Quantum fluctuating antiferromagnetism in insulators and metals:

Intertwining topological order with discrete broken symmetries

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Harvard CMT Kids Seminar

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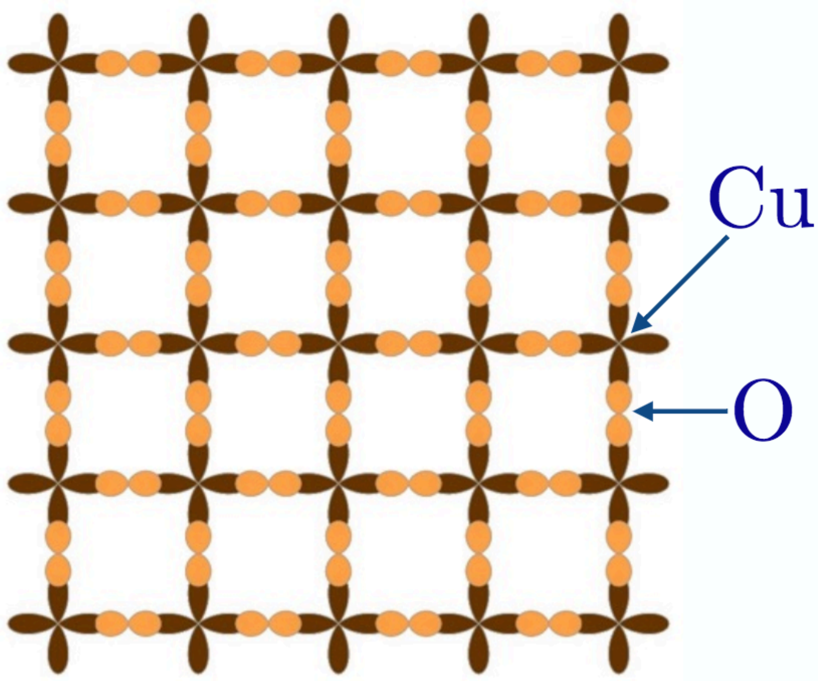
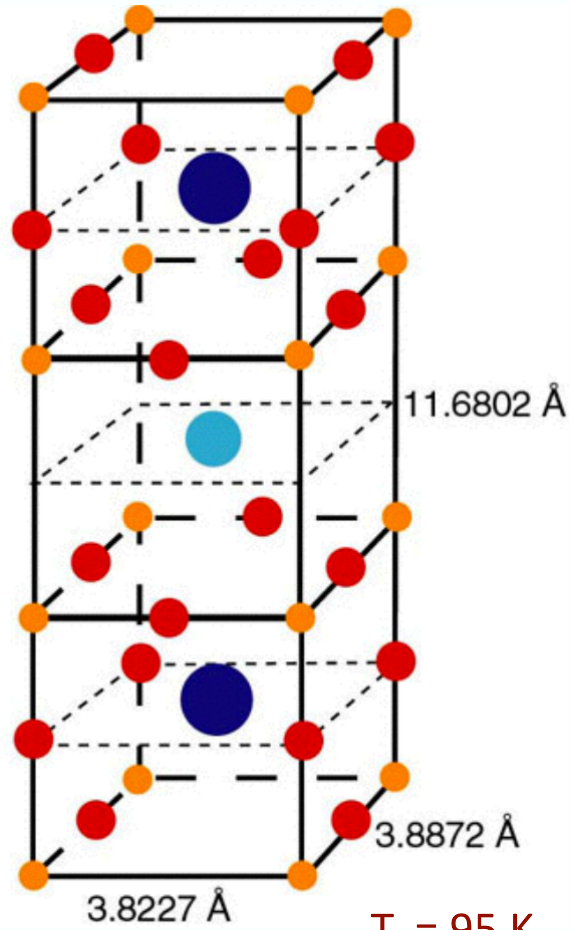
S. Chatterjee and S. Sachdev, **Phys. Rev. B** 95, 2015133, 2017;
S. Chatterjee, S. Sachdev and Mathias S. Scheurer, arXiv: 1705.06289

Structure of cuprate superconductors

High temperature superconductors

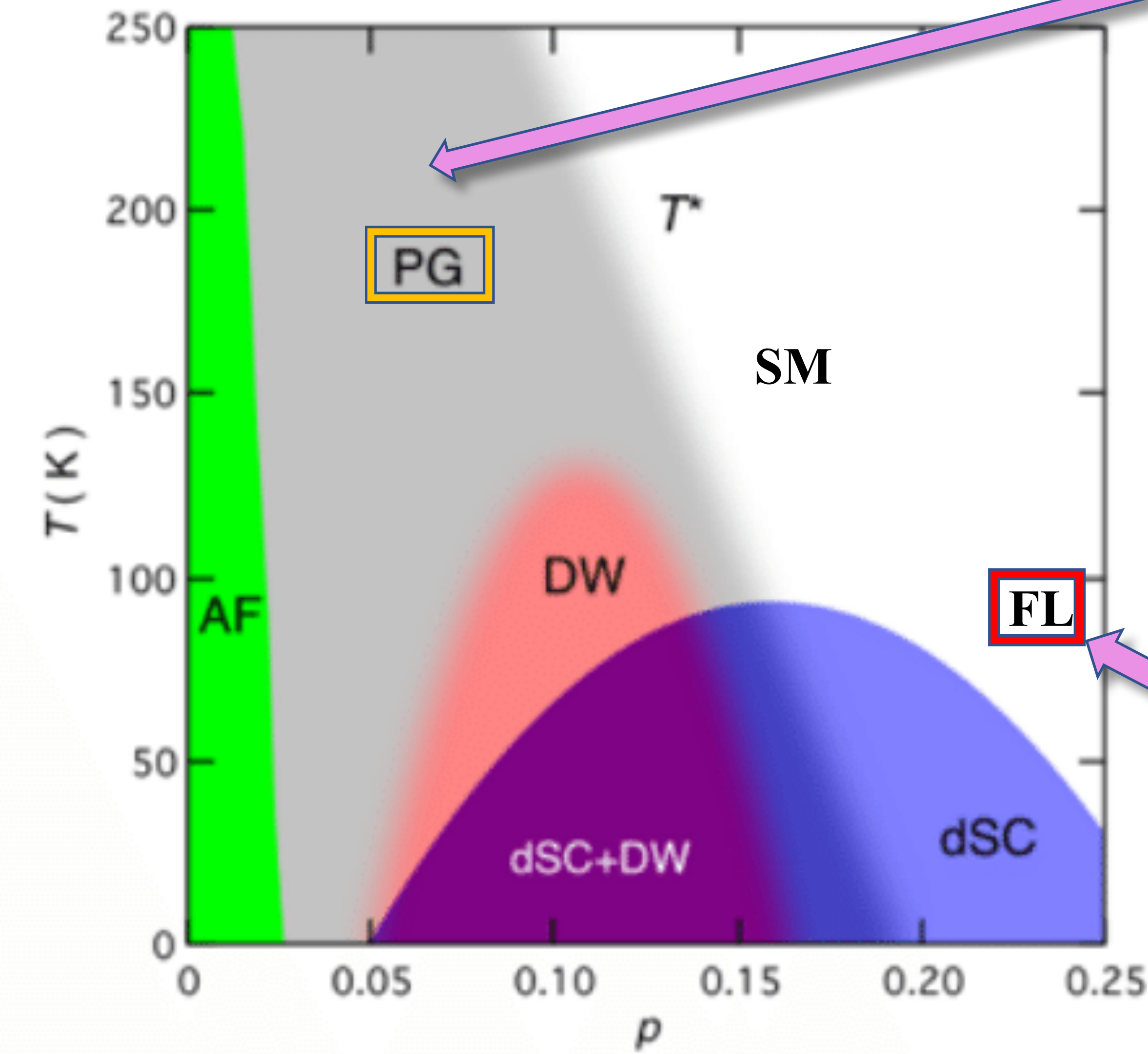


CuO_2 plane

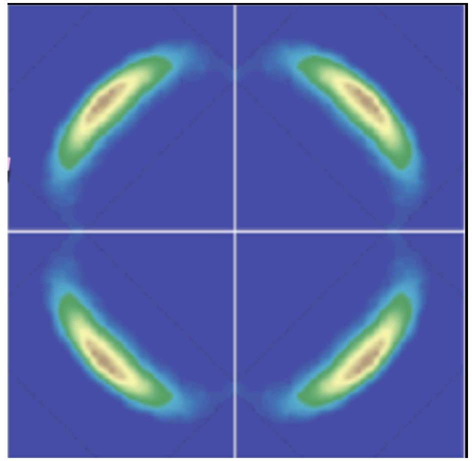


T_c = 95 K

$YBa_2Cu_3O_{6+x}$



Pseudogap metal:
Fermi arcs



Conventional Fermi liquid:
Large hole Fermi surface of size 1 + p.

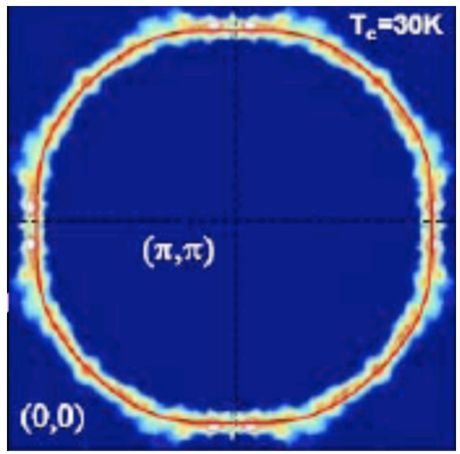


Figure credits: K. Fujita *et al*, Nature Physics **12**, 150–156 (2016)
M. Plate *et al*, PRL. 95, 077001 (2005)

Evidences of metallic behavior in PG phase

- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$, with $1/\tau \sim \omega^2 + T^2$

