Anomalous Hall effect in twisted bilayer graphene

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Sachdev Group Meeting

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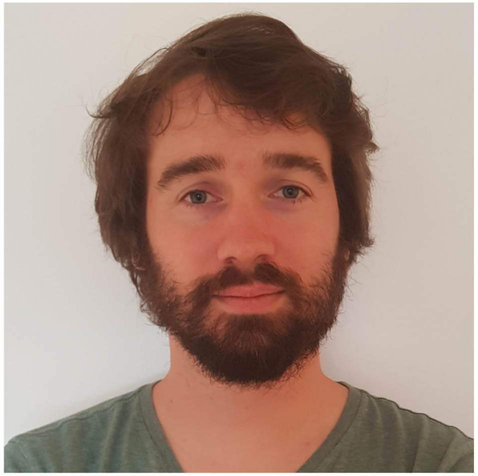
In collaboration with:



Nick Bultinck
UC Berkeley

Michael P. Zaletel
UC Berkeley

N. Bultinck, SC and M. P. Zaletel

arXiv:1901.08110

Twisted bilayer graphene

- Bilayer graphene with a relative twist angle $\boldsymbol{\theta}$
- Nearly flat bands close to the *magic angle* $\boldsymbol{\theta} \sim 1.09°$

Bistritzer, MacDonald, PNAS 2011

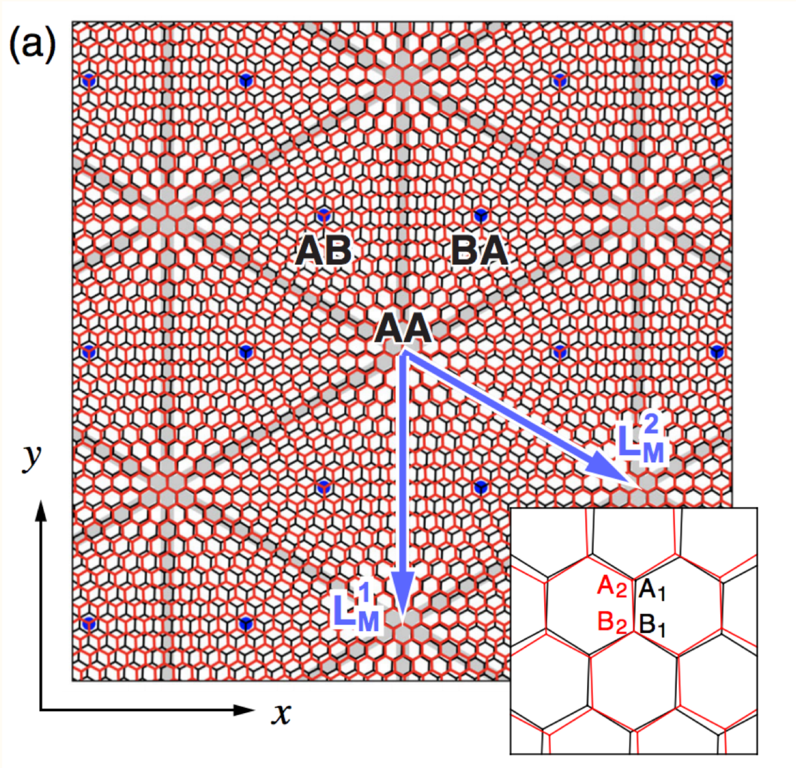
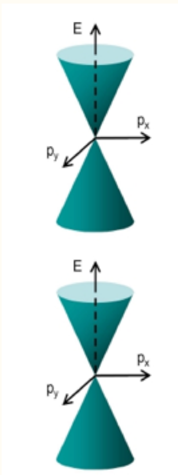
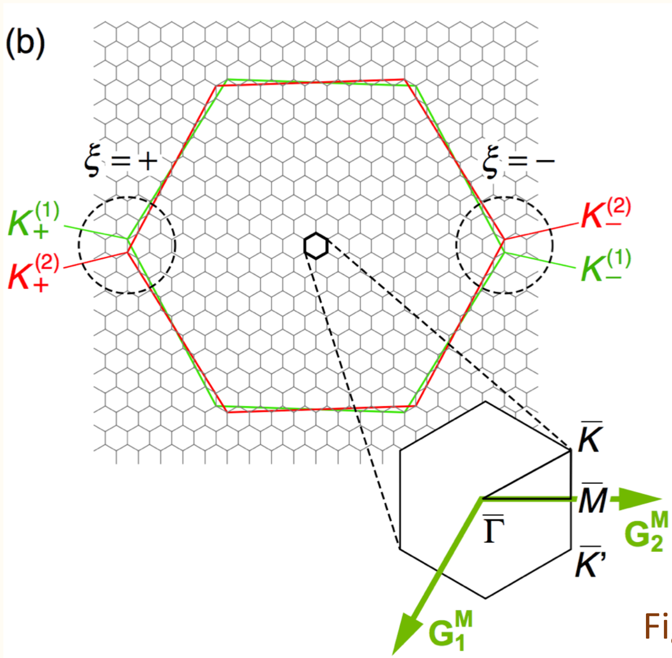


