

The Centrality of Simplicity in Frege's Philosophy

Jim Hutchinson

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Abstract

It is widely recognized that Frege's systematic conception of science has a major impact on his work. I argue that central to this conception and its impact is Frege's *Simplicity Requirement* that a scientific system must have as few primitive truths as possible. Frege states this requirement often, justifies it in several ways, and appeals to it to motivate important aspects of his broader views. Acknowledging its central role illuminates several aspects of his work in new ways.

"Only in systems is science complete. We can never give up on systems."¹ So writes Gottlob Frege, whose commitment to systems is no surprise: the *systematic* conception of science dominated thought about it until well into the 20th century. On this conception, our goal as practitioners of a given science is not only to discover truths about our subject, but to identify some of them as *primitive* and the rest as *theorems*, and to find *proofs* of the theorems from the primitive truths. This is to find a *system*.² While all systematic theorists agree on this basic framework,

¹Frege 1914, 261.

²Frege 1914 reviews the basics: "once we have succeeded in discovering these *primitive truths*...then [the science] will appear as a system of truths that are connected with each other by logical inference." Once each other truth is connected to the primitive truths by chains of such inferences, "these inference-chains constitute the *proof* of our theorem." (220)

they disagree about details: for example, about what something must be like in order to be admissible as a “proof” or a “primitive” truth.³

What motivates a theorist to endorse one account of these details rather than another? And for that matter, why do theorists think we must seek systems in the first place? The answer to both questions is that these theorists think having a system puts us in the best kind of cognitive state that we can achieve: the kind that in the early modern period was often called “*scientia*”.⁴ A theorist who claims that systems must conform to a certain requirement thinks that so conforming has cognitive value, and that only what conforms can put us in that best of cognitive states. Views about scientific systems thus have much in common with contemporary views of the most ambitious cognitive goals.⁵

When we ask why a certain historical figure makes a certain theoretical claim or pursues an inquiry in a certain way, the answer often involves his or her convictions about what the best sort of cognitive state requires: for he or she means to theorize in a way that will achieve that state. Frege is no exception.⁶ My main aim

³See de Jong and Betti 2010 on the influence of the systematic conception—which “count[ed] among its pronounced followers Newton, Pascal, Spinoza, Descartes, Leibniz, Wolff and Kant, and still later Bolzano, Husserl, Frege and Leśniewski”—and some variations in detail.

⁴See, e.g., Sorell, Rogers, and Kraye 2010.

⁵For example, there are many points of contact between the description of *understanding* in chapter 8 of Kvanvig 2003 and the traditional picture of systematic science.

⁶Consider, for example, the citation in Frege 1884 (§5) of a purported “need of reason” concerning primitive truths in scientific systems to guide his inquiry into what numbers are. (That it is *reason* in particular that must strive for scientific systems is a repeated theme in the Appendix to the Transcendental Dialectic of Kant 1781/1787.) Less obviously, Frege’s view on the seemingly peripheral question whether “the primitive truths of a systematic natural [i.e. empirical] science...include supporting data for its laws” or whether they are all themselves laws has proved relevant to the central question *why* he tries to demonstrate logicism about arithmetic. (See Weiner 2004 (123), Jeshion 2001 and Jeshion 2004.)

here is to argue that a certain requirement is of great importance both for Frege's general views about science and for various doctrines of his about logic and other subjects. This is his *Simplicity Requirement* that scientific systems have as few primitive truths as possible.

The first section makes the basic case for the importance of this Simplicity Requirement, showing that Frege explicitly states it, offers reasons to justify it, and claims it as justifying and motivating other important decisions of his. There are, however, reasons to doubt that these passages represent Frege's real views; and the second section addresses these reasons, partly by explaining how the Simplicity Requirement really can justify what Frege says it does. The third section uses the Simplicity Requirement to resolve puzzles about Frege's views on *dependence relations* and a *natural order* among truths. I conclude by discussing how these observations affect our overall picture of the sort of thinker Frege is.

Frege's Simplicity Requirement has not yet received serious attention from interpreters, and this neglect has led to some interpretive missteps and oversights. Interested readers can find fault-finding discussions chiefly in footnotes, while the main text focusses primarily on positive points.

1 Frege's Simplicity Requirement

1.1 Statements of the Simplicity Requirement

In *The Foundations of Arithmetic*, Frege claims that when we can “trace everything back” to “fewer primitive truths...this simplification is in itself a goal worth striving for.”⁷ He does not say here that we *must* strive for such simplification, nor that the cognitive values of *science* call upon us to do so. On its own, then, this *Foundations* passage might express a negotiable guideline, perhaps motivated by the *aesthetic beauty* of simplicity or some other non-cognitive value; or it could be a goal that is specific to work in *mathematics* rather than science in general.

But other passages show that when Frege calls upon us to pursue this sort of “simplification”, he means to invoke a strict requirement of science in general. Here are three such passages—one early, one middle, and one late in his career. (Note that when discussing sciences all of whose primitive truths are *laws*, Frege sometimes puts the requirement in terms of those things.)

1. Early: when it comes to “primitive laws...It is a general, basic principle of science to reduce their number as much as possible”; there is a “basic principle, to shrink the number of primitive laws as much as possible.”⁸

⁷Frege 1884, §2.

⁸Frege 1880–1881, 40-42. A mistake in the standard translation obscures the connection between simplicity and science in this paper. Frege is comparing his *Begriffsschrift*, which is meant to express a scientific system, to Boole's “logical calculus”, which is meant only as a problem-solving tool. Since minimizing the number of primitive truths is a requirement of *science*, Frege observes that since Boole is “only concerned to solve his problems in a brief and practical way” rather than to express a scientific system, it is no surprise that “in [Boole's] case, the striving to

2. Middle: to pursue “the ideal of a strictly scientific method” in mathematics, “one must strive to reduce the number of...primitive laws as much as possible.”⁹
3. Late: “science must strive to shrink the circle of unprovable *primitive truths* as much as possible.”¹⁰

This is the Simplicity Requirement: just as a scientific system requires truths rather than falsehoods and gapless proofs rather than intuitive connections, it requires the minimization of the number of primitive truths as far as is possible—possible without violating other requirements, of course.

To accept some kind of simplicity requirement is not at all unusual: as Elliot Sober notes, it is a historically “influential point of view” on which “the search for simple theories is not optional; rather, it is a requirement of the scientific enterprise...Parsimony is not an optional, aesthetic frill.” But while “to strive for simplicity in one’s theories means that one aims to minimize something...the problem is to figure out what to count.”¹¹ Frege’s Simplicity Requirement is distinguished from many others by what is counted: it is the number of *primitive truths* that must be minimized, rather than that of, say, the entities postulated. We will soon see what Frege thinks about some other simplicity requirements.

manage everything with as few primitive laws as possible is not noticeable.” (“Bei ihm ist ein solches Streben, mit möglichst wenigen Urgesetzen alles zu leisten, nicht bemerkbar.”) But in their translation for Frege 1979, Long and White have him say: “In [Boole’s] case there is nothing remarkable in the attempt to manage everything with the fewest possible primitive laws,” (37) making it sound like what is no surprise is that Boole *does* attempt to minimize their number.