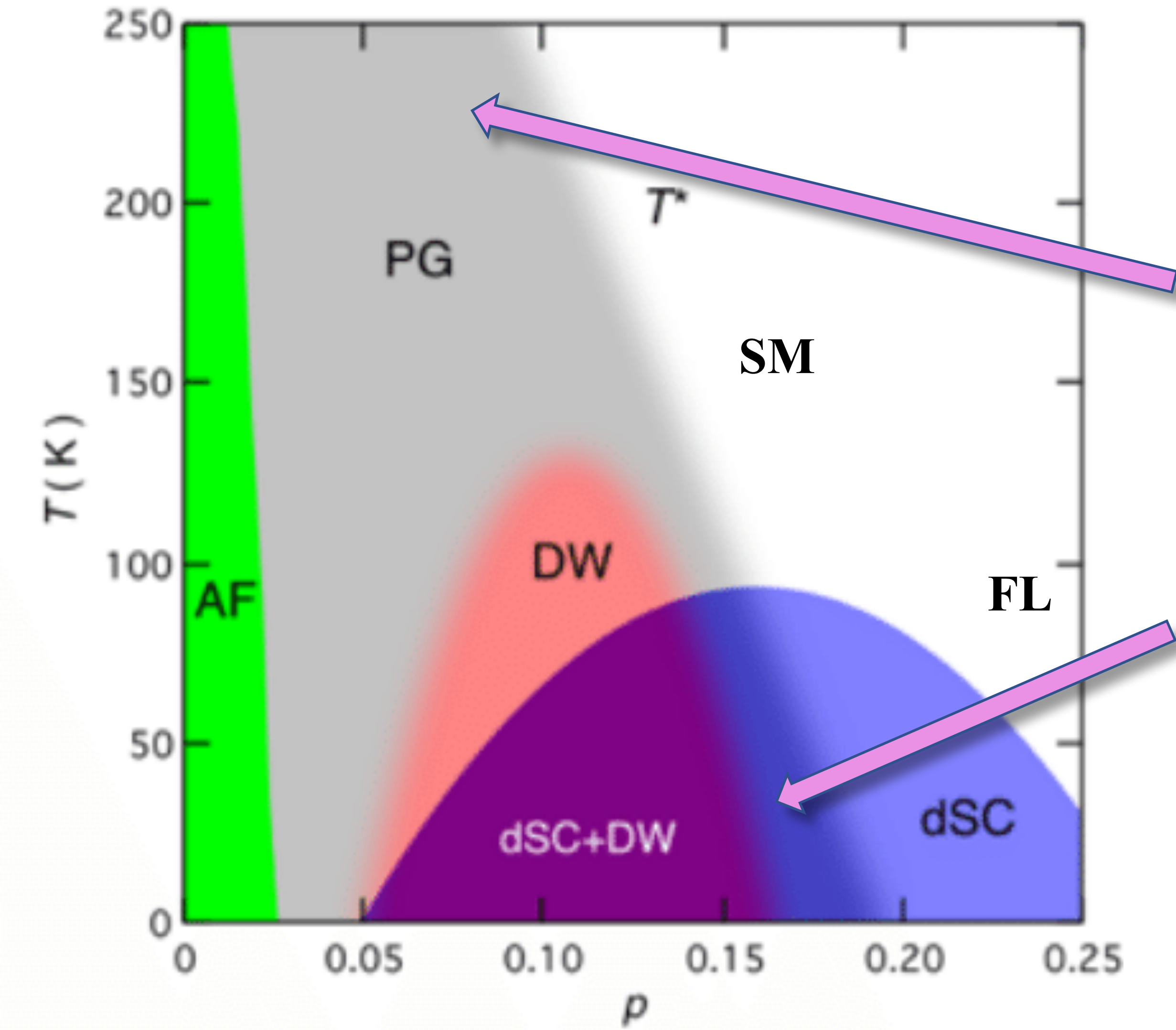
  Mirzaei *et al*, PNAS **110**, 5774 (2013)

- Magnetoresistance $\sim \tau^{-1}(1 + aH^2\tau^2)$ follows Kohler's rule for Fermi liquids

  Chan *et al*, PRL **113**, 177005 (2014)

- T independent Hall coefficient corresponding to a carrier density of *p* in both higher temperature PG and in low T at high magnetic fields

  Ando *et al*, PRL **92**, 197001 (2004), Badoux *et al*, Nature **531**, 210 (2016)



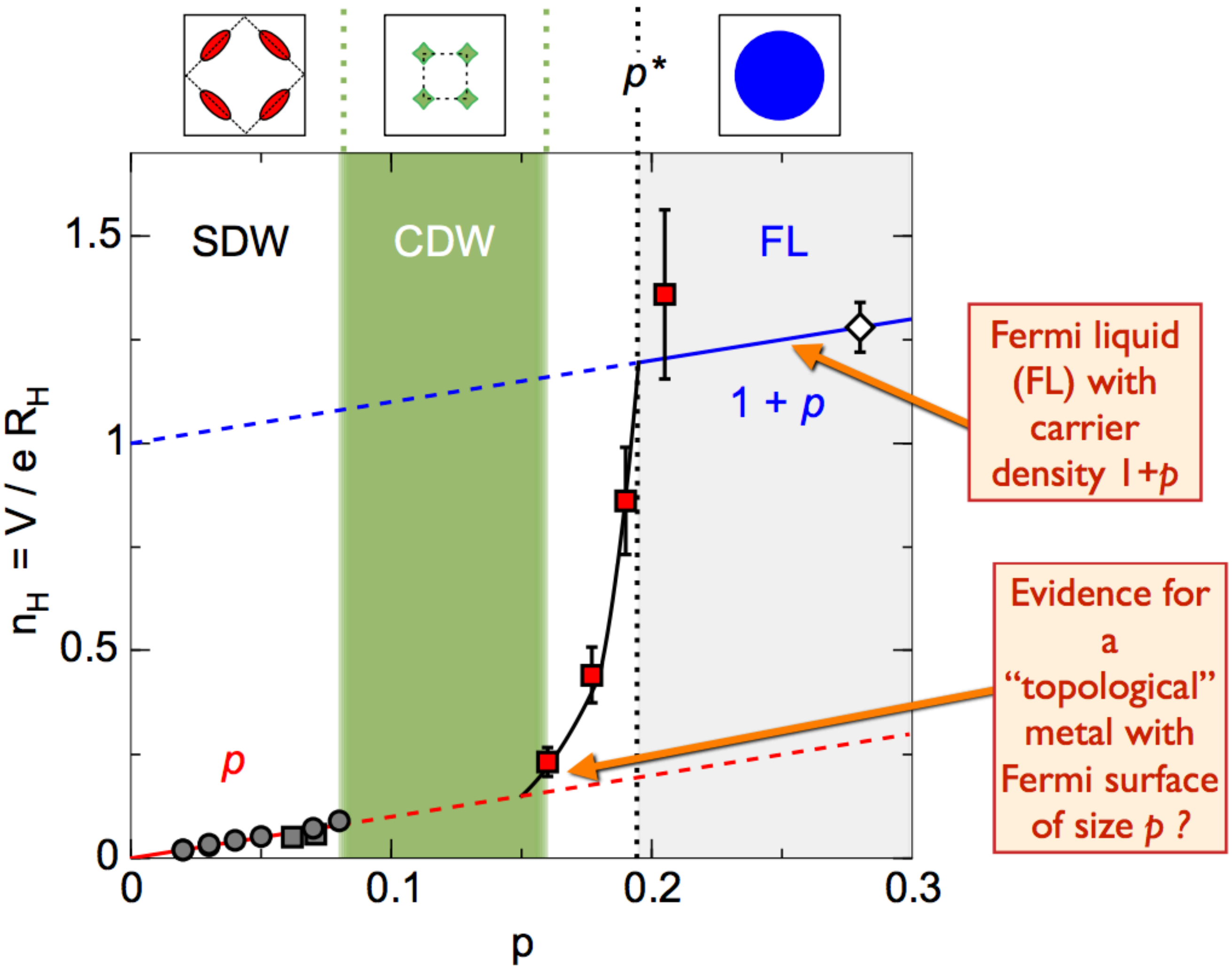
Pseudogap Metal:

Behaves like a Fermi liquid, with a Fermi surface of size p instead of $1 + p$.

Hall effect experiments show that it is also present at high magnetic fields and low temperatures.

Figure credits: K. Fujita *et al*, Nature Physics **12**, 150–156 (2016)
C. Proust *et al*, Nature **531**, 210 (2016).

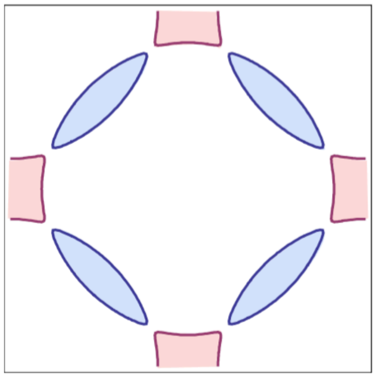
Low T Hall effect measurements in YBCO



Badoux, Proust, Taillefer *et al*, Nature **531**, 210 (2016)

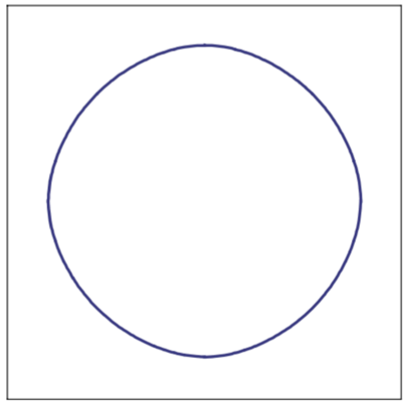
How does the Fermi surface reconstruct?

Possibility 1: Symmetry breaking: Spin density wave (SDW) order



$\langle\vec{\varphi}\rangle \neq 0$

Metal with electron and hole pockets



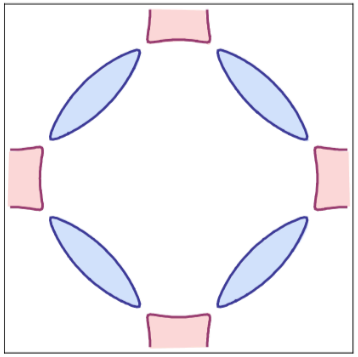
$\langle\vec{\varphi}\rangle = 0$

Metal with "large" Fermi surface

Image credits: S. Sachdev, Harvard

How does the Fermi surface reconstruct?

Possibility 2: Topological order (no symmetry breaking)



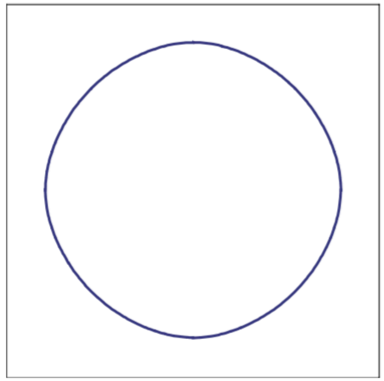
$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

Electron and/or hole Fermi pockets form in "local" SDW order, but quantum fluctuations destroy long-range SDW order

$\langle \vec{\varphi} \rangle = 0$

Algebraic Charge liquid (ACL) or Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and pocket Fermi surfaces



$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

S. Sachdev, E Berg. *S. Chatterjee*, Y. Schattner , PRB **94**, 115147 (2016)

Broken symmetries in the PG metal

- Nematic order: Broken C_4 symmetry   Daou *et al*, Nature **463**, 519 (2010)
- Broken time-reversal symmetry $\boldsymbol{\theta}$

  Mangin-Thro *et al*, Nat. Comms **6**, 7705 (2015), Simon & Varma, PRL **89**, 247003, 2002

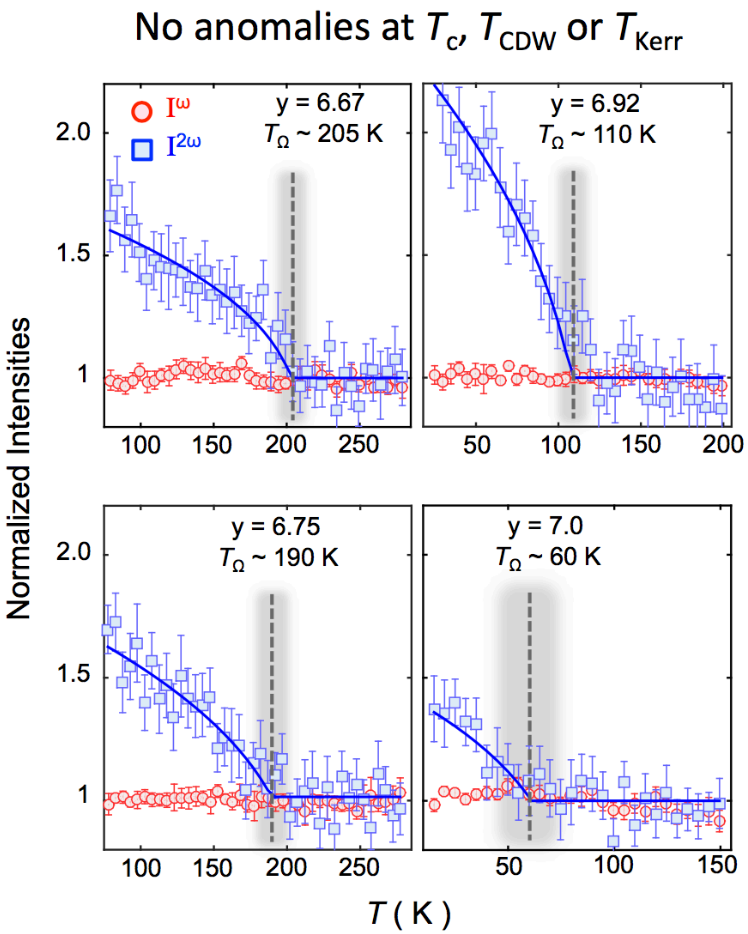
- Broken inversion symmetry C_2. However, $\boldsymbol{\theta}$ C_2, the product of inversion and time-reversal seems to be preserved.