Figure: Koshino et al, PRX 2018

Twisted bilayer graphene

- 8 active low-energy bands (2 layer, 2 valley, 2 spin)
- Inversion C_{2z} and time-reversal T protects Dirac cones   Po *et al*, PRX 2018
- Dirac cones within the same valley have identical chirality





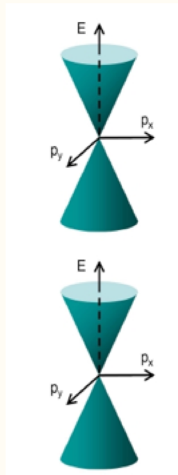
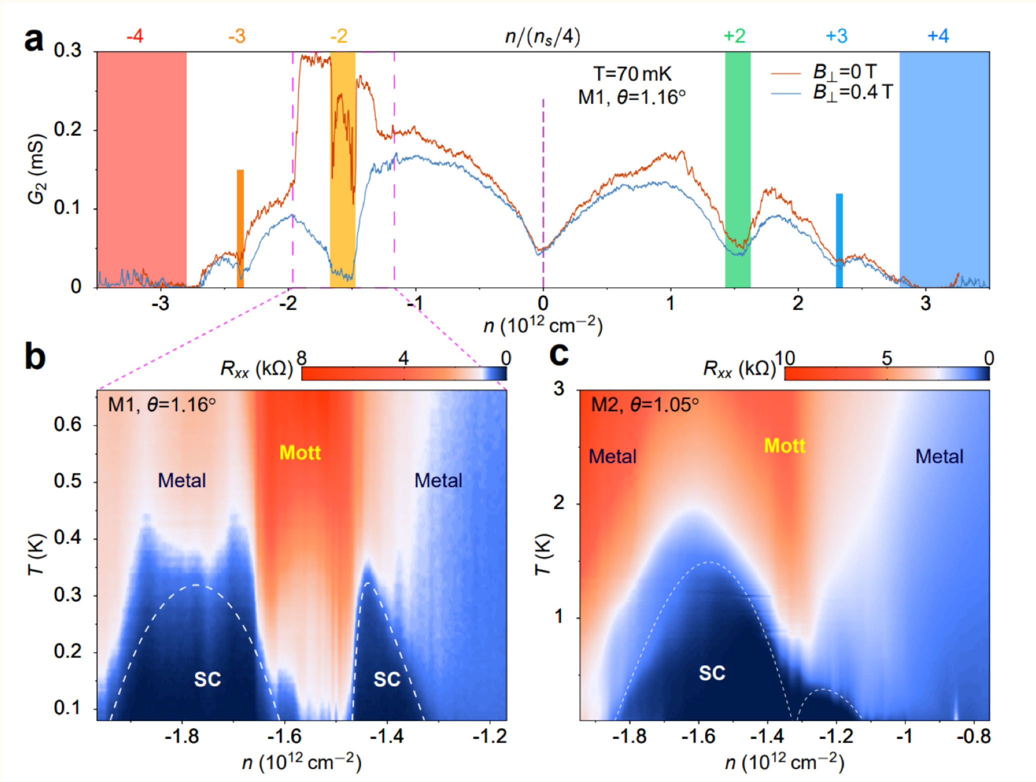


Figure: Koshino *et al*, PRX 2018

Twisted bilayer graphene

- Correlated insulators at integer fillings of Moire unit cells
- Superconductivity on doping away from these insulators



Cao *et al* Nature², 2018
Yankowitz *et al*, Science 2018
Efetov *et al*, KITP talk, 2019

Outline

- Observation of anomalous Hall effect
- Chern bands and valley Zeeman effect
- Metal-insulator competition
- Fate of the inter-valley coherent/excitonic phase
- Conclusions and open questions

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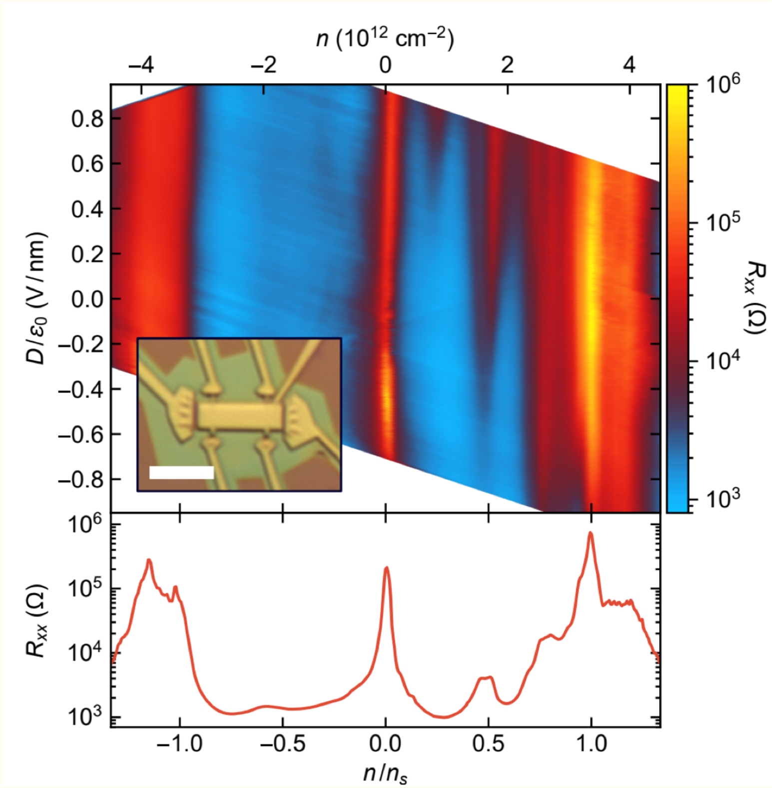
Anomalous Hall effect in TBG

