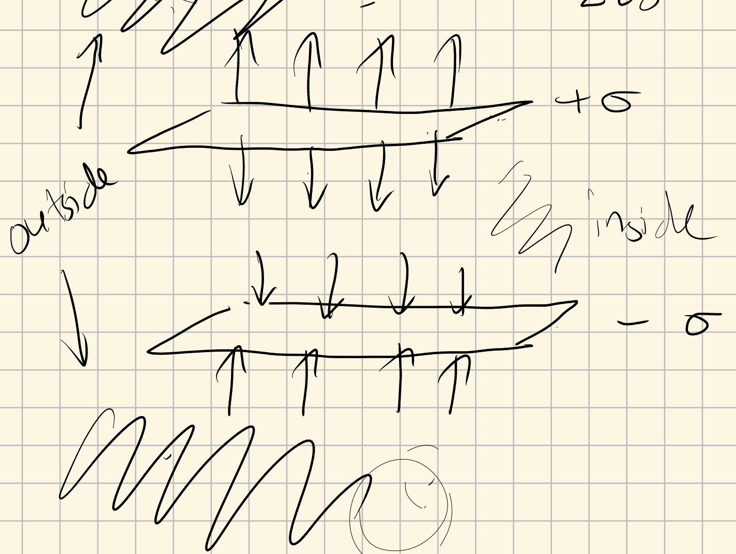


infinitely long/wide

σ

each one

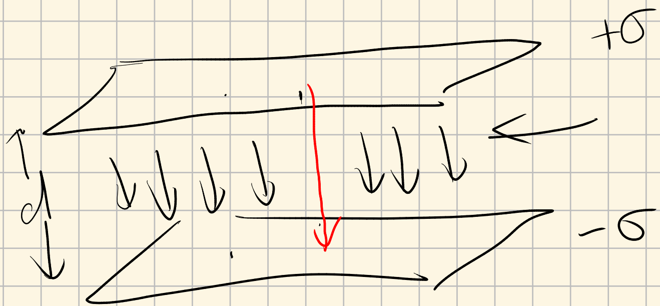
↪ $\vec{E} = \frac{\sigma}{2\varepsilon_0}$



$$\frac{\sigma}{2\varepsilon_0}(\pm\hat{j})$$

$$\vec{E}_{outside} = 0$$

$$\vec{E}_{inside} = -\frac{\sigma}{2\varepsilon_0}\hat{j} - \frac{\sigma}{2\varepsilon_0}\hat{j} = -\frac{\sigma}{\varepsilon_0}\hat{j}$$



$$\vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{j}$$

$$\Delta V = -\int \vec{E}\cdot d\vec{l} = -\int_0^{-d} -\frac{\sigma}{\varepsilon_0}\hat{j}\, d\vec{y}$$

$$= \frac{\sigma}{\varepsilon_0}(-d)$$

Approx: finite plate of area A.

assume the charge is uniform;

if I put $+Q$ on one plate and

$-Q$ on the other $\rightarrow$ infinite sheet.

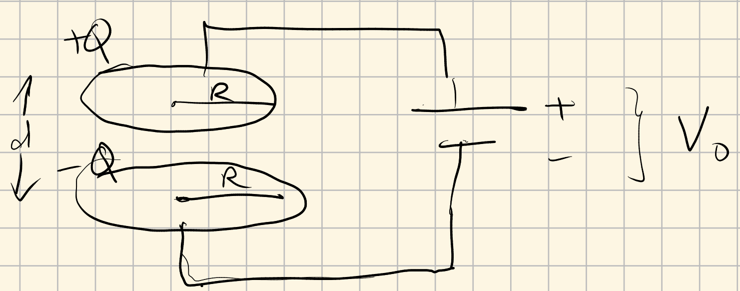
$$\sigma = \frac{Q}{A}$$

$$\Delta V = \underbrace{\frac{\sigma}{\varepsilon_0}(-d)} \rightarrow \boxed{C = \frac{Q}{|\Delta V|}} = \frac{Q}{\frac{Q}{A}\frac{d}{\varepsilon_0}}$$

$$= \frac{Q}{A}\frac{(-d)}{\varepsilon_0} \qquad |\Delta V| = \frac{Q}{C} \qquad = \boxed{\frac{A}{d}\varepsilon_0}$$

Problem

Suppose a parallel plate capacitor is made of 2 circular plates of radius R and separated by a distance d. The capacitor is connected to a battery with voltage V_0 as shown below. $d \ll R$



1) How much charge appears on each plate?
2) Magnitude of electric field between the plates?
3) Suppose the plates are pulled apart to distance $2d$ while still connected to the battery. By what multiplicative factor does the charge on each plate change? $|\vec{E}|$?