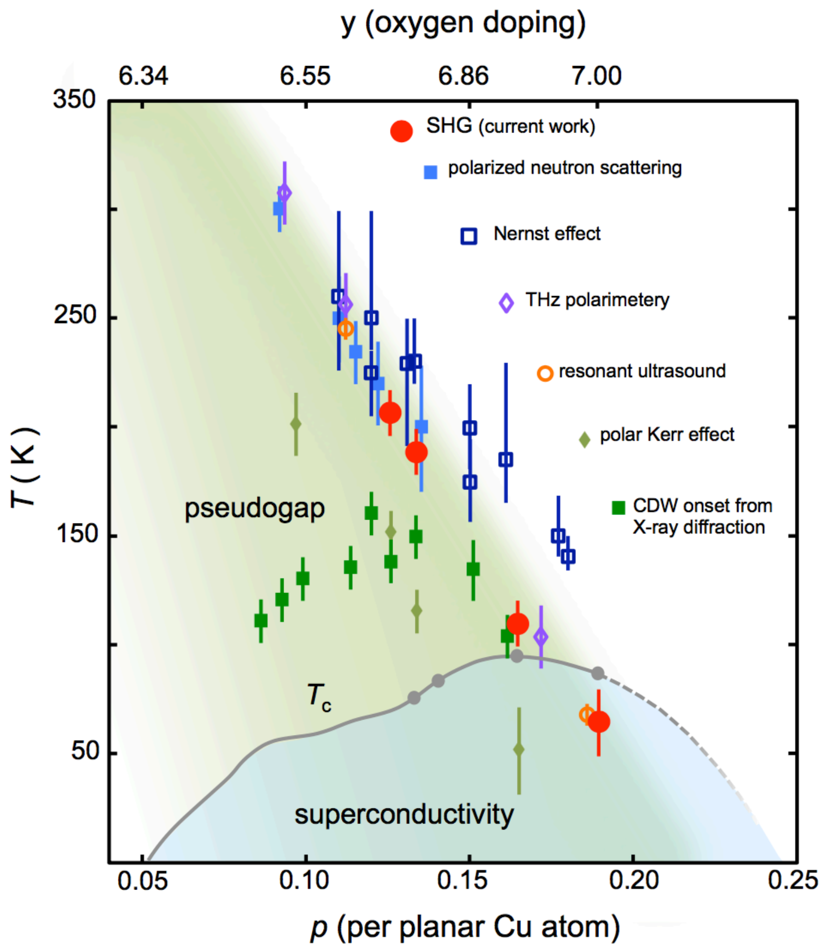
  Zhao, Belvin, Hsieh *et al*, Nature Physics **13**, 250 (2017)

- No evidence of translation symmetry breaking in large parts of the phase diagram: Even with discrete broken symmetries, Small FS violates Luttinger's Theorem and requires *topological order*.

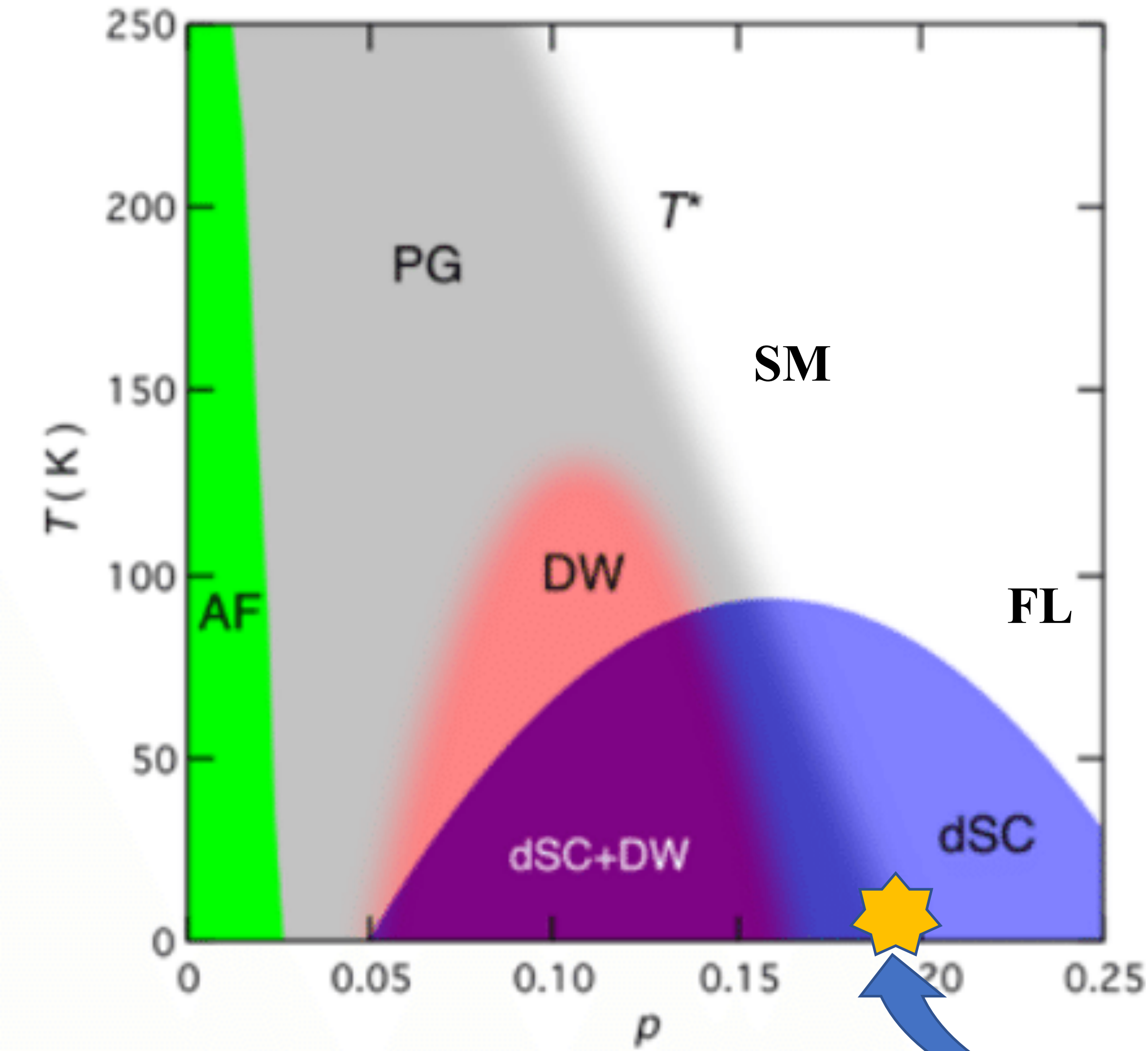
  T. Senthil *et al*, PRL **90**, 216403 (2003)
  Paramekanti *et al*, PRB **70**, 245118 (2004)

Second Harmonic Generation measurements in YBCO





Zhao, Belvin, Hsieh *et al*, Nature Physics **13**, 250 (2017)



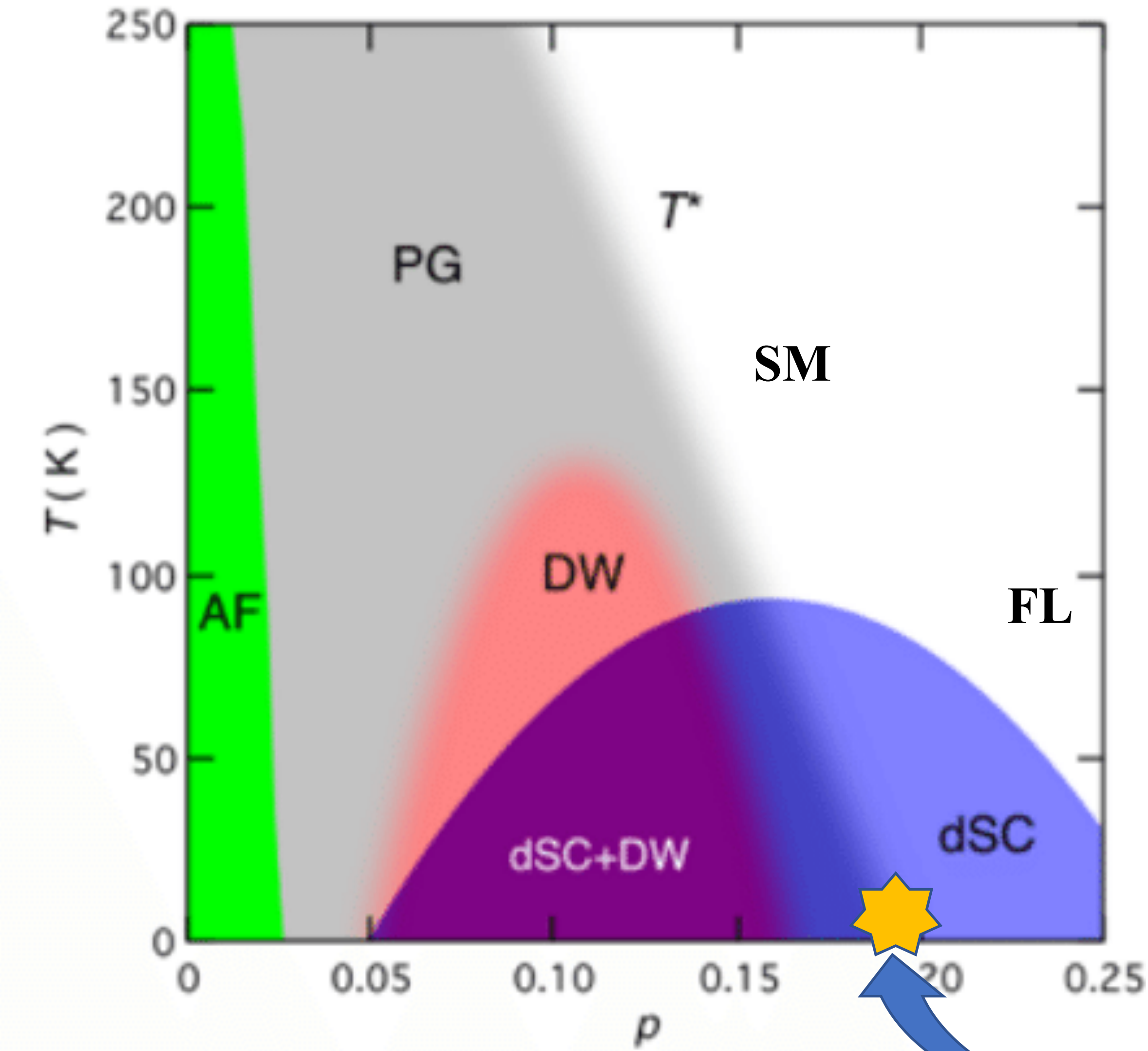
Is there a quantum critical point (QCP) at optimal doping under the superconducting dome?