- Insulator at charge neutrality ($\nu = 0$) – Dirac cones gapped out!
- Large resistance at $\nu = 3$ (but not exactly insulating)



Sharpe *et al*,
arXiv:1901.03520

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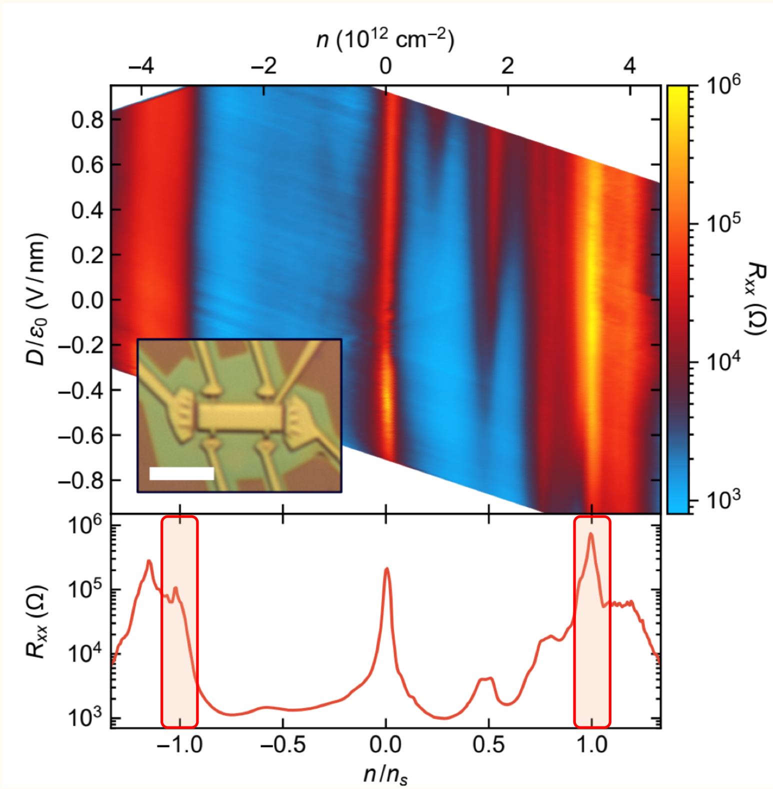

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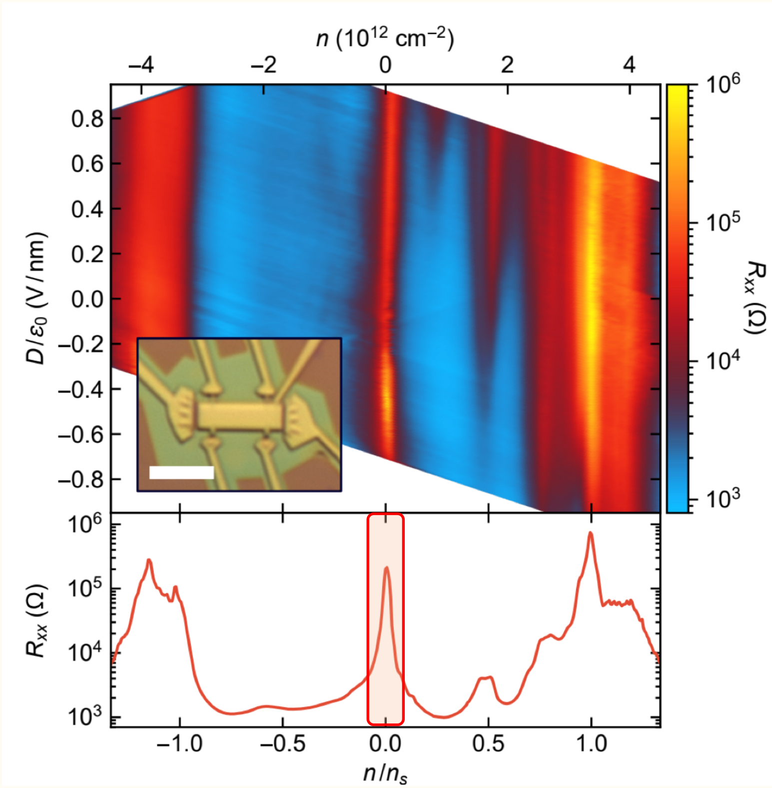