1) $Q = C\Delta V = C V_0$

$$C = \frac{A}{d}\varepsilon_0 = \frac{\pi R^2}{d}\varepsilon_0$$

$$\boxed{Q = \frac{\pi R^2}{d}\varepsilon_0 V_0}$$

2) $$|E| = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{Q}{A} = \cancel{\frac{1}{\varepsilon_0}}\,\cancel{\frac{1}{\pi R^2}}\,\frac{\cancel{\pi R^2}}{d}\cancel{\varepsilon_0} V_0$$

$$= \boxed{\frac{V_0}{d}}$$

3) $Q = C\cancel{\Delta V}$

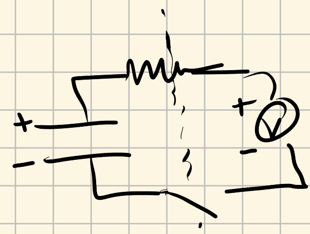
$$C = \frac{A\varepsilon_0}{d}$$

C decreases by a factor 2

Q decreases by a factor 2

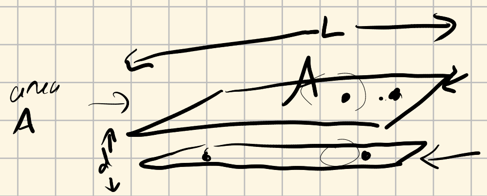
$|E|$ decreases by a factor of 2

Capacitors



$$Q = C V \quad (\Delta V)$$

↳ difference in potential between two conductors of the capacitor



$d/L \ll 1$

Parallel plate

1) imagined that we put some $+Q$ on one plate and $-Q$ on the other

2) $\vec{E}$ due to Q

3) $\Delta V = -\int \vec{E} \cdot d\vec{l}$

4) $C = \dfrac{Q}{\Delta V}$ → Q's cancel

→ C only depends on geometry, dielectric constants

5) $C = \varepsilon_0 \dfrac{A}{d}$

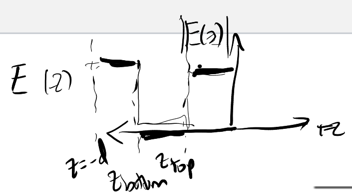
- **Problem 3: A conductor inside a capacitor [3]**

This is problem 7 of WS E6 in your course workbook.

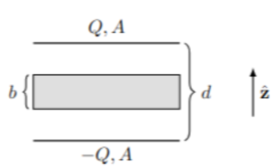
Consider a parallel-plate capacitor with plates of area A separated by some small distance d. It is initially charged up to some charge Q, and then disconnected from the battery. A neutral conducting metal slab, also of area A and with width $b < d$, is then inserted between the plates as shown below. What is the capacitance of this configuration? (Hint: it may help to start by thinking about what the electric field looks like inside the capacitor.)

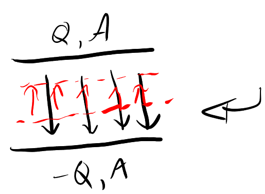
~~[1] We haven't specifically talked about conductors in any worksheet yet, but it's worth noting that conductors have some special properties. Most importantly, $\vec{E} = 0$ in the bulk of any conductor, any net charge on a conductor resides on the surface, and the surface of a conductor is always an equipotential. Review Section 21.9 in your textbook for more details [2].~~