What is the nature of the associated phase transition? Symmetry-breaking or topological?

Figure credits: K. Fujita *et al*, Nature Physics **12**, 150–156 (2016)

Figure credits: Wikipedia

Figure credits: Disney Clip Art



Why not both?

Topological QCP with associated discrete symmetry breaking!

Figure credits: K. Fujita *et al*, Nature Physics **12**, 150–156 (2016)

Plan of the talk

Classical phase diagram of a spin-model with frustrating Heisenberg and ring-exchange

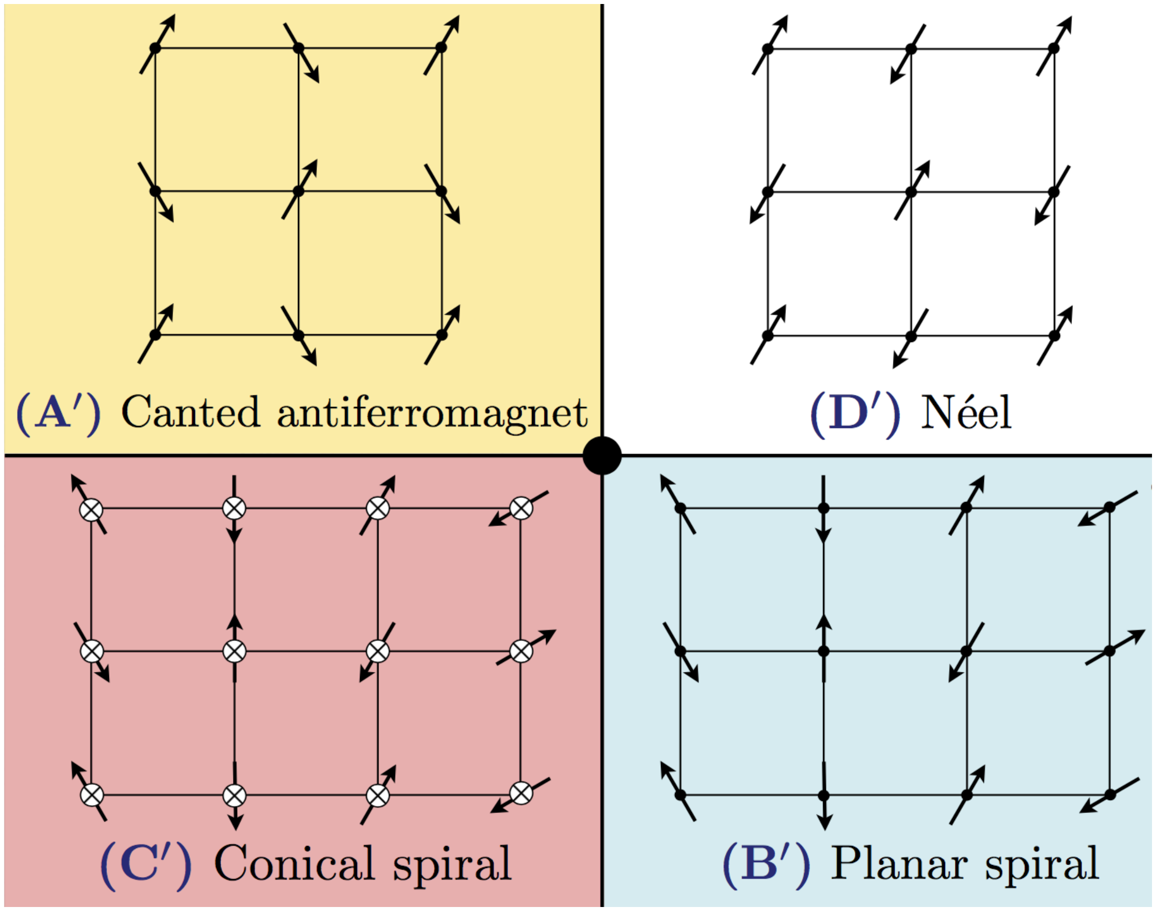
Add charges: Hartree Fock mean-field theory of the Hubbard model

Add topological order: Description in terms of CP^1 model in the insulator

Charges + Topological order: SU(2) gauge theory of the electrons on the square lattice

Classical phase diagram

Square lattice AF with Heisenberg exchanges J_1, J_2, J_3 and J_4 and ring exchange K



Classical phase diagram

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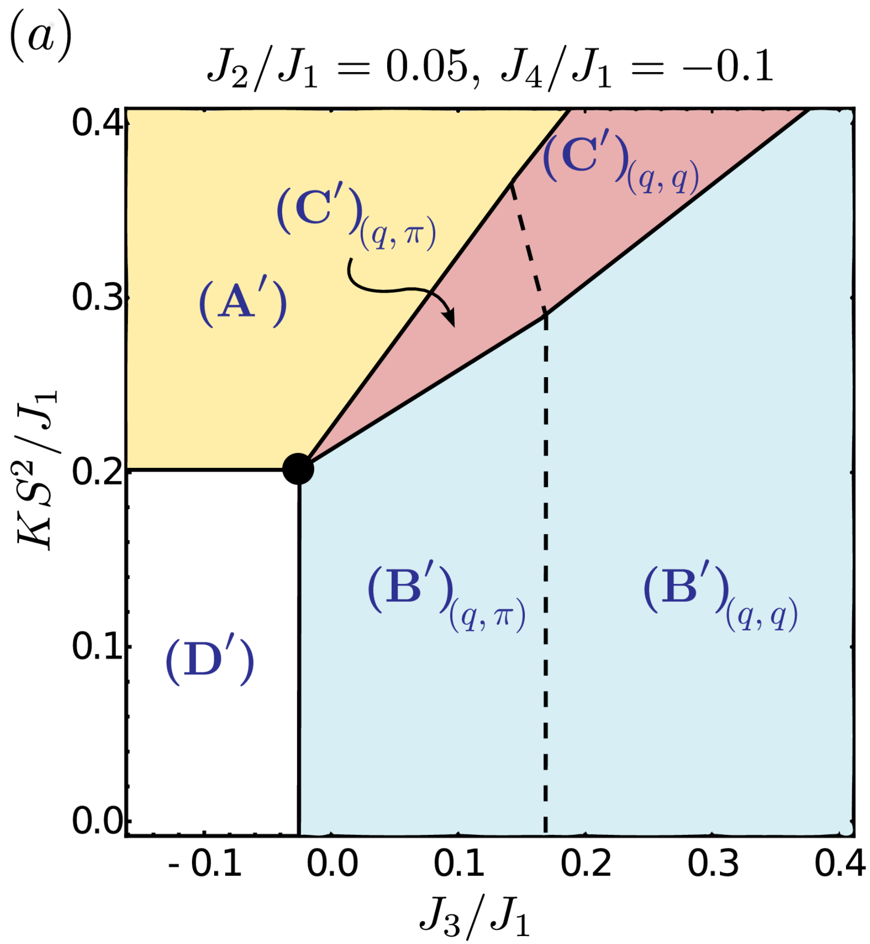


(D'): Neel order

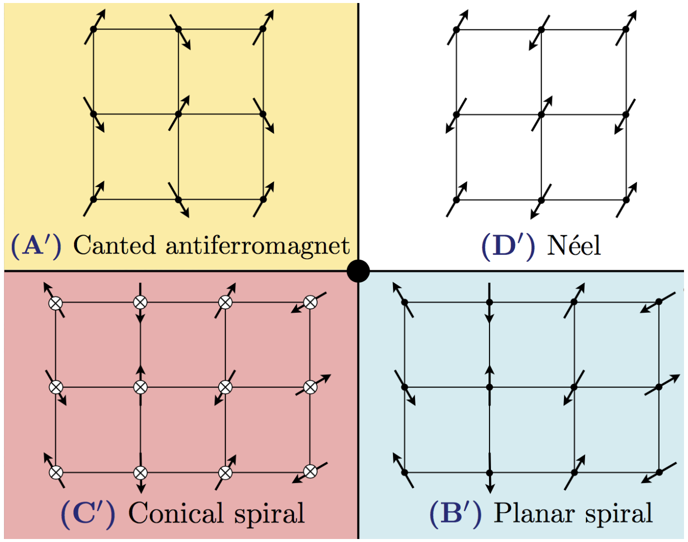
(A'): Canted Neel order

(B'): Planar Spiral order

(C'): Conical Spiral order

Classical phase diagram

Fluctuations of Neel order in the semi-classical non-linear sigma model



$$\hat{\boldsymbol{S}}_i = S\eta_i \boldsymbol{n}_i \sqrt{1 - \boldsymbol{L}_i^2/S^2} + \boldsymbol{L}_i$$

$$\boldsymbol{n}^2 = 1 \quad , \quad \boldsymbol{n} \cdot \boldsymbol{L} = 0 \, ,$$

$\eta_i = \pm 1$ on the two sublattices

Do a gradient expansion in n(**r**,t) and L(**r**,t)

$$\bar{\mathcal{H}}_J = \frac{\rho_s}{2}(\partial_a \mathbf{n})^2 + \frac{1}{2\chi_\perp}\mathbf{L}^2 + C_1(\mathbf{L}^2)^2 + C_2(\partial_a \mathbf{n})^4$$

(A'): $\rho_s, C_1, C_2 > 0, \chi_\perp < 0$

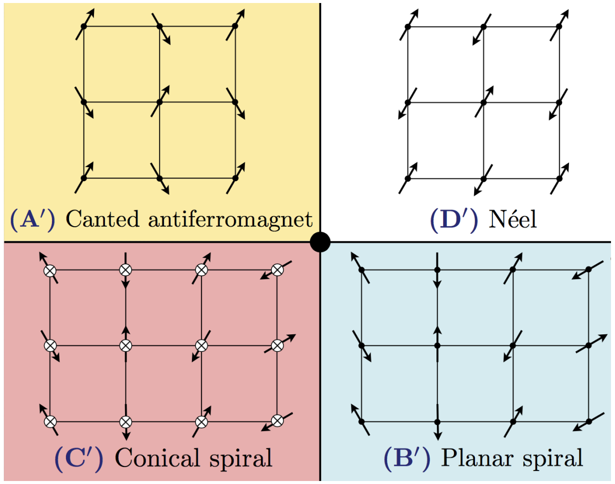
(D'): $\rho_s, \chi_\perp, C_1, C_2 > 0$

(C'): $C_1, C_2 > 0, \rho_s, \chi_\perp < 0$

(B'): $\chi_\perp, C_1, C_2 > 0, \rho_s < 0$

Classical phase diagram

What symmetries are broken in these magnetically ordered phases?