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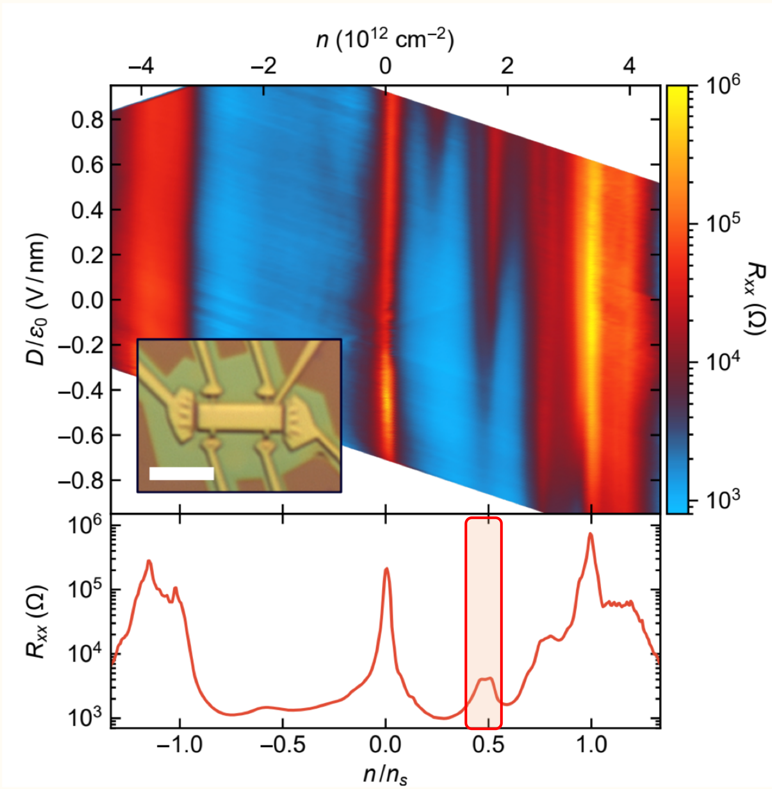

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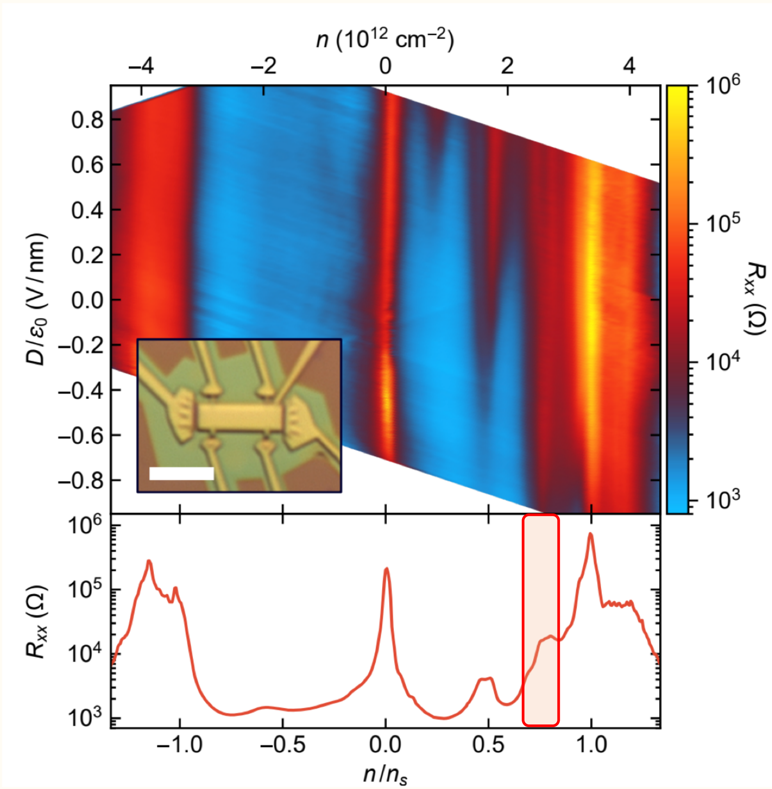

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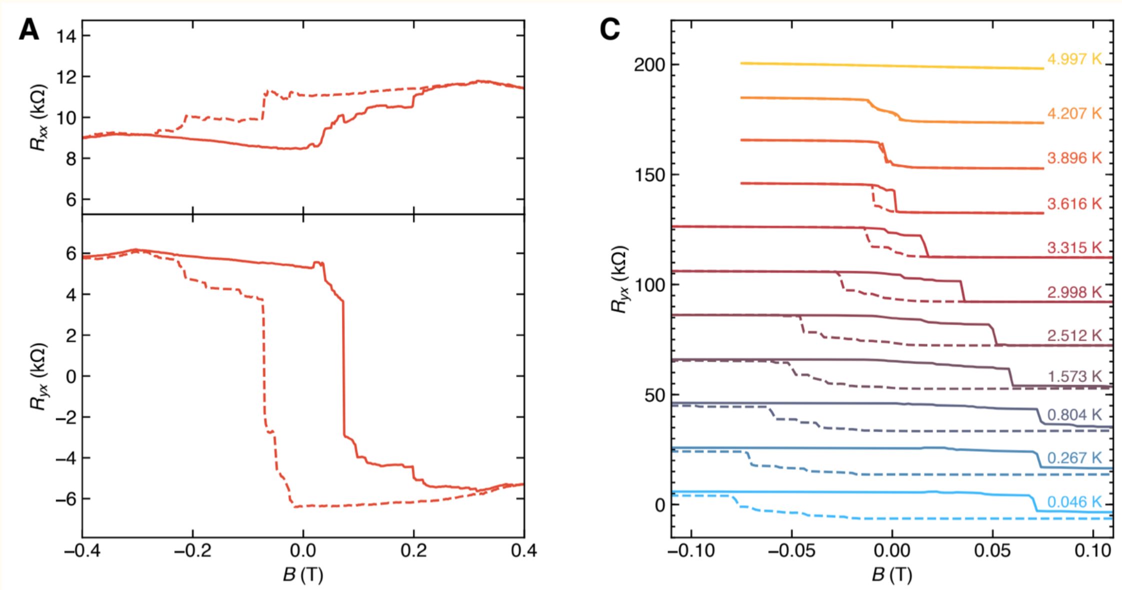
Anomalous Hall effect in TBG