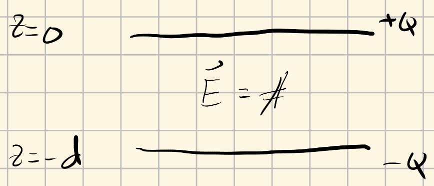
2

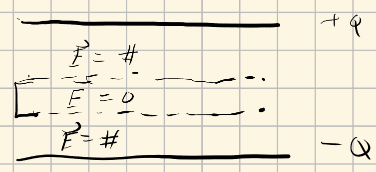


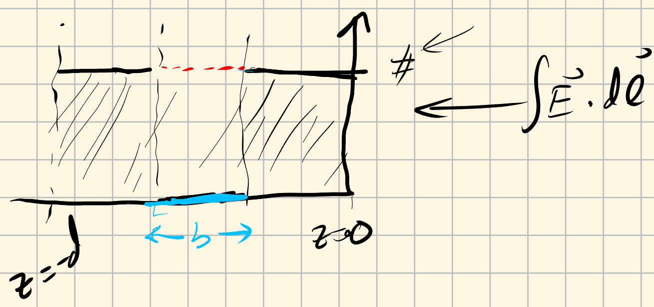
assume $Q > 0$











$$|\vec{E}| = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \quad \times d$$

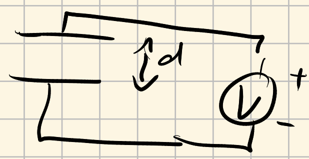
$$\times (d-b)$$

$$\boxed{\Delta V = \frac{Q}{A\varepsilon_0}(d-b)}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Q}{A\varepsilon_0}(d-b)} = \frac{A\varepsilon_0}{(d-b)}$$

Case 1:

Parallel plate capacitor with area A, separation d.



pull the plates apart to $2d$ while battery is connected:

$$Q = C\,\Delta V$$

doesn't change

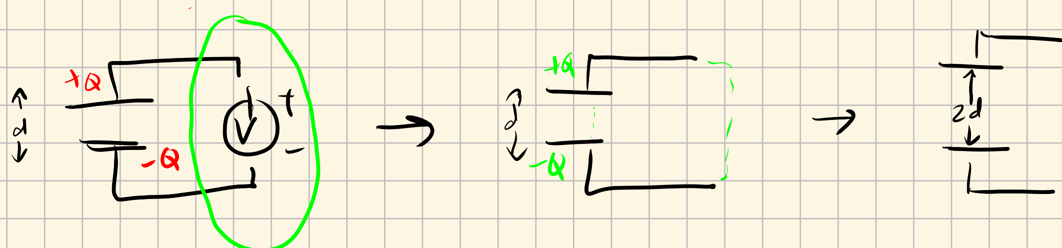
geometric → changes

also changes

$$C = \varepsilon_0 \frac{A}{d}$$

$C \to \frac{1}{2} C$

$Q \to \frac{1}{2} Q$



$$Q = C\,\Delta V$$

No change

$C \to \frac{1}{2} C$

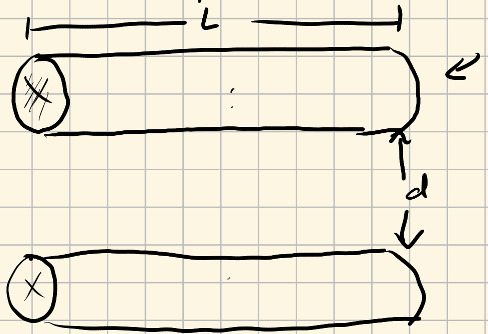
$\Delta V \to 2V$

$$\Delta V = \int \vec{E} \cdot d\vec{\ell}$$

$= |E| \times$ separation

Problem:

A capacitor is formed by two conducting cylindrical shells of length L and radius R, separated by distance d ($d \ll L$). Find capacitance.

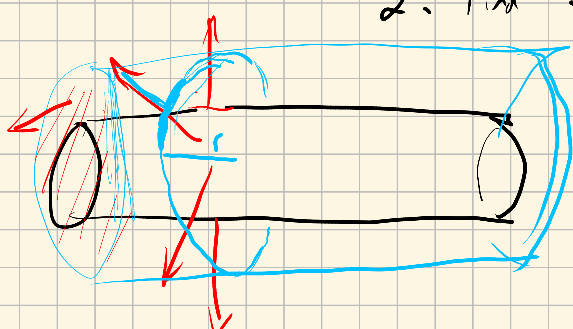


$$C = \frac{Q}{\Delta V}$$

Imagine $+Q$ on one rod, $-Q$ on other.