⁹Frege 1893a, vi.

¹⁰Frege 1914, 221.

¹¹Sober 2015, 2, 4; Sober 2002, 14

1.2 Justification for the Simplicity Requirement

Why would there be a requirement to minimize the number of primitive truths? Why, that is, would achieving the best sort of cognitive state require doing so?

We have seen Frege call the requirement “a general, *basic principle* of science,” and claim minimizing the number of primitive truths to be “*in itself* a goal worth striving for.” The phrases I have italicized suggests that Frege thinks there is such a requirement because satisfying it is of *non-instrumental* cognitive value.

Other passages say more. Frege claims, for example, that the more we assemble the “many details under a more comprehensive point of view,” the “greater surveyability” we obtain; and that “the fewer the number of primitive sentences, the more perfectly they can be mastered.” Most strikingly, he claims that “the essence of explanation consists precisely in the fact that explanation controls a large, possibly unsurveyable, manifold through one or a few sentences,” so that an explanation’s “value can be simply measured by this compression and simplification.”¹² Frege may intend here that in addition to its non-instrumental value, satisfying the Simplicity Requirement also has *instrumental* cognitive value since it secures other valuable things: surveyability, mastery, and optimal explanations. Alternatively, he may see these not as *distinct* valuable things, but only as some familiar labels for the non-instrumental value of minimizing the number of primitive truths. The difference between these two possibilities does not matter much for us. What matters is that Frege justifies the Simplicity Requirement in several

¹²Frege 1914, 261; Frege 1880–1881 44, 40.

places, in ways appropriate to a requirement of science in general: by explicitly describing the *cognitive* value of satisfying it.

Frege also makes the somewhat puzzling justificatory claim that we “must strive to shrink the circle of primitive truths as much as possible; because in these primitive truths is the whole of [the relevant science] contained as in a seed.”¹³ What is puzzling is the “because.” Why must the “seed” which contains the whole science be as small as possible? (Some plants have large seeds, and it would not be obviously better if they were smaller.) I think we must take Frege’s occasional reference to a “seed” of primitive truths as a way of alluding to the cognitive values explicitly identified elsewhere. To have a scientific system is to appreciate the way in which a whole science is contained in its “seed” of primitive truths; and since this appreciation is cognitively valuable in the ways described above, it follows that the smaller the seed, the greater the value.

This, then, is how Frege justifies the Simplicity Requirement. It is important also to notice some ways in which he does *not* try to justify it. For example:

1. The standard justification today for pursuing any kind of simplicity in science is the idea that, as Richard Swinburne puts it, “simplicity is evidence of truth.”¹⁴ Indeed, it is sometimes assumed that *unless* simplicity is evidence for truth, its value could only be aesthetic or otherwise non-cognitive. But while Frege clearly takes the relevant kind of simplicity to have cognitive value, the idea that it is evidence of truth appears nowhere in his writings.

¹³Frege 1914, 221.

¹⁴Swinburne 1997, 1.

2. Arguably, one of Frege's goals in seeking to prove ordinary arithmetical truths was to determine their epistemological status: since it is not clear whether such truths are analytic, a posteriori, or whatever, we can determine that status only by proving them from truths whose epistemological status is already evident.¹⁵ But Frege never cites this goal to justify the Simplicity Requirement. Nor *could* he: for this goal can provide no reason to reduce the number of primitive truths beyond a manageable number, each of whose epistemological status is evident.¹⁶

Is it reasonable to see the sorts of cognitive value Frege does in minimizing primitive truths? In this connection, it helps to make a comparison with Michael Friedman's *unification theory of explanation*. According to Friedman, "this is the essence of scientific explanation—science increases our understanding of the world by reducing the total number of independent phenomena that we have to accept as ultimate or given. A world with fewer independent phenomena is, other things equal, more comprehensible than one with more."¹⁷ Friedman, too, thinks that using something like proof to reduce the number of "ultimate" truths is of cognitive value, increasing *understanding*, *comprehension* and the quality of our

¹⁵Weiner 1990 emphasizes this goal.

¹⁶Schlimm 2017 (19) thinks Frege's Simplicity Requirement is justified by the requirement that proofs be gap-free, since more primitive truths "would make it more difficult to keep track...easier to overlook some...putting at risk the overall goal of gap-free derivations." But I do not see Frege give this justification in the paragraph on which Schlimm is commenting, or anywhere else. Nor could he: again, as long as the number is manageable, there is no real danger of losing "track."

¹⁷Friedman 1974. As with Frege, this reduction is a matter of "deriving" one thing from another, and Friedman sometimes talks (like Frege) of reducing "the total number of...sentences" rather than of "phenomena." Friedman appears to have neglected Frege when he claims that "the only writer that I am aware of who has suggested that this [simplification/unification] is the essence of explanation... is William Kneale." (15)

explanations. (He even echoes Frege’s “essence of explanation” formulation.) The prominence of the unification theory today assures us that Frege’s Simplicity Requirement and his reasons for it cannot be lightly dismissed. (Readers who wish to more closely scrutinize these views in a non-historical way can consult discussions surrounding the unification theory, and those already familiar with some of its challenges will recognize Frege addressing forms of them below.¹⁸)

1.3 Justification by the Simplicity Requirement

That Frege states and justifies the Simplicity Requirement does not, on its own, guarantee that it has much further impact on his views about science in general, or about other things. But Frege also explicitly claims this requirement to justify several important theoretical choices. Here are four examples.

1. Frege requires a proper system to *prove as many things as possible*, including things that are obviously true.¹⁹ He claims this requirement to be justified by the Simplicity Requirement. The *Foundations* passage, for example, begins: though “it may seem almost ridiculous” to give proofs of “things which anyone would concede...without question,” we must do so because “the further one pursues these enquiries, the fewer the primitive truths to which we trace everything back; and this simplification is in itself

¹⁸For example, §§2.1-2.2 discusses challenges posed by the idea that we can trivially reduce primitive truths by adding inference rules or by conjoining primitive truths. Addressing the latter, which had been pointed out by Hempel and Oppenheim 1948 (footnote 28), was a major goal of Friedman 1974. Recent discussions of the unification theory largely focus on the variation provided by Kitcher 1989, which aims to preserve Friedman’s core idea while avoiding technical problems that arose from his attempt to address such challenges. (see, e.g., 431-432)

¹⁹e.g., Frege 1914, 221. “Science demands that we prove whatever is susceptible of proof.”

a goal worth striving for.”²⁰ The Middle passage indicates this connection more briefly: “one must strive to reduce the number of these primitive laws as much as possible *by proving everything that is provable.*”²¹ (My italics.)

2. Frege requires the language used to express a proper system to have *as few primitive signs as possible*. The Early passage justifies this requirement: “the more primitive signs are introduced, the more primitive laws will be needed. But it is a general, basic principle of science to reduce their number as far as possible.” He then notes that his *Begriffsschrift* compares favourably with alternatives because it “needs...fewer primitive signs and therefore also fewer primitive laws.”²²
3. Frege requires a system to employ as few patterns of proof—as few *modes of inference*—as possible. The Early passage goes on to treat “the basic principle, to shrink the number of primitive laws as much as possible” as lying behind this requirement: this is what “determined” him to use “only a single [mode of inference]” in his early work. The Middle passage goes on to claim that one must “reduce[] the modes of inference and consequence to

²⁰Frege 1884, §2.