	$\mathcal{T}$	T_x	T_y	I_x	I_y
$\vec{n}$	−	−	−	+	+
$\vec{L}$	−	+	+	+	+
J_x	−	+	+	−	+
J_y	−	+	+	+	−

All phases break spin-rotation, translation and time-reversal

(B’): Has additional nematic order, breaks lattice rotation

(C’): Breaks both lattice rotation and inversion

$$\mathbf{O} = \vec{L} \cdot (\vec{n} \times \nabla \vec{n}), \quad \langle \mathbf{O} \rangle \neq 0$$

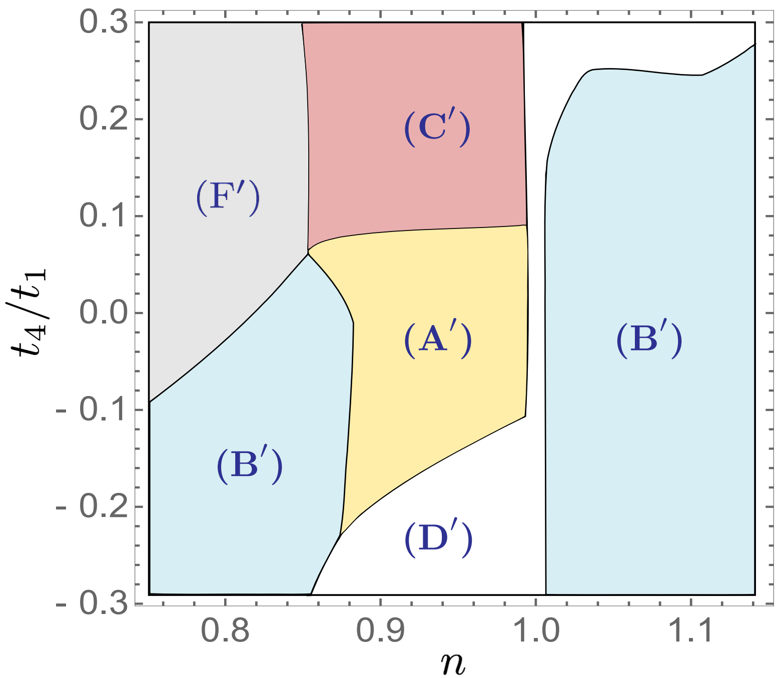
Add charges: Hartree-Fock theory

Hubbard model on the square lattice: Mean-field theory of magnetism preserving translation invariance in the charge sector

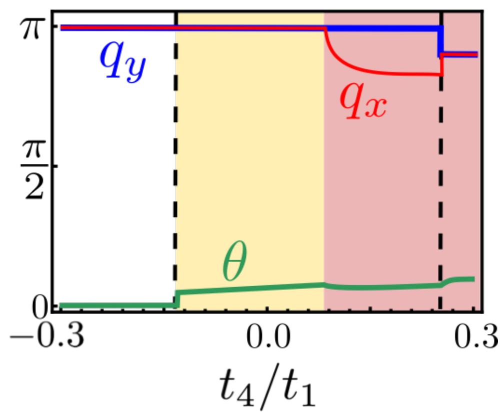
$$\mathcal{H}_U = -\sum_{i<j,\alpha} t_{ij} c^\dagger_{i,\alpha} c_{j,\alpha} - \mu \sum_{i,\alpha} c^\dagger_{i,\alpha} c_{i,\alpha} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

$$\left\langle \hat{\boldsymbol{S}}_i \right\rangle = N_0 \left[\cos\left(\boldsymbol{K}\cdot\boldsymbol{r}\right)\cos(\theta)\,\hat{\boldsymbol{e}}_x + \sin\left(\boldsymbol{K}\cdot\boldsymbol{r}\right)\cos(\theta)\,\hat{\boldsymbol{e}}_y + \sin(\theta)\,\hat{\boldsymbol{e}}_z\right]$$

$t_2/t_1 = -0.3,\ t_3/t_1 = -0.2,\ U/t_1 = 8.0$



$n = 0.96$

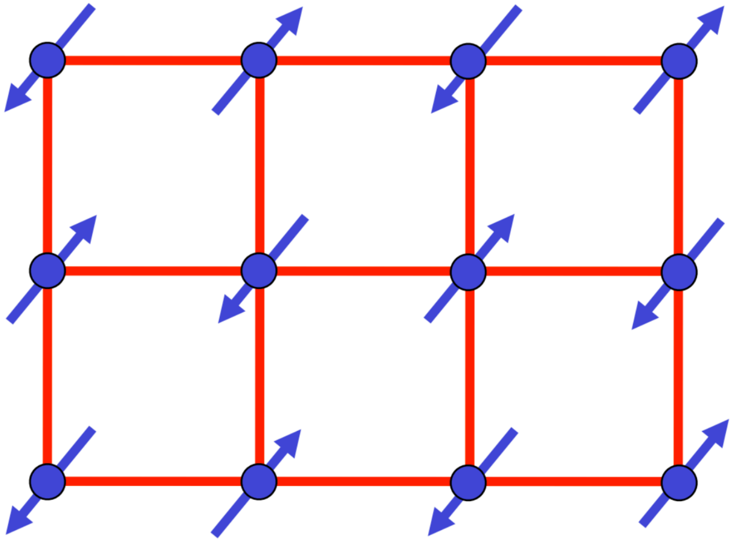


- Same phases in the doped system
- Phase diagram is particle-hole asymmetric

Add topological order: CP^1 theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

$$\mathbf{n} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \text{ with } \alpha, \beta = \uparrow, \downarrow, \ |z_\alpha|^2 = 1$$



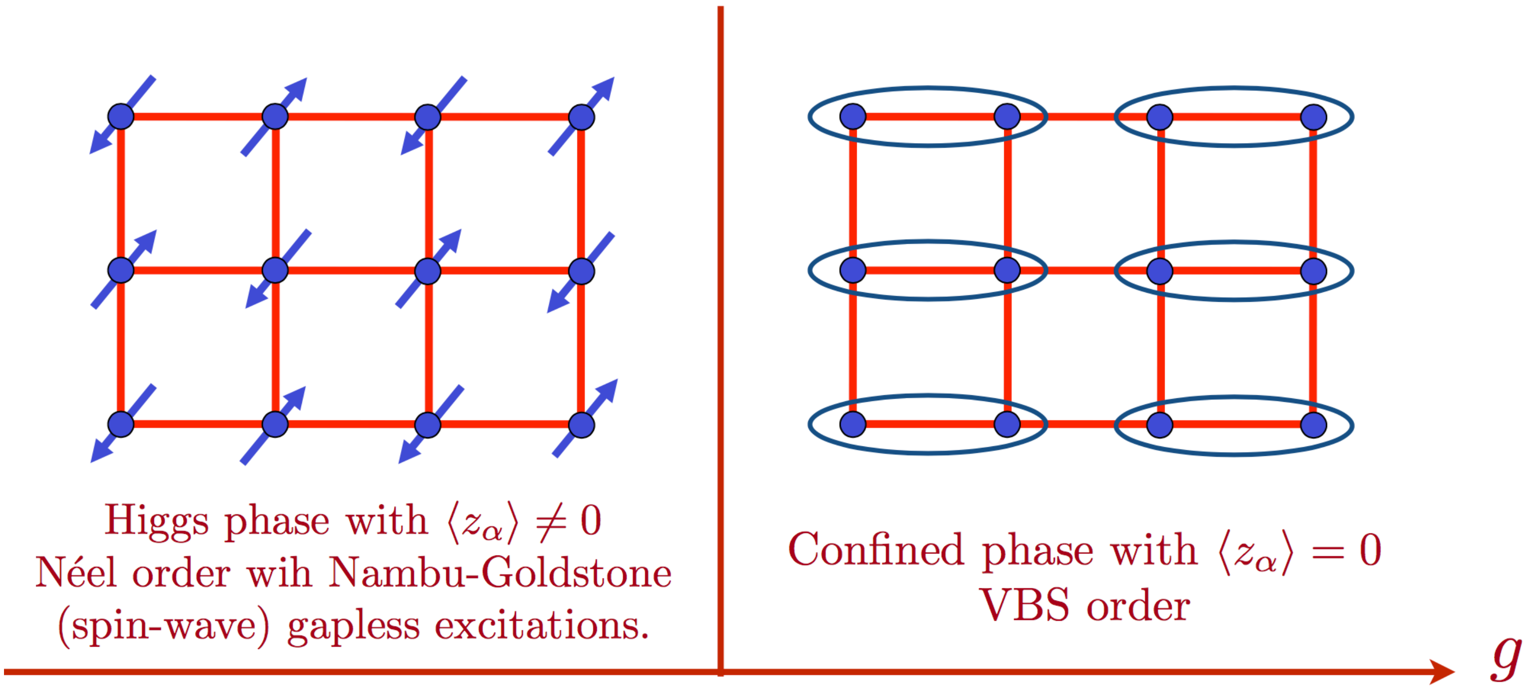
$$S = \frac{1}{2g} \int d^2 r dt \ \left(\partial_\mu \mathbf{n}\right)^2$$
$$\rightarrow \frac{1}{2g} \int d^2 r dt \, |(\partial_\mu - i a_\mu) z_\alpha|^2$$

The CP^1 theory an has emergent U(1) gauge field a_μ

Add topological order: CP^1 theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

For S = ½ , additional Berry phase term for the U(1) gauge field



Read and Sachdev, PRL **62**, 1694 (1989)

Add topological order: CP^1 theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

For Z_2 topological order, need to condense Higgs fields with charge 2 under emergent U(1) gauge field

Simplest candidates: Spin rotation invariant long-wavelength spinon pairs:

$$P \sim \varepsilon_{\alpha\beta} z_\alpha \partial_t z_\beta \quad , \quad Q_a \sim \varepsilon_{\alpha\beta} z_\alpha \partial_a z_\beta \text{ with } a = x, y$$

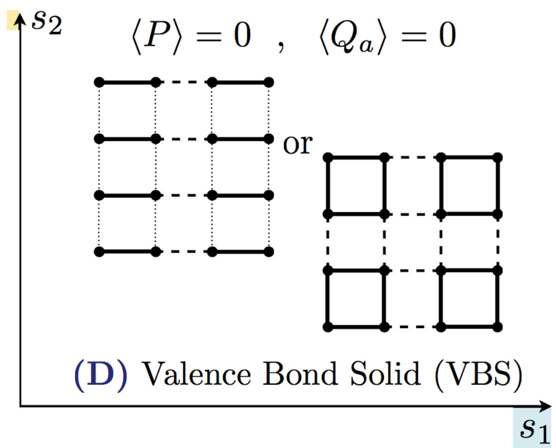
Gauge invariance + Symmetry

$$\mathcal{L} = \frac{1}{g}|(\partial_\mu - i a_\mu) z_\alpha|^2 + s_1 |P|^2 + s_2 |Q_a|^2$$

Add topological order: CP^1 theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Phase diagram at large g with $\langle z_\alpha \rangle = 0$