1. Find E due to $\pm Q$
   a. approximate the rod as infinitely long
   b. Gauss's law
2. Find $\Delta V = -\int \vec{E} \cdot d\vec{\ell}$

1\.



~ infinitely long

~ charge is uniformly

$\Phi =$

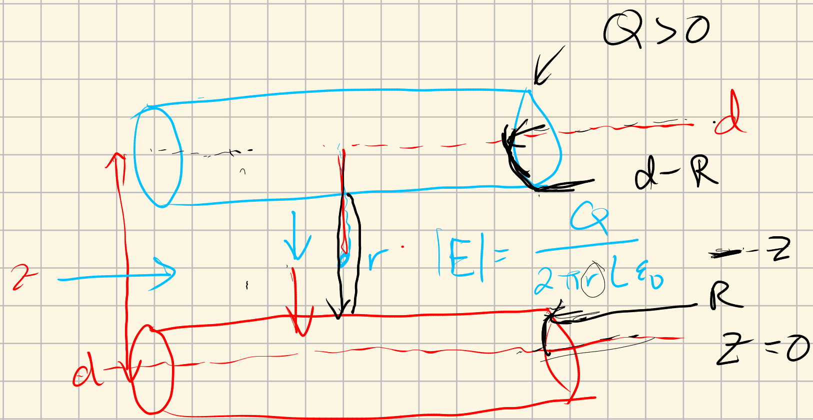
$$\int \vec{E} \cdot d\vec{A} = \int_{\substack{\text{lateral} \\ \text{surface}}} \vec{E} \cdot d\vec{A} + \cancel{\int_{\substack{\text{left} \\ \text{end cup}}} \vec{E} \cdot d\vec{A}} + \cancel{\int_{\substack{\text{right} \\ \text{end cup}}} \vec{E} \cdot d\vec{A}}$$

$0 \qquad 0$

$$= \int_{\substack{\text{lateral} \\ \text{surface}}} \vec{E} \cdot d\vec{A} = \int_{\substack{\text{lateral} \\ \text{surface}}} |E||dA|$$

$$= |E| \int_{\substack{\text{lateral} \\ \text{surface}}} |dA| = |E|(2\pi r L)$$

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \longrightarrow |E|(2\pi r L) = \frac{Q}{\varepsilon_0} \rightarrow |E| = \frac{Q}{2\pi r L \varepsilon_0}$$



$R < d/2$

$$\vec{E} = -\frac{Q}{2\pi z L\varepsilon_0}\hat{k} - \frac{Q}{2\pi(d-z)L\varepsilon_0}\hat{k}$$

$$\Delta V = -\int_{\text{top}}^{\text{bottom}} \vec{E}\cdot d\vec{l} = -\int_{d-R}^{z=R}\left(-\frac{Q}{2\pi z L\varepsilon_0} - \frac{Q}{2\pi(d-z)L\varepsilon_0}\right)dz$$

$$= \frac{Q}{2\pi L\varepsilon_0}\int_{d-R}^{R}\left(\frac{1}{z} + \frac{1}{d-z}\right)dz$$

$$= \frac{Q}{2\pi L\varepsilon_0}\left[\ln\left(\frac{R}{d-R}\right) - \ln\left(\frac{d-R}{R}\right)\right]$$

$$= \frac{Q}{2\pi L\varepsilon_0}\ln\left[\frac{R^2}{(d-R)^2}\right]$$

$$= \frac{Q}{\pi L\varepsilon_0}\ln\left[\frac{R}{d-R}\right]$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Q}{\pi L\varepsilon_0}\ln\left[\frac{R}{d-R}\right]} = \frac{\pi L\varepsilon_0}{\ln\left(\frac{R}{d-R}\right)}$$

$R < d/2$

$$\frac{R}{d-R} > 1$$

$$\frac{d/4}{d-d/4} = \frac{d/4}{3d/4} = \frac{1}{3}$$

fixing sign

$$= \frac{\pi L\varepsilon_0}{\ln\left(\frac{d-R}{R}\right)}$$