²¹That is: “Es muss danach gestrebt werden, die Anzahl dieser Urgesetze möglichst zu verringern, indem man Alles beweist, was beweisbar ist.” (Frege 1893a, vi.)

²²Frege 1880–1881, 40–44, 52. In discussing Frege’s effort to minimize the number of primitive signs, Bellucci, Moktefi, and Pietarinen 2017 fail to note that he does so in order to minimize the number of primitive truths, which leads their comparison with Peirce (10) to miss a potentially illuminating connection to Peirce’s notion of *analysis*, on which “the fewer the primitive symbols, the fewer the axioms; and the fewer the symbols and axioms the more analytic the system.” Schlimm 2017, by contrast, notices this justification, and partly for this reason, I think his brief discussion (18–19) is the best that exists so far of Frege’s Simplicity Requirement. (Though see footnotes 16 above and 36 below.)

a minimum” if one is to comply with the requirement to “reduce the number of...primitive laws as much as possible.”

4. Frege holds truth-values—the True and the False—to be objects that are the referents of sentences. After stating the Simplicity Requirement in the Middle passage, he justifies this view in terms of it: by “how much simpler...everything becomes by the introduction of the truth-values.”²³

I will discuss these justifications below. For now, I note two things about them.

In the first three examples, Frege claims that the Simplicity Requirement motivates other requirements he places on systems. One who is especially interested in the effects of the Simplicity Requirement on Frege’s doctrines about things other than science should note that he also claims that these other requirements directly affect that theorizing. For example: having said that the Simplicity Requirement justifies a requirement to have as few primitive signs as possible, Frege explains that “to obtain as few primitive signs as possible, I had to choose the simplest possible meanings for them,” which led him to choose the conditional as a primitive logical concept; and he similarly justifies his treatment of *negation* by

²³Frege 1893a, x. Dummett 1973 thinks Frege’s talk of simplicity here instead concerns a “simplification in...ontology,” (183) while Burge 1986 thinks it is about making available “analogies that are quite natural within a formal context,” and allowing “the simplest construal of the Composition Principle.” (113-115) But these readings are not very plausible. For unlike minimizing the number of primitive truths, Frege has not mentioned the importance of such “simplifications” in this passage—nor indeed, to my knowledge, ever. Moreover, since there is no *requirement* to pursue them—at least, Frege certainly never endorses any—Dummett is probably right that it would be a “blunder” to achieve these sorts of “simplification...at the price of a highly implausible analysis of language.” Frege surely has not made this blunder; surely he had his *Simplicity Requirement* in mind instead.

claiming that it leads to “an economy of logical primitives.”²⁴ Since the choice of these concepts affects the whole shape of his logical system, it follows that if Frege chooses these concepts because of a requirement to minimize the number of primitive signs which he endorses because of the Simplicity Requirement, the Simplicity Requirement has a tremendous influence on his theorizing in logic.

Also: in the second and third examples, what is justified are actually other simplicity requirements: requirements that the number of primitive signs and inference rules also be minimized (the former of which leads, in turn, to another simplicity requirement concerning meanings). There is a general fact here worth noticing: that when Frege calls for other kinds of simplicity, he does not talk of basic principles of science, of what is in itself worth doing, of the essence of explanation, etc.; he explicitly claims each such requirement to follow from the Simplicity Requirement. His concern for other kinds of simplicity traces back to his concern for the minimization of primitive truths, which he justifies directly.

To summarize the story so far, then: explicitly and repeatedly, Frege endorses the Simplicity Requirement and offers justification for it. In turn, he cites it as justifying and motivating other significant theoretical moves. Taken at face value, this implies that the Simplicity Requirement should have a central role in any attempt to understand Frege’s general thinking about science, as well as many

²⁴The discussion of the conditional is at Frege 1880–1881, 40–44; of negation, at Frege 1918–1919, 384–385. Interestingly, his justification for including the concept of generality may also appeal implicitly to the Simplicity Requirement. He claims that the “value” that justifies the choice of this concept is found in the way it enables us to make a claim that “contains many—indeed infinitely many—particular facts as special cases.” (Frege 1923/1925, 278.) He does not say why this is valuable, but one obvious answer is that doing so “controls a large, possibly unsurveyable, manifold through one or a few sentences.”

aspects of his theorizing about other subjects.²⁵

We now turn, however, to some surprisingly plausible reasons *not* to take it at face value.

2 Reasons for Doubt

Could the passages that we have been considering be “all talk”? Could Frege’s claims to actually be motivated by the Simplicity Requirement be *untrue*?

Such a suspicious interpretive possibility might seem paranoid. But Frege’s claims that the Simplicity Requirement justifies and motivates other decisions appear not in the thick of his reasoning, but either in introductory material or a single article specifically aimed at drumming up interest in his already-completed work by providing reasons to prefer it to Boole’s.²⁶ Moreover: Kant, who declares allegiance to the systematic conception by identifying a “science” with “every doctrine that is supposed to be a system,” also claims “parsimony of principles [i.e. primitive truths]” to be an aspect of “systematic unity”; and following Kant’s

²⁵Previous footnotes have pointed out failures to recognize the role of the Simplicity Requirement in justifying particular decisions. General discussions of Frege’s version of the systematic conception of science exhibit a neglect which sometimes quite perplexing. de Jong 1996, for example, introduces Frege’s view of systematic science with a large block quote, in the course of which Frege says “we must try to diminish the number of these primitive laws as far as possible.” Soon after, de Jong apparently glosses this as the claim that “every proposition should be proved from a limited number of principles,” which becomes, later, “there are...a (finite) number of fundamental propositions.” That is not the Simplicity Requirement. Despite being quoted, it has been misinterpreted or (at best) passed over without comment. If mentioned at all, it is either misinterpreted or breezed over by Detlefsen 1988, Jeshion 2001, Shapiro 2009 and Macbeth 2016.

²⁶Schröder 1880 charged that Frege’s *Begriffsschrift* was not very interesting because it “does not differ essentially from Boole’s formula language.” The Early passage is from Frege’s response. See Sluga 1987 for discussion.

lead, the Simplicity Requirement was widely endorsed in the Neo-Kantian movement. But Frege sometimes seems to go out of his way to say things that he hopes will appeal to those in that then-dominant movement. (Recall his attempt to soften his criticisms of Kant with effusive praise of that “spirit...to whom we can only look up with grateful admiration.”)²⁷ In this context, it is not paranoid to worry that rather than revealing what *motivates* Frege’s decisions, the relevant passages are merely attempts to *advertise* them.

I know of two serious reasons in favour of seeing things this way.