- Large anomalous Hall effect at ν = 3 (out of 4 conduction bands), not quite quantized with $\boldsymbol{\sigma}_{xy} \sim 0.5\ e^2/h$
- Hysteresis loop in presence of a coercive magnetic field
- Signatures of chiral edges from non-local resistance (?)



Sharpe *et al*, arXiv:1901.03520

Anomalous Hall effect in TBG

- *Key feature of the device*: One graphene layer is very closely aligned with hexagonal Boron Nitride (h-BN) substrate

- Effects of h-BN

1. Broken inversion (C_{2z}) symmetry (Dirac cones get gapped)
2. Additional Moire potential (weaker than the original one)

- Hypothesis: Gapped bands can have Chern numbers +C for valley K and -C for valley K′

- Interaction-driven valley polarization $\longrightarrow$ anomalous Hall effect.

- Full valley + spin polarization can cause quantized anomalous Hall conductance

$$\text{At } \nu = 3 : (K, \uparrow), (K, \downarrow), (K', \uparrow)$$

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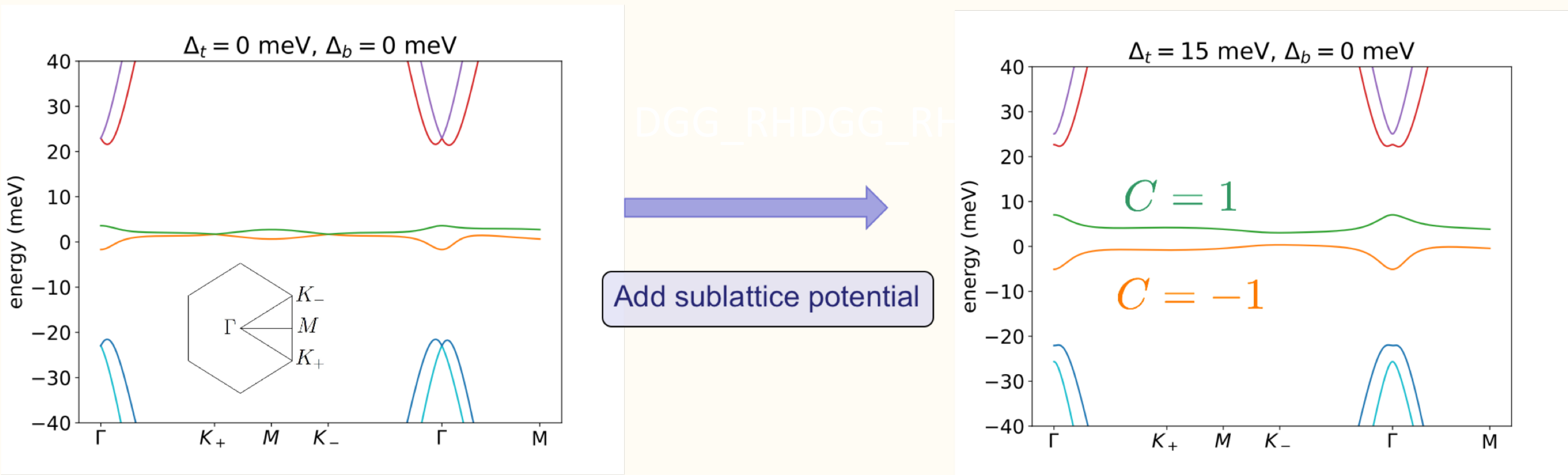
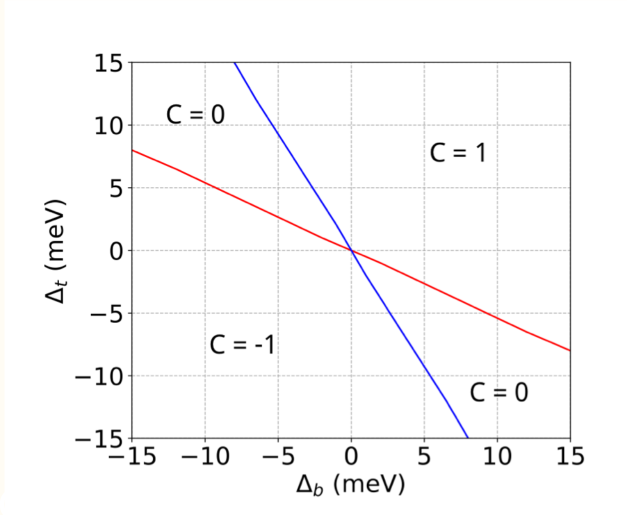
Chern bands and Valley Zeeman effect