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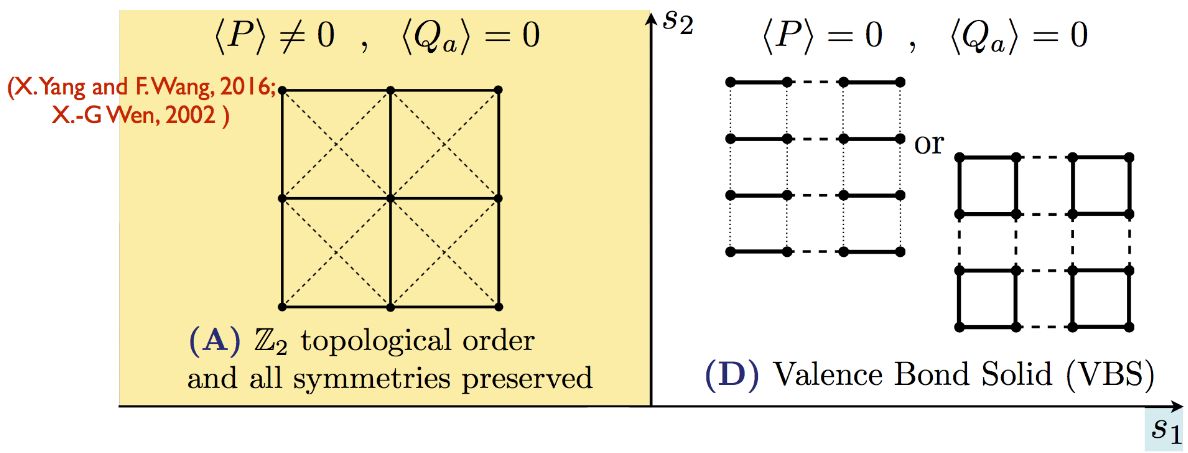

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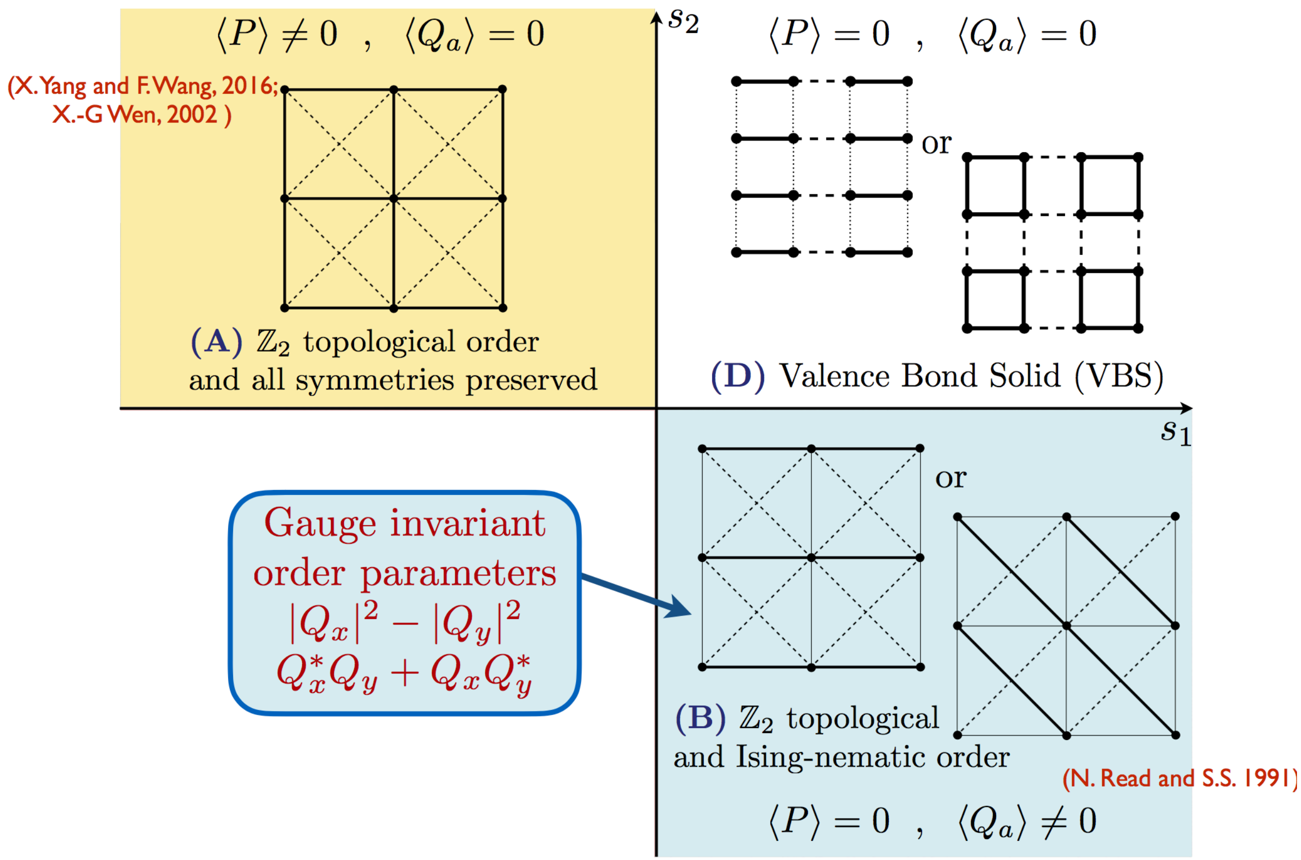
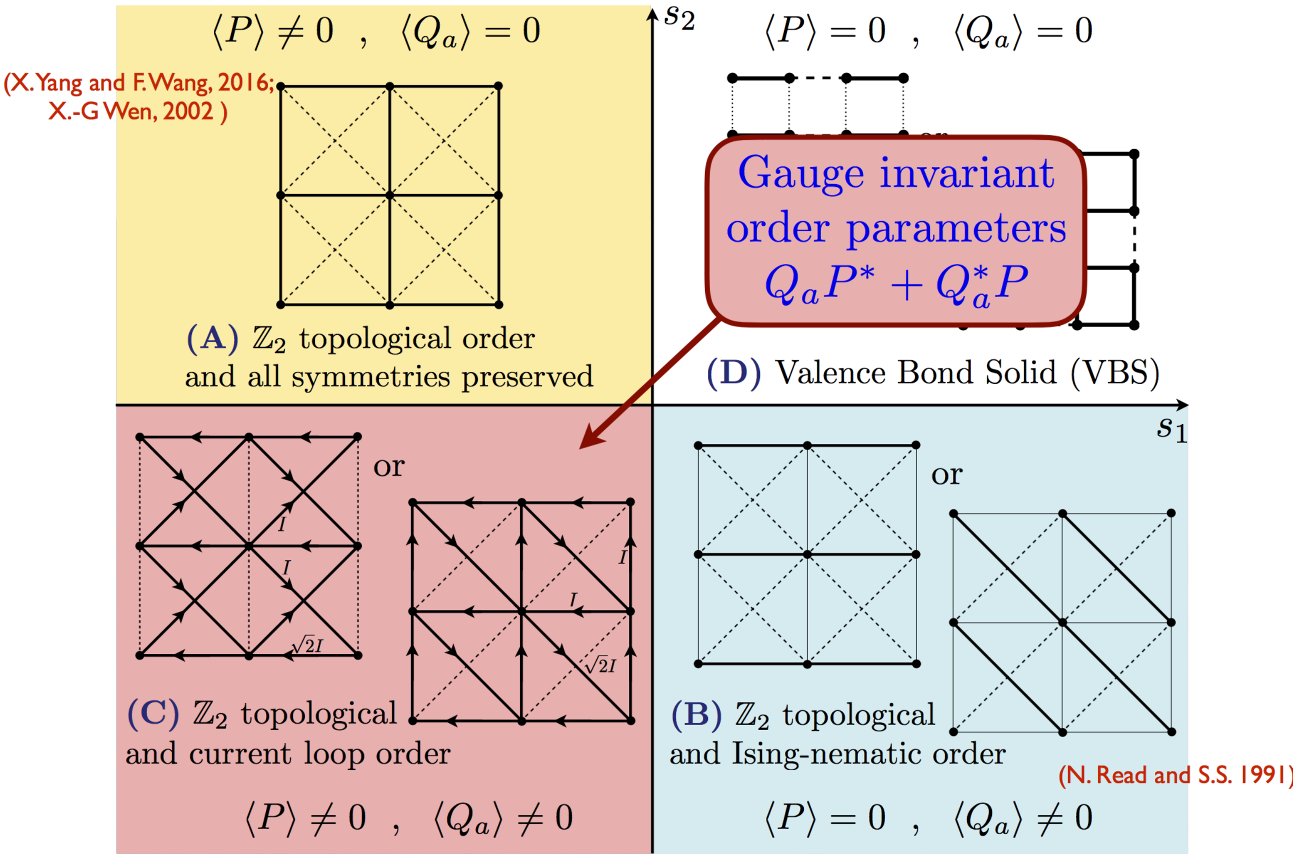


Add topological order: CP^1 theory

Quantum disorder the spins: Spin-rotation and translation invariance regained. Discrete symmetries remain broken.