1. It is not clear how the Simplicity Requirement *could* justify some of what Frege says it does—and though anyone can make a mistake, the fewer of Frege’s justificatory claims are plausible, the more likely it becomes that he has not considered them very carefully and they do not reflect his real motivations.
2. Even if there are genuine justificatory connections between the Simplicity Requirement and Frege’s decisions, one passage suggests that they nonetheless cannot really *motivate* those decisions; because in this passage, Frege appears to admit that he has not tried to satisfy the Simplicity Requirement in his work.²⁸

²⁷Frege 1884, §89. For Kant’s endorsement of the systematic conception and of the minimization of the number of “principles,” see Kant 1781/1787, A649/B677-A650/B678. On the dominance of the Neo-Kantians, see, e.g., the General Introduction to Beiser 2014. Such influential Neo-Kantian figures as Liebmann 1876 had affirmed (my italics) that “No science can be counted as perfect and completed...until it...forms a logical whole, in which...an *absolutely minimized number of...primitive sentences...*[a] narrow tip of primitive thoughts...flows into the broad—indeed infinite—group of...details.” (8)

²⁸Thanks to [excised for anonymity] for suggesting the relevance of this point.

To address the first concern, I will discuss two of Frege's more puzzling justifications, showing that they make good sense. This not only removes the first reason for doubting that what Frege says reflects his motivations, but the details are also interesting in their own right, and illustrate how considering the Simplicity Requirement in the context of fuller discussions of the relevant issues promises to illuminate them in new ways. I will then address the second reason by explaining why the relevant passage does not imply that Frege's decisions are not motivated by the Simplicity Requirement.

2.1 Primitive Truths and Inference Rules

Inference rules and primitive truths are different. So how could a requirement to minimize the number of *inference rules* be justified by the Simplicity Requirement to minimize that of primitive *truths*? Why would one have to minimize the number of the former in order to minimize that of the latter? In fact, the opposite seems true. For any logical primitive truth, there is a "corresponding" inference rule—one enabling proofs of everything we can prove using the truth. (To get the idea, compare the truth that *if P, then $\neg\neg P$* with an inference rule allowing one to always conclude $\neg\neg P$ from P .) It looks, then, like *adding* inference rules is the way to reduce the number of primitive truths as far as possible. This suggests not only that Frege is wrong to think the Simplicity Requirement justifies a requirement to minimize the number of inference rules, but also that this requirement would in fact call for a very different system of logic than the one Frege offers: one with

more inference rules.²⁹

But Frege is well aware of this close connection between inference rules and primitive truths: he himself claims that each inference rule “contains” its corresponding truth, of which it is the “image” or “reflection.”³⁰ He also says that every inference rule is “subject to” its corresponding truth; that this truth must be “counted among” the truths in the system; and even that an inference rule and its corresponding truth are really “the same thing in two different forms.”³¹ This connection matters for Frege’s understanding of the Simplicity Requirement. For example: in a passage discussing the sense in which a whole science is contained in the primitive truths—the fact that enables a grasp of this “seed” to yield the sort of “mastery” of the whole which is better the fewer primitive truths there are, thus justifying the Simplicity Requirement—Frege clarifies that strictly speaking, the whole science is contained *not just* in the “small number of laws” identified as primitive truths, but in those laws *together with* “those [laws] contained in the [inference] rules.”³² The relevant mastery, then, comes not from grasp of the primitive truths *alone*, but of them together with the laws which correspond to the inference rules. This means, at least, that we should *count* those laws along with the primitive truths for the purposes of the Simplicity Requirement, which then really will justify minimizing the number of inference rules.³³

²⁹Many contemporary logicians prefer to operate *exclusively* with inference rules rather than with Hilbert-and-Frege-style axiomatic systems. Dummett 1973 (433-434) credits Gentzen as “the first to correct [Frege’s] distorted perspective.”

³⁰Frege 1879, §13; Frege’s term is “Abbild.”

³¹Frege 1914, 219; Frege 1880–1881, 42.

³²Frege 1879, §13.

³³Depending on how seriously we take Frege’s claim that inference rules and primitive truths

2.2 Primitive Truths and Truth-Values

How could introducing truth-values as the referents of sentences be called for by a requirement to minimize the number of primitive truths? How does introducing truth-values shrink that number?

The first thing to note is that Frege could not state his logical primitive truths as he does if sentences did not refer to truth-values.³⁴ For taking sentences to refer to these objects is what allows the *identity function* to play the role of a biconditional function, which it does in some of these truths. Of course, Frege could always replace these uses of the identify function by introducing a new biconditional function and a corresponding new symbol—say, “ \leftrightarrow .” But since the identity function and its symbol (“=”) are already needed for other reasons, the truth-values provide (something which serves as) a biconditional function without introducing a new symbol. And this matters for the number of primitive truths, assuming Frege is right that “the more primitive signs are introduced, the more primitive laws will be needed.” The upshot is that *if* the minimization of the number of primitive truths is somehow facilitated by the availability for use in primitive truths of a biconditional, then it is also facilitated by the introduction of the truth-values, because this provides a biconditional without a new symbol.

Is the minimization of primitive truths somehow facilitated by the availability of a biconditional, then? It depends on what Frege would do if there no bicondi-

are the “same thing”, it may be that in the end, he really accepts not the Simplicity Requirement as he states it, but a requirement to minimize the number of *primitive truths and inference rules*.

³⁴Or at least, unless all true sentences refer to one object, and all false sentences to another.

tional were available. One might naïvely suggest that he would have to replace any primitive truth currently involving a biconditional with *more than one* primitive truth—say, replacing any truth involving something of the form “ $A = B$ ” with one truth involving something of the form “ $B \rightarrow A$,” and another involving the form “ $A \rightarrow B$.” If so, then the availability of a biconditional obviously *would* help to minimize the number of primitive truths. But it seems not to be so; for it also seems possible to replace truths involving a biconditional with a single truth that is logically equivalent: replacing “ $A = B$ ” with, say, “ $\neg((A \rightarrow B) \rightarrow \neg(B \rightarrow A))$ ”.³⁵ Though more complex, this is still just *one* truth. Having a biconditional would only lead to fewer primitive truths along the lines of the naïve suggestion, then, only if such logically equivalent replacements were somehow forbidden.

But they *are* forbidden. To see why, consider why the Simplicity Requirement does not motivate reducing the number of primitive truths by (something logically equivalent to) a *conjunction* of truths currently counted as primitive. This—as Wittgenstein observes in the *Tractatus*—is ruled out by an independent requirement of Frege’s on primitive truths: that they be *known without being proved*.³⁶ For primitive truths specific to empirical sciences, this requirement is met because

³⁵Recall that *Begriffsschrift* has no primitive conjunction function. Frege recognizes distinct but logically equivalent truths when he identifies truths with thoughts and takes logical equivalence not to imply sameness of thought. (See Frege 1902 and Frege 1914, 253.)