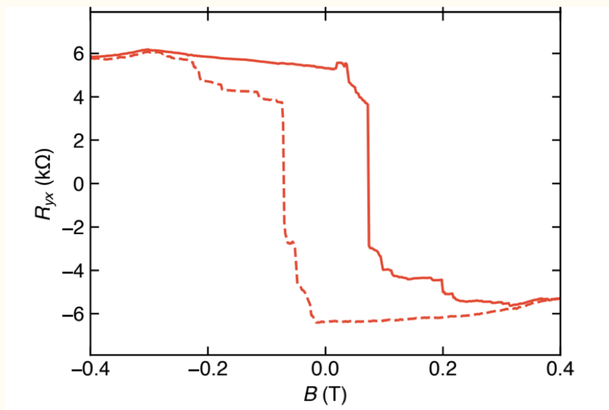
- Band structure calculations using monolayer free-electron bands coupled via interlayer tunneling
- Added sublattice potential + phenomenological lattice relaxation



Chern bands and Valley Zeeman effect

- Generically, topologically non-trivial bands in large parts of the phase diagram
- $C = \pm 1$ for valleys related by time-reversal symmetry
- Large valley Zeeman effect due to orbital magnetic moment, $g_v = -2$ to -6
- Chern bands + spontaneous polarization + valley Zeeman lead to anomalous Hall effect and hysteresis in R_{xy}





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Metal insulator competition

- Toy model: Two bands with opposite Chern numbers ±1 described by dispersive lowest Landau levels, with screened isotropic Coulomb interaction u_0

$$H = H_p + H_{int}$$

$$H_p = \sum_{\mathbf{k},\tau=\pm} \varepsilon_{\mathbf{k}}\, c^{\dagger}_{\mathbf{k},\tau} c_{\mathbf{k},\tau}$$

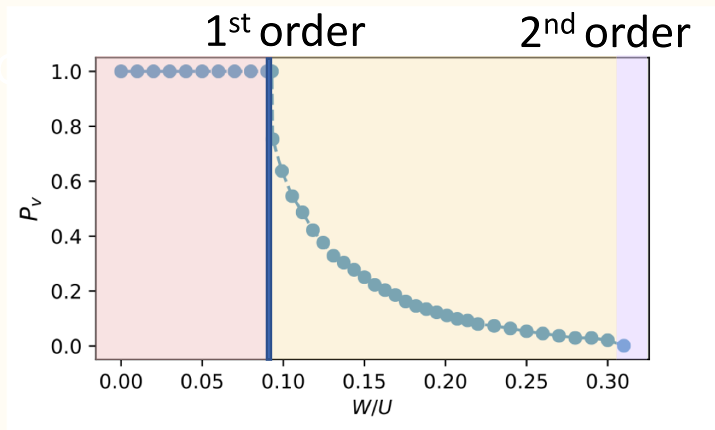
$$H_{int} = \frac{1}{2N_\phi} \sum_{\mathbf{q},\tau,\tau'} V_{\tau,\tau'}(\mathbf{q}) : n_\tau(\mathbf{q}) n_{\tau'}(-\mathbf{q}) :$$

Screened Coulomb

- Metal → Valley polarized metal → Valley polarized insulator transition at half-filling when W/u_0 is tuned

Metal insulator competition

- Toy model: Two bands with opposite Chern numbers ±1 described by dispersive lowest Landau levels, with screened isotropic Coulomb interaction u_0
- Metal → Valley polarized metal → Valley polarized insulator transition at half-filling when W/u_0 is tuned
- Anomalous Hall effect if valley-polarization is non-zero, quantized if fully valley + spin polarized



Valley polarized insulator	Valley polarized metal	Unpolarized metal

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Fate of inter-valley coherence

- In bilayer quantum hall systems, inter-layer coherence (also called exciton condensate) is preferred in presence of small interaction anisotropy between layers

$$|\psi\rangle \sim \prod_{\mathbf{k}} \frac{1}{\sqrt{2}}(c^\dagger_{\mathbf{k},+} + c^\dagger_{\mathbf{k},-})|0\rangle \qquad \Delta_{\mathbf{k}} = \langle c^\dagger_{\mathbf{k},+} c_{\mathbf{k},-}\rangle \neq 0$$

- What is the fate of an analogous inter-valley coherent state?

- IVC state not break time-reversal symmetry → should show no Hall effect at zero field