Phase diagram at large g with $\langle z_\alpha \rangle = 0$

Three phases with Z_2 topological order



Add topological order: CP^1 theory

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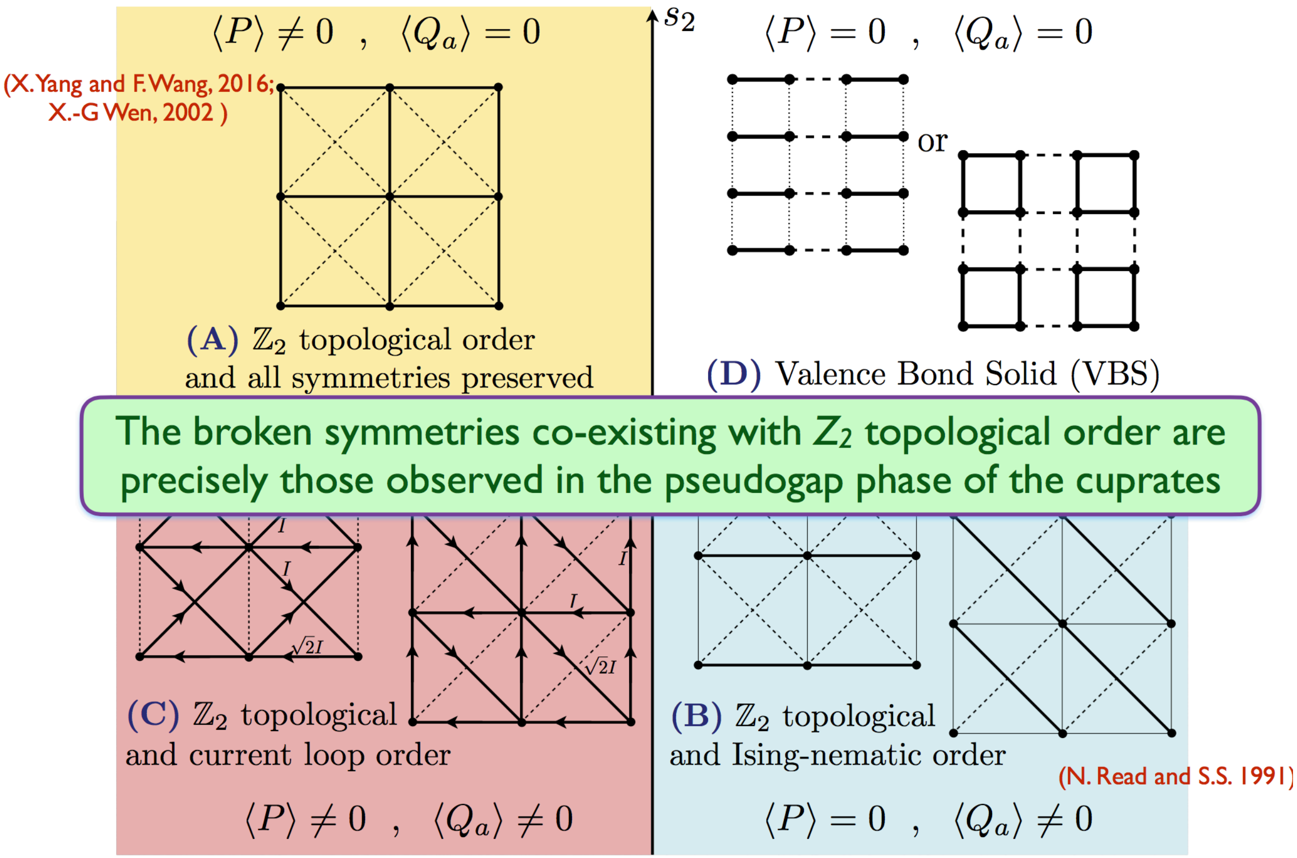
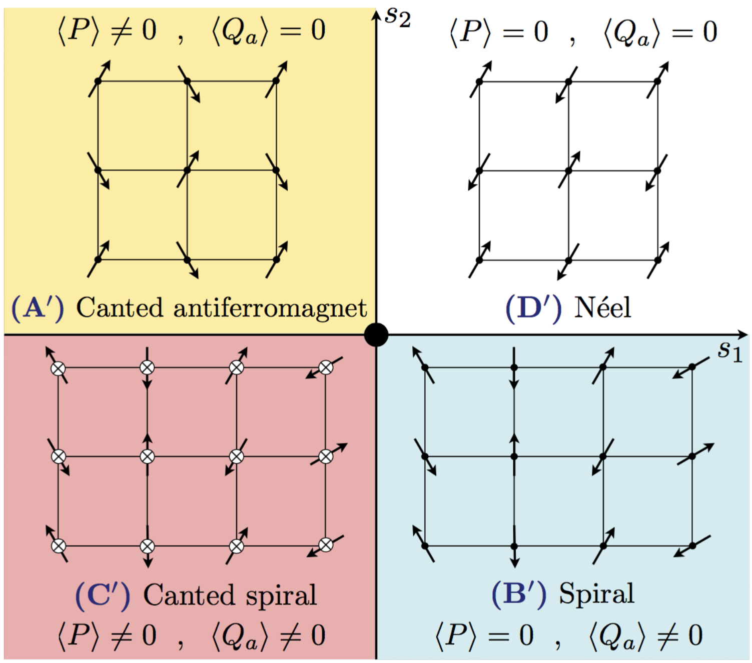


The broken symmetries co-existing with Z_2 topological order are precisely those observed in the pseudogap phase of the cuprates

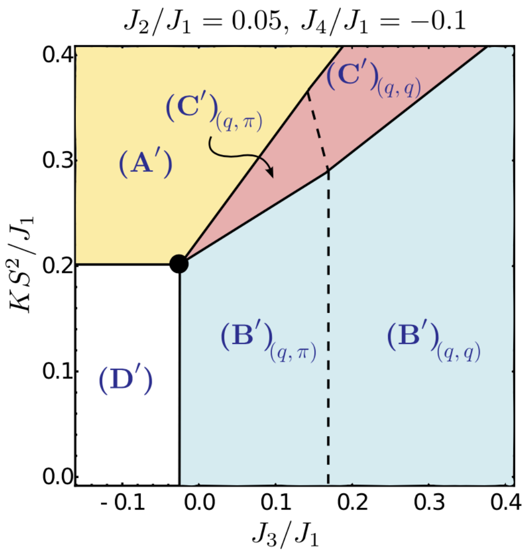
Add topological order: CP^1 theory

Describes ordered phases at small g : break translation and spin-rotation symmetries, and have no topological order.

Phase diagram of $\mathbb{CP}^1$ model at small g, coupled to Higgs fields P and Q_a $(a = x, y)$. All phases have $\langle z_\alpha \rangle \neq 0$



Classical phase diagram of square lattice antiferromagnet with near-neighbor exchanges J_1, J_2, J_3, J_4 and ring-exchange K



Charges + Topological Order: SU(2) gauge theory

Intertwining topological order and discrete symmetry breaking in the PG metal

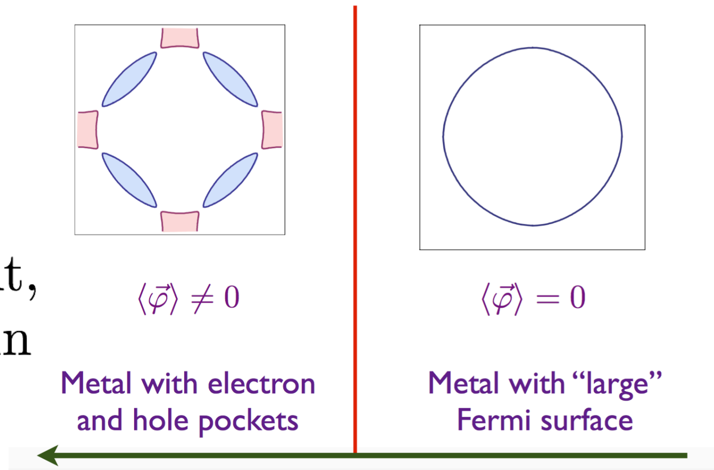
Spin-fermion model : Electrons on a square lattice

$$H = -\sum_{i<j} t_{ij} c_{i,\alpha}^{\dagger} c_{j,\alpha} - \mu \sum_i c_{i,\alpha}^{\dagger} c_{i,\alpha} + H_{int}$$

Couple to AF order parameter

$$H_{int} = -\lambda \sum_i \eta_i \vec{\phi}(i) \cdot c_{i,\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i,\beta}$$

When $\vec{\phi}$ is a site-independent constant, we have long range AF order and a gap in the anti-nodal spectrum



Charges + Topological Order: SU(2) gauge theory

Intertwining topological order and discrete symmetry breaking in the PG metal

Locally well-developed AF order parameter + angular fluctuations

Transform to a rotating reference frame using SU(2) rotations R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right)$$

Degrees of freedom: Spinless chargons (psi) and Higgs Field $\mathbf{H}_i$

$$\sigma^\ell \Phi^\ell(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

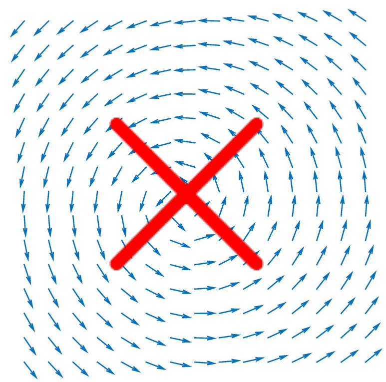
Charges + Topological Order: SU(2) gauge theory

Intertwining topological order and discrete symmetry breaking in the PG metal

Simplest effective Hamiltonian for the chargons is identical to the electrons: Higgs field replaces AF order

$$H_\psi = -\sum_{i<j} t_{ij} \psi^\dagger_{i,s} \psi_{j,s} - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} + H_{int}$$

$$H_{int} = -\lambda \sum_i \eta_i \vec{H} \cdot \psi^\dagger_{i,s} \vec{\sigma}_{ss'} \psi_{i,s'} + V_H$$



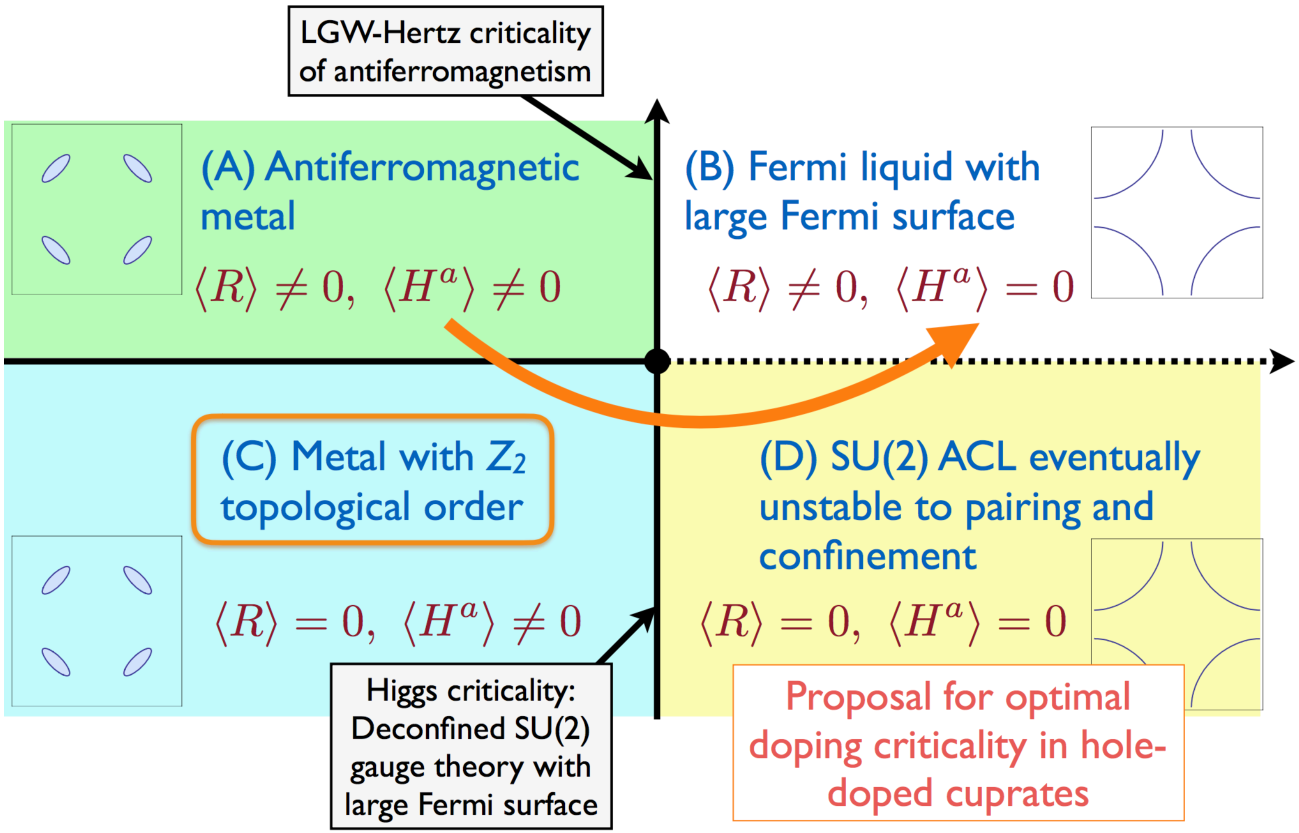
The chargons will inherit the anti-nodal gap only if such a transformation R_i can be found. Need to suppress Z_2 vortices of SO(3) Higgs field $\Longrightarrow$