³⁶Wittgenstein 1922, 6.1271. See, e.g., Frege 1879–1891 (3) and Frege 1884 (§5) on this requirement, and Weiner 2020, 142 and Weiner 2007, 681 for discussion. Schlimm 2017 (19) seems to treat the constraint on minimizing the number of primitive truths provided by this other requirement as a reason to downplay the Simplicity Requirement: to describe Frege, for example, as merely “hinting” at a “quantitative assessment” of a system’s contribution to cognitive value. But it no more does so than does the constraint placed by the requirement that primitive truths be *true*, without which many new possibilities for reduction would appear.

sense-perception affords the relevant knowledge,³⁷ which prevents reduction in the number of primitive truths by conjunction because the conjunction of two perceived truths is rarely itself a perceived truth. Similarly, Frege claims that the “logical source of knowledge” affords us the relevant knowledge of the primitive truths of logic by rendering those truths “self-evident”; and a truth that is logically equivalent to the conjunction of several self-evident truths will typically not itself be self-evident. The same is true of a highly complex truth that is logically equivalent to a self-evident one.³⁸ This means that since any biconditional-free logically equivalent reformulations of Frege’s primitive truths which use the identity-sign-as-biconditional will be more complex, their admissibility as primitive would be in question. Since Frege was already worried that at least one of these truths might not be “as self-evident as...must really be required of a [primitive] logical law,”³⁹ he surely thought that any equivalent reformulation without the biconditional would lack the self-evidence required to be known without proof through the logical source of knowledge. This means that without a biconditional, he really could only replace these truths with *multiple* primitive truths; which means that the availability of a biconditional *does* reduce the number of primitive truths; which means that the truth-values do help to minimize the number of primitive truths.⁴⁰

³⁷E.g., Frege 1884, §3; Frege 1924/1925a, 286-288

³⁸Frege 1884, §5; Frege 1924/1925, 288-292. Though what Frege means by “self-evidence” is a difficult interpretive question, on no reasonable construal (e.g.: Burge 1998, Jeshion 2001) does the fact that two truths are self-evident guarantee that a truth conjoining them is, or the fact that a truth is self-evident guarantee that more complex but logically equivalent truths are too.

³⁹This is Basic Law V; see the Appendix to Frege 1893b.

⁴⁰They may also do so in other ways. For example: where Frege 1879 (§§20-21) had relied on *two* primitive truths involving identity, both are *proved* from the single Basic Law III in §50 of

2.3 Frege Chooses Not to Satisfy the Simplicity Requirement

Even if the Simplicity Requirement can justify what Frege says it does, that does not guarantee that it *motivates* those decisions. And in the *Basic Laws*, Frege claims not only that his own formulation of the scientific system of logic does not fully satisfy the “commandment of scientific parsimony,” but that “practical reasons” led him to *choose* not to satisfy it: had he done so, his book would have exhibited “too great a length.”⁴¹ Apparently, then, he simply ignores this requirement when convenient. Could he, then, really accept it at all? And if he could, doesn’t this at least mean that when he chooses to conform to it, his *motivation* for doing so must be something other than this acceptance?

But suppose that a friend who regularly professes that one morally ought each year to donate 10% of one’s income to charity says one day that he will not be making this year’s donation, for pressing “practical reasons”. Only an absurd philosophy of action would conclude that he must not really accept that one morally ought to make the donation every year, or that some other motivation is needed to explain his making the donation in other years. To take a somewhat closer analogy: in order to have a piece of suitable length, one sometimes publishes work including claims which one recognizes as strictly speaking *false*, since one has left out necessary hedges and qualifications. This shows neither that one does not think truth is required for full correctness from the cognitive point of view, nor

Frege 1893a—and the truth-values are implicated in these proofs by allowing the negation and horizontal functions as substitutions for a second-level variable. ([Excised for anonymity].)

⁴¹Frege 1893a, Foreword and §14.

that the literal truth of other claims in the work does not motivate one in making them. It is just that, for practical reasons, one has written something that one recognizes as falling short of full correctness from the cognitive point of view. (A strong indication that one still accepts that truth is required for correctness is if one *points out* that additional qualifications would be required for truth, and gives some indication of what these qualifications are.)

This is just what we see in Frege. Frege's point is that the system expressed in his book is not fully correct by cognitive standards, because full correctness would make the book too long: too long to write, or too long to secure a publisher, etc. This does not show that he does not really hold the relevant views about what is correct, or that those views do not motivate other aspects of his work. (A strong indication that he holds these views is that he points out that his system does not meet the Simplicity Requirement, and encourages readers to consult his earlier work to see "how one could satisfy the strictest demands."⁴²) So there is no reason here to doubt that the requirement motivates what Frege says it does.

3 Simplicity and Dependence

We have seen how taking seriously Frege's explicit claims that various decisions are justified and motivated by the Simplicity Requirement promises to illuminate those decisions. I will now present an example of how even when Frege does not signal this requirement's role as clearly, keeping its centrality firmly in mind helps

⁴²Frege 1893a, vi.

us to understand his thinking.

3.1 Frege's Dependence Requirement

We have noted (§1.3) that Frege thinks we must *prove* as many things as possible. Let us call this his “Proof Requirement.” We have also seen him appeal to the Simplicity Requirement to justify the Proof Requirement. This justification is very straightforward: since whatever truth is proved thereby becomes not primitive, proving as many truths as possible will leave the minimum number as primitive.

Frege also offers what looks like a second justification for the Proof Requirement. In explaining why we should prove even things that are already certain without proof, he claims that “proof has not only the goal to raise the truth [of what is proved] beyond all doubt, but also to grant insight into the dependence of truths upon one another. After we have convinced ourselves that a boulder is immovable, by trying unsuccessfully to move it, there remains the further question, what is it that supports it so securely?”⁴³ Apparently, truths *depend* upon one another, and proofs give us insight into these dependence relations—presumably by deriving truths from those on which they depend. To place such insight alongside removing doubt as a “goal” of proof is to take it to have cognitive value. So Frege's point seems to be this: since many provable things are not subject to doubt, the goal of removing doubt does not justify the Proof Requirement, but since proofs give cognitively valuable insight into dependence relations, the goal of gaining such insight *does* justify it. Of course, for this to justify proving *what-*

⁴³Frege 1884, §2.

ever we can, it must be that *all* admissible proofs grant insight into dependence relations. (If some did not, it would not justify giving *those* proofs.) When he says that proofs grant insight into dependence, then, Frege must mean that *all* proofs must follow dependence relations. Call this Frege's "Dependence Requirement".

So we seem to have two independent justifications for the Proof Requirement: one resting on the Simplicity Requirement, and another resting on the Dependence Requirement and the cognitive value of insight into dependence relations.

3.2 Frege's Treatment of Dependence is Puzzling

There is something puzzling here, rooted in the fact that Frege does not say anything about dependence relations before claiming that it is a goal of proof to reveal them, and also says almost nothing about them elsewhere. Let me first discuss relevant other passages, and then say why this is puzzling.

There are two other passages which closely mirror the one just discussed, by contrasting a goal of proof which concerns something like removing doubt with another goal which concerns revealing relations among truths and justifies the Proof Requirement. But these passages, at most, *repeat* what is said in the one just discussed; no significant information about dependence relations is added.⁴⁴

Shortly after the passage mentioning dependence, Frege approvingly quotes

⁴⁴Frege 1914 (220) contrasts the goal to "make [truths] more certain" with that of letting "the relations of the [truths] to one another emerge," while Frege 1884 (§17) contrasts the goal to "convince us of the truth of what is proved" with that to "bring out logical connections among truths." Though Frege talks only of "relations" and "logical connections" in these passages, the goal of proof is presumably not to reveal *any old* relations among truths, nor even any *logical* relations. He is presumably talking here about the same relation that he elsewhere describes in terms of dependence.