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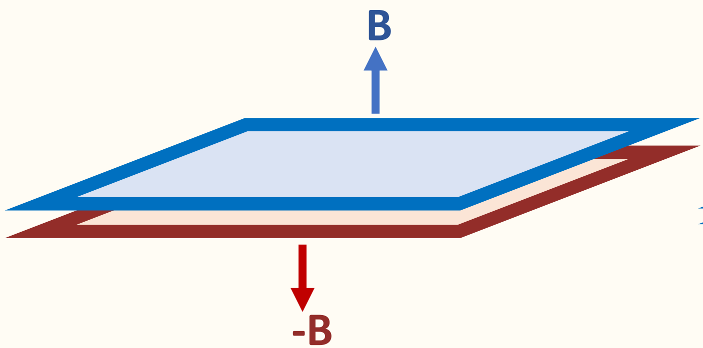
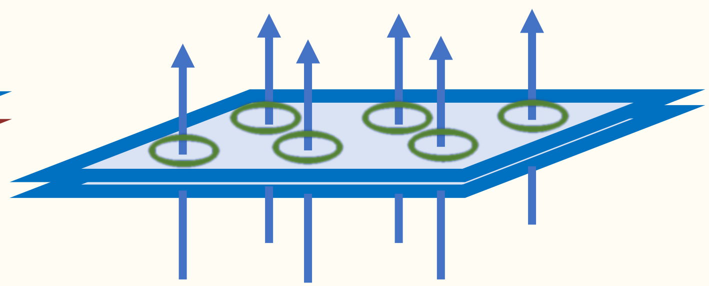
- Twist in the story: The two valleys have opposite Chern numbers

Fate of inter-valley coherence

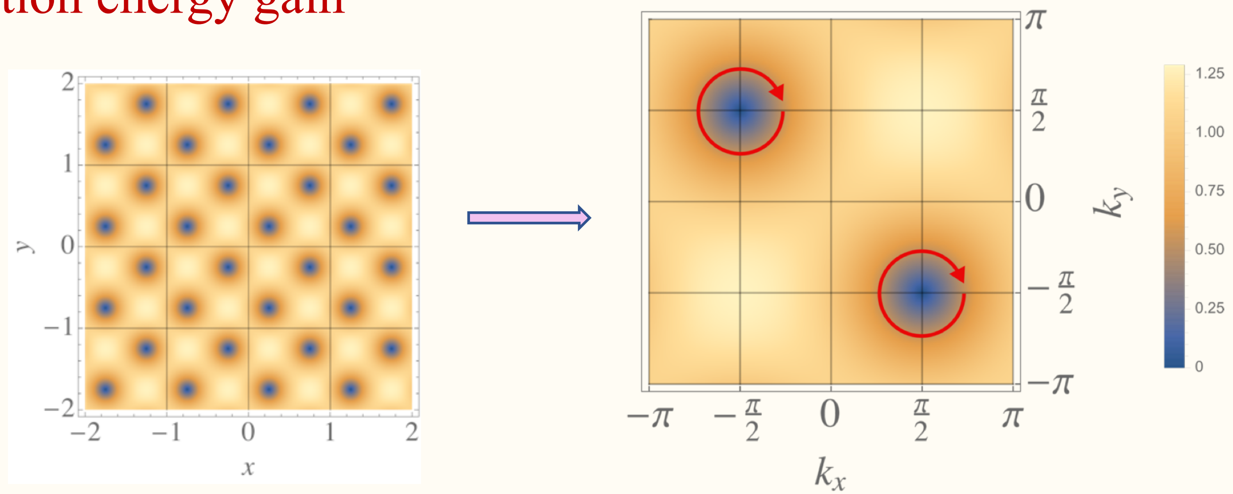
- Lack of IVC: Real space picture
- Electrons moving in Landau levels with opposite Chern numbers see opposite magnetic fields
- The IVC order parameter $\mathbf{\Delta}(\mathbf{r})$ therefore behaves as a superconducting order parameter in a uniform magnetic field
- The flux must penetrate (no Meissner effect allowed)
- Vortices in $\mathbf{\Delta}(\mathbf{r})$ lower correlation energy gain





Fate of inter-valley coherence

- Lack of IVC: Momentum space picture
- Recall: $\Delta_{\mathbf{k}} = \langle c^\dagger_{\mathbf{k},+} c_{\mathbf{k},-} \rangle$
- The phases of the electron creation operators in ± valleys wind around the BZ by $\pm 2\pi$
- Phase of $\boldsymbol{\Delta}_{\mathrm{k}}$ must wind by 4π around the BZ → Nodes in $\boldsymbol{\Delta}_{\mathrm{k}}$ lower correlation energy gain



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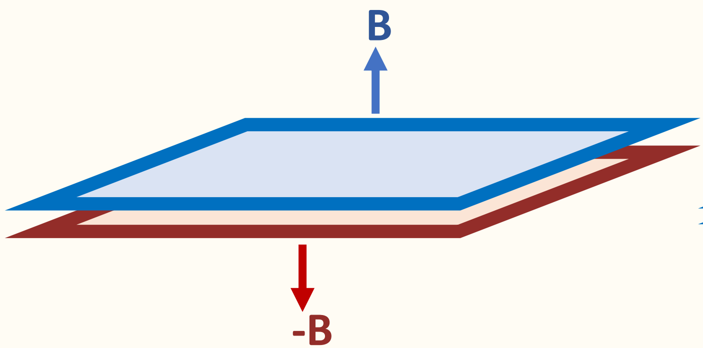
$$V(\mathbf{q}) = u_0(\mathbf{q}) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + u_1(\mathbf{q}) \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