Metal with Z_2 topological order and a pseudogap

Charges + Topological Order: SU(2) gauge theory

Intertwining topological order and discrete symmetry breaking in the PG metal

Global phase diagram



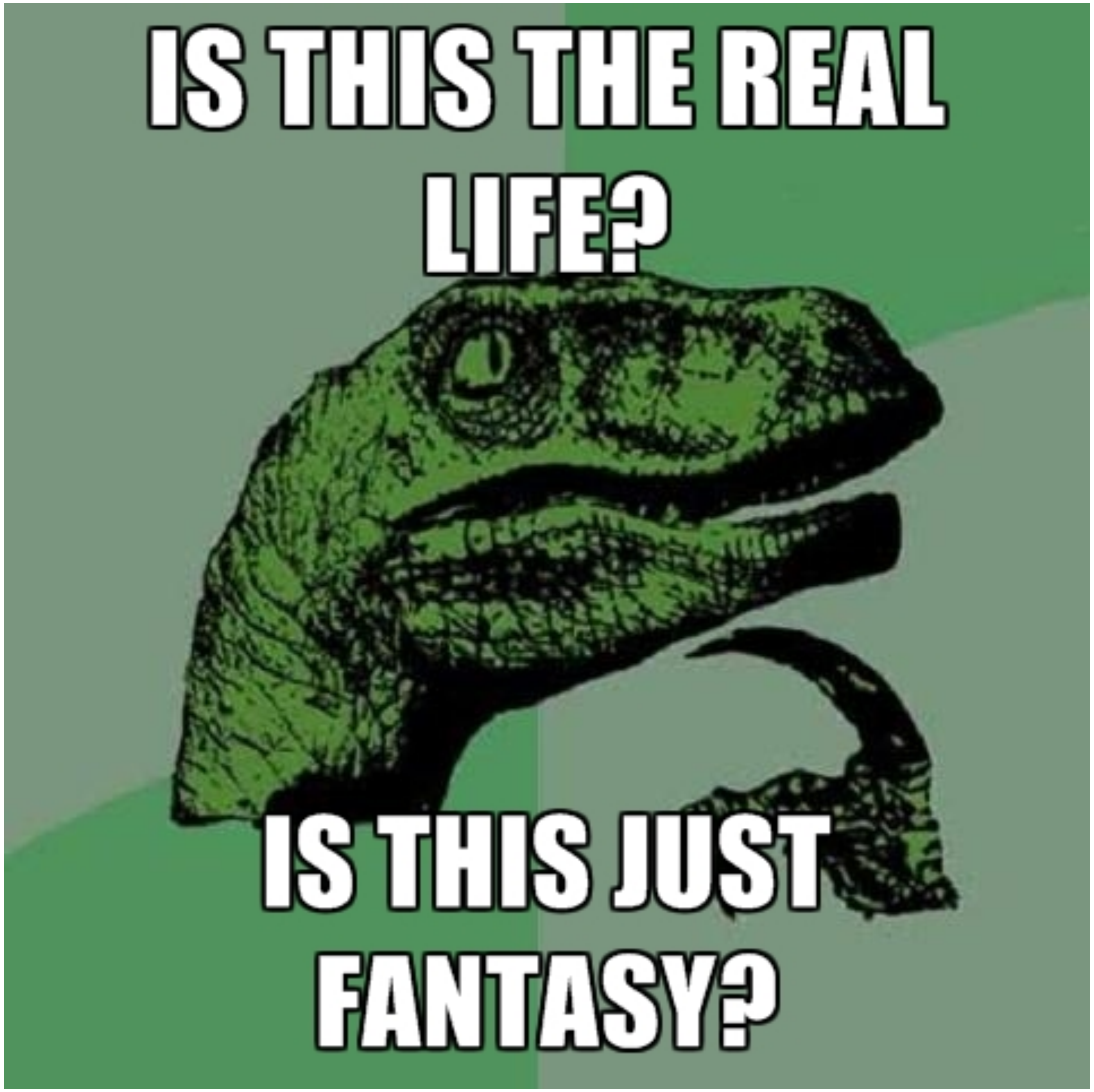
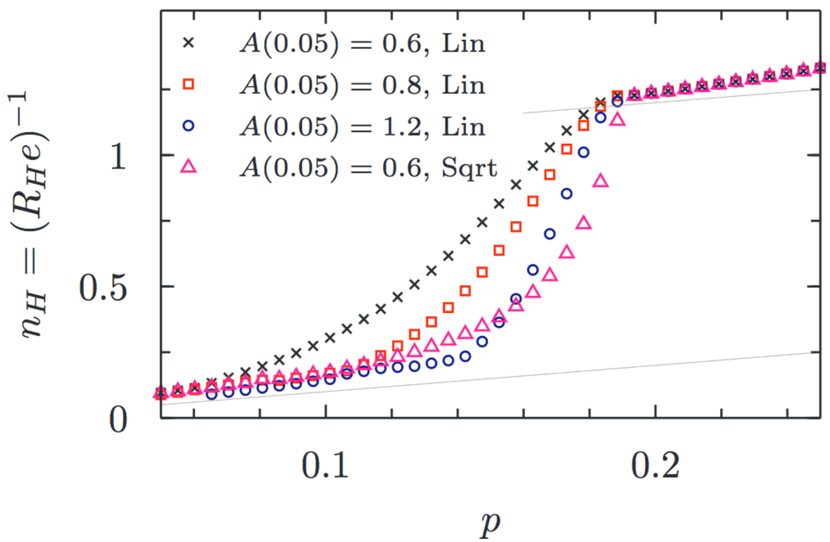


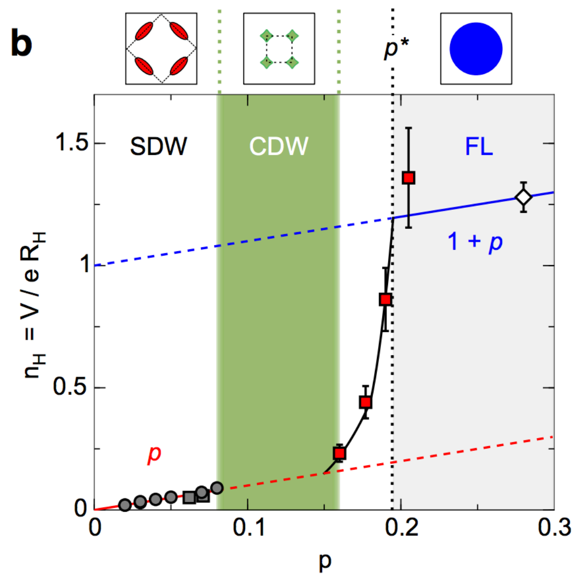
Figure credits: http://creatememe.chucklesnetwork.com/memes/16712

Comparisons with experiments

Hall data shows good qualitative agreement, as do data on longitudinal thermal and electric transport



A. Eberlein *et al*, PRL, **117,** 187001 (2016)

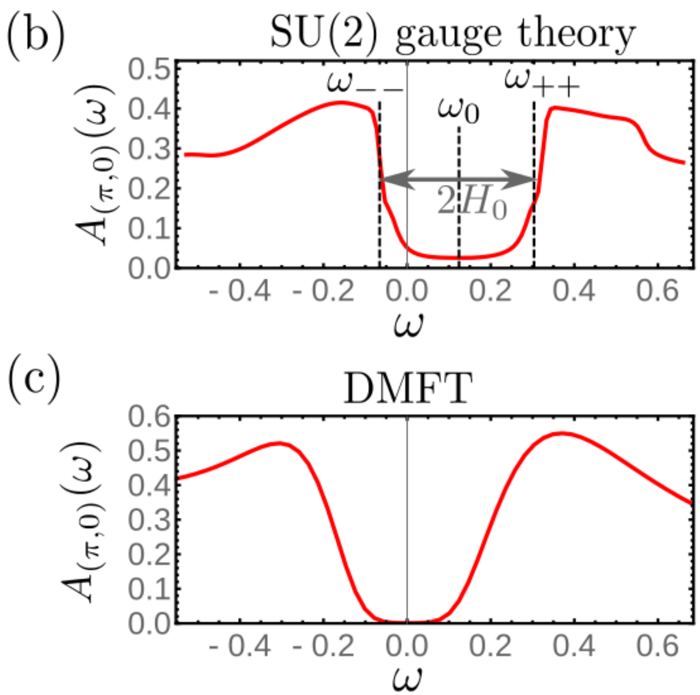


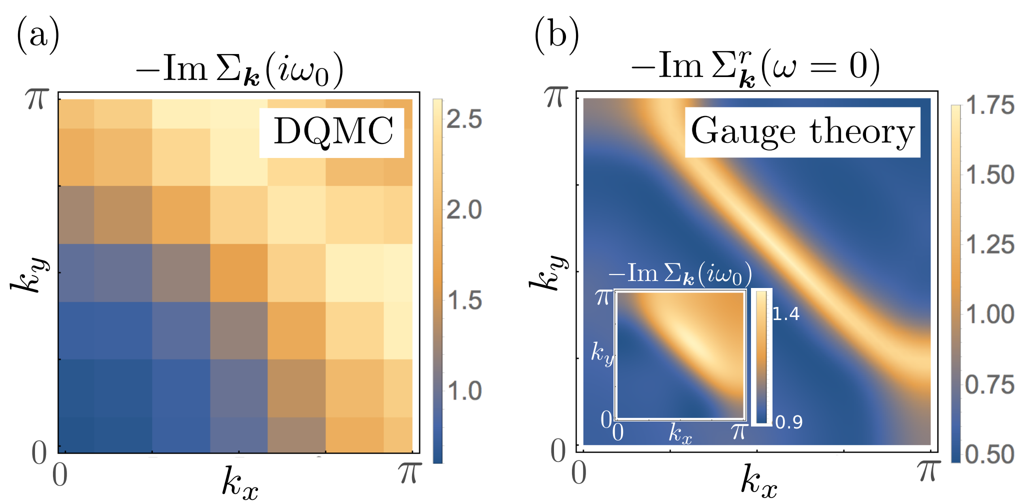
Badoux, Proust, Taillefer *et al*, Nature **531**, 210 (2016)

S. Chatterjee, S. Sachdev and A. Eberlein, PRB, **96,** 075103 (2017)

Comparisons with numerics

Electron spectral functions / self-energies from the SU(2) gauge theory closely resemble those from DMFT/QMC on 2d Hubbard model





M. Scheurer, *S. Chatterjee*, M. Ferrero, A. Georges, S. Sachdev and W. Wu, to appear

Summary

SU(2) gauge theory of metals with Z_2 topological order can explain the concurrent appearance of anti-nodal gap and discrete broken symmetries in the hole-doped cuprates

Topologically ordered phases energetically proximate to the Neel state have the desired broken symmetries

Thermal/electric transport and spectroscopic data for such models are consistent with experiments

Ongoing work: Comparison with DMFT/QMC on the 2d Hubbard model. Preliminary agreements seem encouraging!

Thank you for your attention!