Leibniz's claim that proofs must follow the "connection and natural order of truths." This "natural order," then, is plausibly induced by dependence relations.⁴⁵ But Frege says almost nothing here about these relations or the order they induce. His point, like Leibniz's in the quoted passage, is just that the order which proofs must follow is *not* the order in which any individual person happens to discover them.⁴⁶ Leibniz himself says much more about the natural order, and if we could assume that Frege endorsed these views, it would tell us more about his views of dependence relations. But Frege does not explicitly endorse any more of what Leibniz says, and cannot endorse much of it. (For example, Leibniz famously holds that every truth can be proved from "identities" of the form "*a* is *a*" or "*a* is not not-*a*": "all other truths are reduced to [identities] with the aid of definitions or by the analysis of concepts."⁴⁷ He takes such proofs to reflect the natural order and be of great cognitive value, providing "what I value most highly...a glimpse of the true *source* of [these] truths."⁴⁸ But Frege denies that every truth can be proved from identities and that these are the best proofs; he even takes Leibniz's other claims to "lose some weight" in the light of these problematic views.⁴⁹)

⁴⁵Frege 1879, §13. Detlefsen 1988 98, Jeshion 2001, 945, and Shapiro 2009, 183 read it this way.

⁴⁶"The question here does not concern the history of our discoveries, which is different in different men, but the connection and natural order of truths, which is always the same." (Leibniz 1765, 470.) The view that proofs should follow the order of discovery was attributed by Leibniz to his opponent, Locke. (See Wilson 1967.)

⁴⁷Leibniz 1680–1684, 267.

⁴⁸Leibniz 1765, 523. The speaker is Theophilus, who represents Leibniz's view.

⁴⁹Frege 1884 (§15). It is worth mentioning that Leibniz 1765 thinks humans cannot do most proofs from identities: I cannot, for example, prove that I exist from the claim that I am me, because "only God can see how the two items I and existence are connected." (469) He thus contrasts a natural order which only God can follow with a *second* natural order which he recommends for us. Frege cannot think of dependence relations as inducing Leibniz's *second* order either, because

Similarly, Frege's dependence relation is reminiscent of Bernard Bolzano's "grounding" relation, which leads Bolzano to a "grounding" requirement on proof. ("As the establishment of the objective ground is something so useful that we should communicate it...as often as possible...the proven truth [should be] derived from its objective ground."⁵⁰) If we could assume that Frege thinks of dependence relations the way Bolzano does grounding relations, it might tell us more about his views. But Frege never cites Bolzano, nor does he apply the Bolzanian term "grounding," to dependence relations.⁵¹

The lack of information about dependence relations is somewhat obscured by secondary literature, in which the dependence relation is among the most widely-discussed aspects of Frege's view of science.⁵² But this is possible only because the claims in these discussions lack adequate textual support. For example: it is regularly claimed that Frege thinks dependence relations are *asymmetric*, in that if A depends on B, then B does not depend on A.⁵³ But no textual evidence is

in it, we do not prove anything that is already maximally certain unless we can do so from identities. (This is why the "Cartesian Principle...*I am*, is an axiom, and... a *primitive truth*... in the natural order of our knowledge.")

⁵⁰Bolzano 1837, §525.

⁵¹Frege does talk of "on what, in the deepest grounds, the justification for holding something to be true rests," "grounds of proof," "grounds of judgement," and "grounds of justification," but this talk is not distinctively Bolzanian, and he never talks of grounding relations among *truths*. At one point, he mentions in passing the "relation of ground and consequent," but never elaborates. (Frege 1884 §3, §17; Frege 1879–1891, 3; Frege 1880–1881, 42.) Some commentators on Frege talk interchangeably of "dependence" and "grounding," but they only mislead themselves by doing so. (Jeshion 2001, for example, argues that Frege sees an important difference between *a priori* sciences and empirical ones partly because he "never says that primitive truths of empirical sciences 'ground' other non-basic truths." That is no difference: he never says that *any* truths "ground" any others.)

⁵²It receives serious attention from, e.g., Detlefsen 1988, Jeshion 2001, and Shapiro 2009.

⁵³Shapiro 2009 claims that "like Bolzano's...ground-consequence relation, Frege's dependency relation is asymmetric: if proposition A depends on proposition B, then B does not depend on A."

cited for this claim, which hardly follows merely from Frege’s word “dependence” (“*Abhängigkeit*”); like stones in an arch, things can depend upon one another. And indeed, Frege is indirectly committed to denying such asymmetry. For he says:

it is conceivable that there is a truth A and a truth B, of which each could be proved from the other in conjunction with truths C, D, E, F, while the truths C, D, E, F, are not enough on their own for a proof of A or a proof of B...[In such a case], we have the choice whether to regard A, C, D, E, F as axioms and B as a theorem, or B, C, D, E, F as axioms, and A as a theorem...the possibility of one system does not rule out the possibility of another.⁵⁴

That is: sometimes, we can pick between proving A from B, and B from A; either is acceptable. But if the dependence relation between truths on which the admissibility of a proof depended were asymmetric, then this would be impossible—the acceptability of one system would rule out that of the other. He must, then, think that sometimes, two truths depend on each other.⁵⁵ Here and elsewhere, commentators’ claims about dependence are unmoored from Frege’s own.⁵⁶

(184) Detlefsen 1988 agrees that “the...relation is asymmetrical.” (endnote 7.)

⁵⁴Frege 1914, 205. He makes the same point at Frege 1879, §13 and Frege 1923–1926 (49).

⁵⁵Perhaps in an effort to resolve the conflict between this passage and the asymmetry assumption, Jeshion 2001 claims that cases like these are ones in which *neither A nor B* depends on the other: whichever truth we prove “still does not admit of proof in the sense that its truth is not grounded on any other propositions.” (951) But she gives no evidence that Frege thinks this, and indeed he cannot: the whole reason he brings up dependence relations is in discussing why we should prove precisely truths like these, which are acceptable as axioms (and hence not in doubt).

⁵⁶To take one more example: Jeshion 2001 observes that Frege 1885 claims that primitive truths involve simple *concepts*, and extrapolates that the order induced by the dependence relation “is a structuring of propositions...according to their relative simplicity and complexity.” (945) But even

Frege says, then, says almost nothing about dependence relations, nor does he endorse anyone else's views about them. This is puzzling. For without knowing the first thing about dependence relations, how are we supposed to determine which truths depend on which others—and without knowing that, how are we to adhere the Dependence Requirement, or check whether Frege does? And equally importantly: without making clear what sort of relation he is talking about, how could Frege expect us to agree that the cognitive value of insight into it justifies a requirement to prove as many things as possible?

3.3 The Simplicity Requirement Again

I think these puzzles can be resolved by returning to the “dependence” passage with the centrality of the Simplicity Requirement in mind. Here it is as a whole:

Proof has not only the goal of removing all doubt in the truth of a proposition, but also, to grant insight into the dependence of truths upon one another. After we have convinced ourselves that a boulder is immovable, by trying unsuccessfully to move it, there remains the further question, what is it that supports it so securely? The further one pursues these enquiries, the fewer the primitive truths to which we trace everything back; and this simplification is in itself a goal worth striving for.

if Frege thinks this order starts from truths involving only simple concepts, this does not imply that the *rest* of the order continues in order of increased complexity. A chain of proof in mathematics often starts from primitive truths involving only simple concepts, detours through truths involving complex ones, and ends with truths involving only simple ones again.