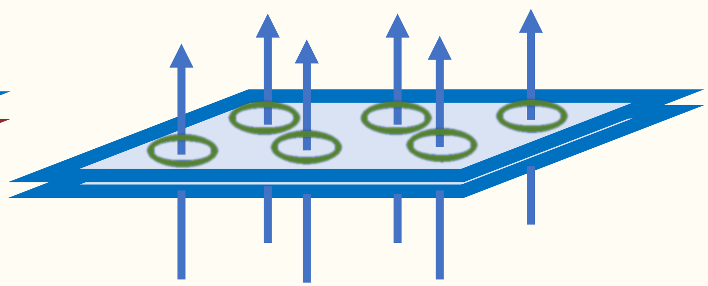
$$e^{HF}(P_v = 1, \Delta_0) - e^{HF}(P_v = 1, 0) \propto \int_{\mathbf{k},\mathbf{q}} u_0(\mathbf{q}) |\Delta_+ - \Delta_-|^2 + \int_{\mathbf{k},\mathbf{q}} u_+(\mathbf{q}) |\Delta_+ + \Delta_-|^2 - 4u_1(\mathbf{0}) \int_{\mathbf{k}} |\Delta_{\mathbf{k}}|^2$$

$$\Delta_\pm = \Delta_{\mathbf{k} \pm \mathbf{q}/2}$$

Fate of inter-valley coherence

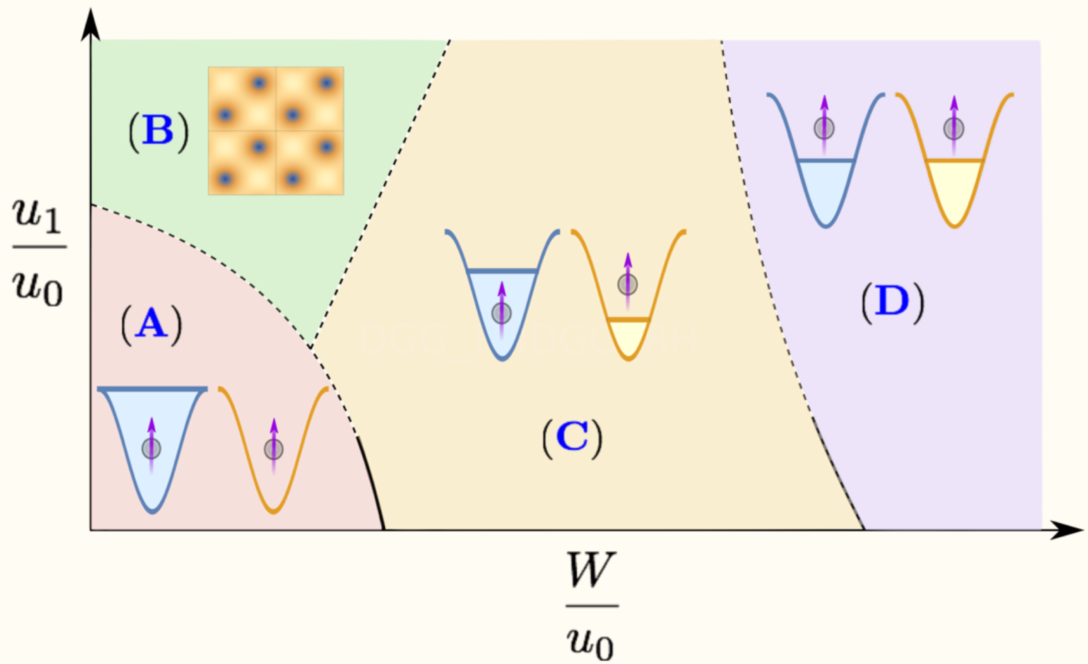
- Lack of IVC: Symmetry argument
- Valley Chern bands with opposite Chern numbers do not have SU(2) valley rotation symmetry
- Valley polarized state breaks a discrete (time-reversal) symmetry and is gapped
- Therefore must be stable to small perturbations
- A novel excitonic vortex lattice phase is possible at large anisotropy





Fate of inter-valley coherence

- Lack of IVC at low interaction anisotropy; excitonic vortex lattice phase is possible when anisotropy is cranked up.



(A) Valley-polarized insulator (B) Exciton vortex lattice
(C) Partially polarized metal (D) Unpolarized metal

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Conclusions and open questions

- Twisted bilayer graphene *aligned* with hexagonal Boron Nitride (h-BN) substrate can host non-trivial Chern bands
- Interaction-driven valley polarization → anomalous Hall effect
- Full valley + spin polarization can cause quantized anomalous Hall conductance

- The hysteresis loop in R_{xy} and its sign can be explained by a large valley Zeeman effect
- The valley polarized state is stable to inter-valley coherence due to topological features of the band structure

Conclusions and open questions

- Nature of the insulator at $\nu = 2$? (Also need to consider spin degrees of freedom)

- Puzzling transport behavior: Gap increases with perpendicular magnetic field and then decreases, while it decreases in an in-plane field quadratically. Why?

Sharpe *et al*, arXiv:1901.03520
Yankowitz *et al*, Science 2018

- What about Landau fans seen in quantum oscillations?

- Phenomenology of the exciton vortex lattice phase? Signatures in spectroscopic or transport probes?

Related work: Y. Zhang, D. Mao, T. Senthil, arXiv:1901.08209,
M. Xie, A. Macdonald, arXiv:1812.04213,
Y. Lin, R. Nandkishore, arXiv:1901.00500

Thank you for your attention!