Above, I read this passage as containing two independent reasons for the Proof Requirement: one in the last sentence based on the Simplicity Requirement, and another in the other two, based on the Dependence Requirement. This reading generates the puzzles just described.

An alternative reading is suggested when we recall that Frege often justifies other requirements by tracing them back to the Simplicity Requirement, and note the emphasis here on the *non-instrumental value* of minimizing the number of primitive truths and the *lack* of any such emphasis for insight into dependence relations. Perhaps the last sentence tells us *why* it is a goal of proof to have insight into dependence relations: because proofs that give such insight help to satisfy the Simplicity Requirement. Then there would be no independent justification here for the Proof Requirement from the Dependence Requirement. Instead, the Proof Requirement *and* the Dependence Requirement would be justified here by the Simplicity Requirement: *because* we must minimize the number of primitive truths, proofs must follow dependence relations *and* we must do as many of them as possible.

How could the Dependence Requirement be justified by the Simplicity Requirement? The key is Frege's claim that "the further one pursues these enquiries"—enquiries into *dependence relations*—"the fewer the primitive truths to which we trace everything back." With this statement, Frege actually gives us useful information about dependence relations, attributing to them a feature *not* shared by most logical implication relations. There are many logical implication relations whose discovery cannot lead to a reduction of the number of primitive truths,

since the corresponding proof could not appear in a system with the fewest possible primitive truths.⁵⁷ Frege's claim is that dependence relations are not like this; that there is such a relation *only* where there is a proof that could appear in a system with the fewest possible primitive truths. This is to claim that a dependence relation is a *local* representative, at the level of individual truths, of the *global* feature of systems called for by the Simplicity Requirement. Given this claim, the Simplicity Requirement does imply the Dependence Requirement, for in following the former, we will necessarily end up following the latter.

This alternative reading fits with the passage, and is recommended by the fact that it avoids the puzzles. For on this reading, the Dependence Requirement is not part of an independent argument for the Proof Requirement, so there is no puzzle how it could do so when we know so little about dependence relations. And on this reading, we do not need to be told more about dependence relations to be able to recognize them: for we already know how to minimize the number of primitive truths, and doing so will necessarily lead us to dependence relations. This, I think, makes it the best reading. It does, however, generate two new questions worth discussing.

First: why bother mentioning dependence relations at all, if the point of the passage is to justify the Proof Requirement, and the Simplicity Requirement can straightforwardly do so all on its own? (Recall: since whatever truth is proved

⁵⁷Suppose that a domain comprises just three truths: T1, T2, and T3, all of which are knowable without proof and otherwise suitable to be primitive. T1 is logically implied by T2 and T3 together, while T1 alone logically implies T2 and T3 both. The Simplicity Requirement requires us to prove T2 and T3 from T1. Recognizing that T1 is logically implied by T2 and T3, then, does not help to minimize the number of primitive truths.

thereby becomes not primitive, proving as many truths as possible will leave the minimum number as primitive.) Part of the answer may be that talk of dependence relations serves to shift emphasis in the way just described: from a global feature of systems to a local one. Such a shift is appropriate when, as in this passage, the question under discussion will often face us as a local one: why we ought to prove a particular truth that is already certain.

Second: by emphasizing Frege's claim that there is a dependence relation only where there is a proof that can occur in a maximally simple system, this reading raises the question *why* that would be so: it looks like a coincidence in need of explanation.⁵⁸ Those worried about such a coincidence should recall a possibility which Bolzano raises (but does not endorse) about his own grounding relation: "the concept of ground and consequence...may turn out to be none other than the concept of an ordering of truths which allows us to deduce from the smallest number of...premises the largest possible number of the remaining truths."⁵⁹ A deflationary picture of dependence along these lines is always available: the coincidence will be removed if the dependence relation *just is*—is *defined as*—the local representative of the global feature called for by the Simplicity Requirement.

⁵⁸Shapiro 2009 makes the related observation that for Frege's Dependence Requirement to be compatible with his requirement that we know primitive truths without proving them, some set of truths must *both* stand at the head of chains of dependence leading to every other truth *and* be among those of which we can know without proof. Shapiro sees a "large dose of preestablished harmony" in the idea that the truths are "structured in such a pleasing way, a way designed to facilitate proper...knowledge...by beings just like us." (186) [Excised for Anonymity.]

⁵⁹Bolzano 1837, §221.

4 Conclusion

My main goal in this paper has been to show the centrality of Frege's Simplicity Requirement to his thinking. We have seen that he accepts it and why, and we have seen how it motivates him in placing additional requirements on science, choosing his logical concepts, introducing the truth-values, etc. I believe several other issues in Frege's philosophy are illuminated by recognizing the centrality of the Simplicity Requirement,⁶⁰ but let me close by considering how it affects our understanding of Frege's work as a whole.

When Michael Dummett discusses Frege's appeal to simplicity in introducing the truth-values, he claims that for Frege to achieve "a great simplification...at the price of a highly implausible analysis of language" would be a "ludicrous deviation" as well as a "gratuitous blunder."⁶¹ Setting aside the question whether there is any blunder here, it is at least clear that it is no *deviation* from what Frege usually does: the Simplicity Requirement pervades Frege's work. By contrast, one is hard-pressed to find evidence that Frege assigns any comparable importance to achieving a plausible "analysis of language".⁶² Dummett's remark, then, seems to betray a misapprehension about Frege's priorities and goals. This misapprehension bears on a broader question faced by readers concerning the extent to which Frege is to be understood as a "scientist"—in particular, a mathematician—and

⁶⁰[Excised for anonymity.]

⁶¹Dummett 1973, 183-184; see footnote 25 above.

⁶²See Weiner 1996.

that to which he is to be understood as a “philosopher.”⁶³ We are pushed in the latter direction whenever we find Frege motivated by what we take to be distinctively “philosophical” concerns—for Dummett, the central such concern being the analysis of language. By contrast, the more we find him motivated by general scientific concerns, the more we are pushed toward reading him as a “scientist.”

The considerations in this paper may seem to push us in the latter direction. After all, the Simplicity Requirement is a general requirement on scientific systems, and it motivates a great deal of what Frege does. But we have also seen Frege engage in explicit reflection on *why* there is a Simplicity Requirement; and this is to reflect explicitly on cognitive values and on what it is to be in the best cognitive state, naturally leading Frege, for example, to take a stand on what the “essence of explanation” is. These are paradigmatic philosophical issues. In emphasizing the Simplicity Requirement, then, the most immediate effect of this paper is not to shift our interpretation in the scientific direction, but to highlight new aspects of his work to focus on when we ask what kind of thinker he is.

References

Beiser, Frederick (2014). *The Genesis of Neo-Kantianism, 1796-1880*. Oxford: Oxford University Press.

Bellucci, Francesco, Amirouche Moktefi, and Ahti-Veikko Pietarinen (2017). “Simplex sigillum veri: Peano, Frege, and Peirce on the Primitives of Logic”. In:

⁶³See, for example, the first section of Jeshion 2001.

- History and Philosophy of Logic*. DOI: 10.1080/01445340.2017.1358414, pp. 1–16.
- Bolzano, Bernard (1837). *Wissenschaftslehre*. Sulzbach: Seidel.
- Burge, Tyler (1986). “Frege on Truth”. In: *Frege Synthesized*. Ed. by Haaparanta and Hintikka. Dordrecht: Kluwer Academic Publishers, pp. 97–154.
- (1998). “Frege on Knowing the Foundation”. In: *Mind* 107, pp. 305–347.
- de Jong, Willem (1996). “Gottlob Frege and the analytic-synthetic distinction within the framework of the aristotelian model of science”. In: *Kant-Studien* 87, pp. 290–324.
- de Jong, Willem and Arianna Betti (2010). “The Classical Model of Science: A Millenia-old model of Scientific Rationality”. In: *Synthese* 174, pp. 185–203.
- Detlefsen, Michael (1988). “Fregean Hierarchies and Mathematical Explanation”. In: *International Studies in the Philosophy of Science* 3, pp. 97–116.
- Dummett, Michael (1973). *Frege Philosophy of Language*. London: Harper and Row.
- Frege, Gottlob (1879). *Begriffsschrift*. Halle: Louis Nebert Verlag.
- (1879–1891). “Logik”. In: Frege 1969, pp. 1–8.
- (1880–1881). “Booles rechnende Logik und die Begriffsschrift”. In: Frege 1969, pp. 9–52.
- (1884). *Die Grundlagen Der Arithmetik*. Breslau: Wilhelm Koebner Verlag.
- (1885). “Über formale Theorien der Arithmetik”. In: *Jenaische Zeitschrift für Naturwissenschaft* 19, Heft II, pp. 94–104.

- Frege, Gottlob (1893a). *Grundgesetze der Arithmetik: Begriffsschriftlich Abgeleitet*. Vol. 1. Jena: Hermann Pohle Verlag.
- (1893b). *Grundgesetze der Arithmetik: Begriffsschriftlich Abgeleitet*. Vol. 2. Jena: Hermann Pohle Verlag.
- (1902). “Frege an Russell, 20.10.1902”. In: Frege 1976, pp. 231–232.
- (1914). “Logik in der Mathematik”. In: Frege 1969, pp. 219–270.
- (1918–1919). “Die Verneinung”. In: *Beiträge zur Philosophie des deutschen Idealismus* 1, pp. 143–157.
- (1923–1926). “Gedankengefüge”. In: *Beiträge zur Philosophie des deutschen Idealismus* 3, pp. 36–51.
- (1923/1925). “Logische Allgemeinheit”. In: Frege 1969, pp. 278–281.
- (1924/1925). “Erkenntnisquellen der Mathematik und der mathematischen Naturwissenschaften”. In: Frege 1969, pp. 286–294.
- (1969). *Nachgelassene Schriften*. Ed. by Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach. Hamburg: Felix Meiner.
- (1976). *Wissenschaftlicher Briefwechsel*. Ed. by Gottfried Garbiel et al. Hamburg: Felix Meiner.
- (1979). *Posthumous Writings*. Ed. by Hermes, Kambartel, and Kaulbach. Oxford: Blackwell.
- Friedman, Michael (1974). “Explanation and Scientific Understanding”. In: *The Journal of Philosophy* 71, pp. 5–19.
- Hempel, Carl G. and Paul Oppenheim (1948). “Studies in the Logic of Explanation”. In: *Philosophy of Science* 15, pp. 135–175.

- Jeshion, Robin (2001). "Frege's Notions of Self-Evidence". In: *Mind* 110, pp. 937–976.
- (2004). "Frege: Evidence for Self-Evidence". In: *Mind* 113, pp. 131–138.
- Kant, Immanuel (1781/1787). *Critique of Pure Reason*. Trans. by Kemp Smith. 1929. London: Macmillan.
- Kitcher, Philip (1989). "Explanatory Unification and the Causal Structure of the World". In: *Minnesota Studies in the Philosophy of Science* 13, pp. 410–505.
- Kvanvig, Jonathan (2003). *The Value of Knowledge and the Pursuit of Understanding*. Cambridge: Cambridge University Press.
- Leibniz, Gottfried Wilhelm (1680–1684). "First Truths". In: *Philosophical Papers and Letters*. Ed. and trans. by Leomker. 1992. Dordrecht: Kluwer Academic Publishers, pp. 267–271.
- (1765). *New Essays Concerning Human Understanding*. Trans. by Langley. 1896. London: Macmillan.
- Liebmann, Otto (1876). *Zur Analysis der Wirklichkeit*. Straßburg: Karl Trübner Verlag.
- Macbeth, Danielle (2016). "Frege and the Aristotelian Model of Science". In: *Early Analytic Philosophy - New Perspectives on the Tradition*. Ed. by Costreie. Dordrecht: Springer, pp. 31–48.
- Schlimm, Dirk (2017). "On Frege's Begriffsschrift Notation for Propositional Logic: Design Principles and Trade-Offs". In: *History and Philosophy of Logic*. DOI: 10.1080/01445340.2017.1317429, pp. 1–27.

- Schröder, Ernst (1880). “Rezension von Frege, *Begriffsschrift*”. In: *Zeitschrift für Mathematik und Physik* 25: Historisch-literarisch Abteilung, pp. 81–94.
- Shapiro, Stewart (2009). “We Hold These Truths To Be Self-Evident: But What Do We Mean By That?” In: *The Review of Symbolic Logic* 2, pp. 175–207.
- Sluga, Hans (1987). “Frege Against the Booleans”. In: *Notre Dame Journal of Formal Logic* 28, pp. 80–98.
- Sober, Elliot (2002). “What is the problem of simplicity?” In: *Simplicity, Inference and Modelling*. Ed. by Keuzenkamp Zellner and McAleer. Cambridge: Cambridge University Press, pp. 13–31.
- (2015). *Ockham’s Razors*. Cambridge: Cambridge University Press.
- Sorell, Tom, G.A.J Rogers, and Jill Kraye, eds. (2010). *Scientia in Early Modern Philosophy*. Dordrecht: Springer.
- Swinburne, Richard (1997). *Simplicity as Evidence of Truth*. Milwaukee: Marquette University Press.
- Weiner, Joan (1990). *Frege in Perspective*. Ithaca: Cornell University Press.
- (1996). “Has Frege a Philosophy of Language?” In: *Early Analytic Philosophy: Frege, Russell, Wittgenstein*. Ed. by Tait. Chicago: Open Court, pp. 249–272.
- (2004). “What was Frege Trying to Prove? A Response to Jeshion”. In: *Mind* 113, pp. 115–129.
- (2007). “What’s in a Numeral? Frege’s Answer”. In: *Mind* 116, pp. 677–716.
- (2020). *Taking Frege at his Word*. Oxford: Oxford University Press.

- Wilson, Margaret Dauler (1967). “Leibniz and Locke on First Truths”. In: *Ideas and Mechanism*. 1999. Princeton: Princeton University Press, pp. 353–372.
- Wittgenstein, Ludwig (1922). *Tractatus Logico-Philosophicus*. Trans. by Ogden. London: Routledge.