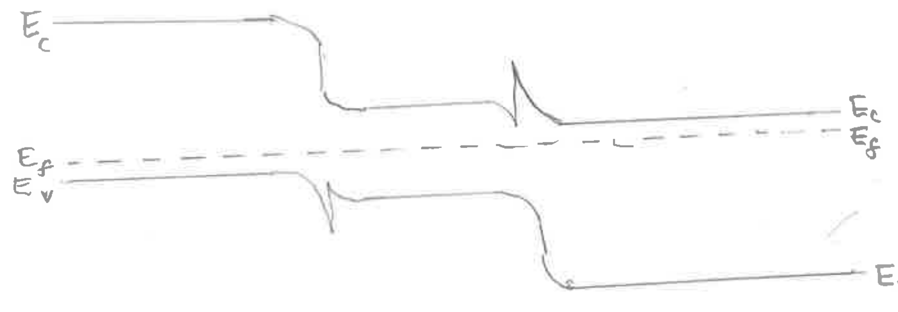
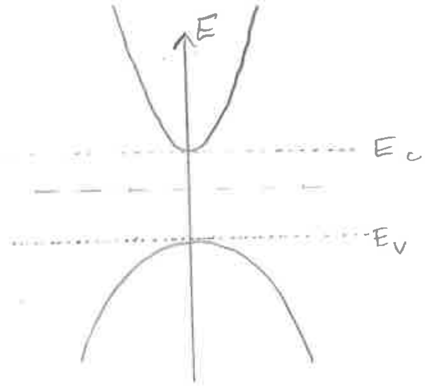
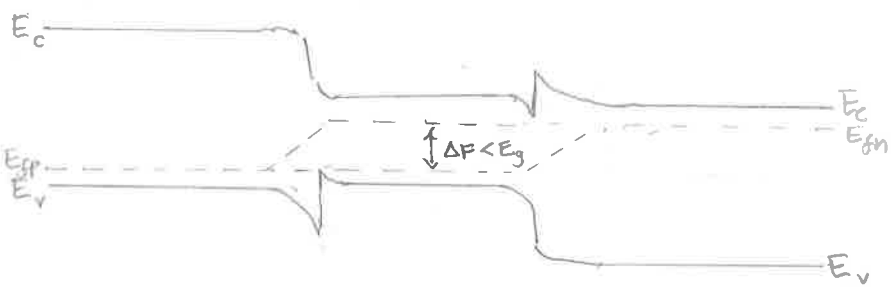
# Transparency Concentration & Current

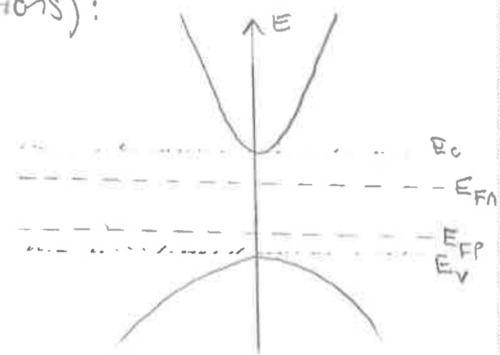
We begin by analyzing the band structure of a double-heterostructure before and after forward bias in an attempt to understand the carrier dynamics. Let us assume that the quantum well is intrinsic for the time being:

① 

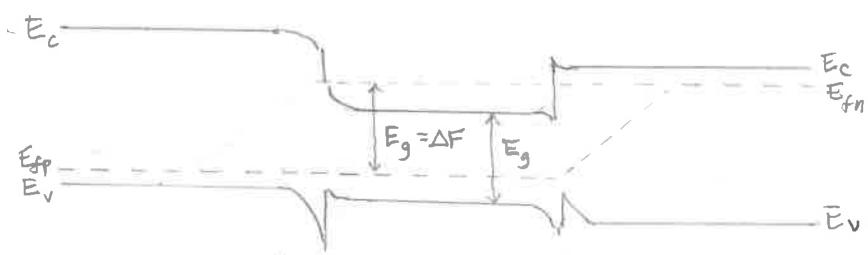



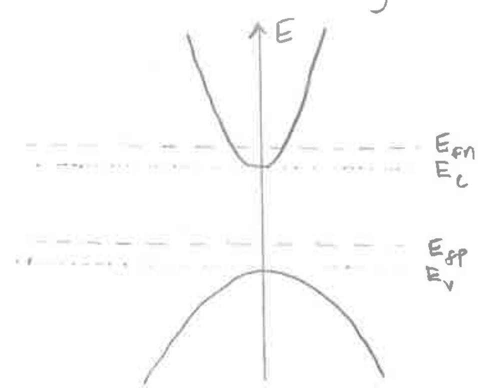

Next, we apply a small Forward bias (Assume voltage dropped in junctions):

② 

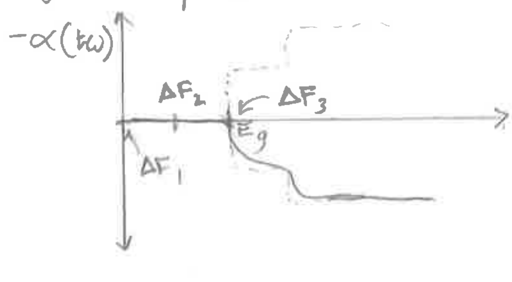




Finally, we apply a Forward bias equal to $qE_g$, ie we satisfy the Bernard Duraffourg gain condition:

③ 


These three bias conditions correspond to the following points on the gain spectrum curve:



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Foremost, there are a few subtle characteristics to highlight in these band diagrams. When drawing the diagrams, a more p-doped well was assumed. The doping of the well has a strong impact on the way in which the bands of the different regions move relative to one another under bias. In the case of a p-type well, band bending will be most significant at the interface of the well and the N-type semiconductor. This means that the largest built-in field will exist between these two regions and thus an applied bias will act to reduce this field more than the P-well junction. As a result, the relative positioning of the P-SC and QW bands will change very little while the N-SC bands will be lifted up (like shown). This has a strong impact on carrier concentration which brings us to a trend:

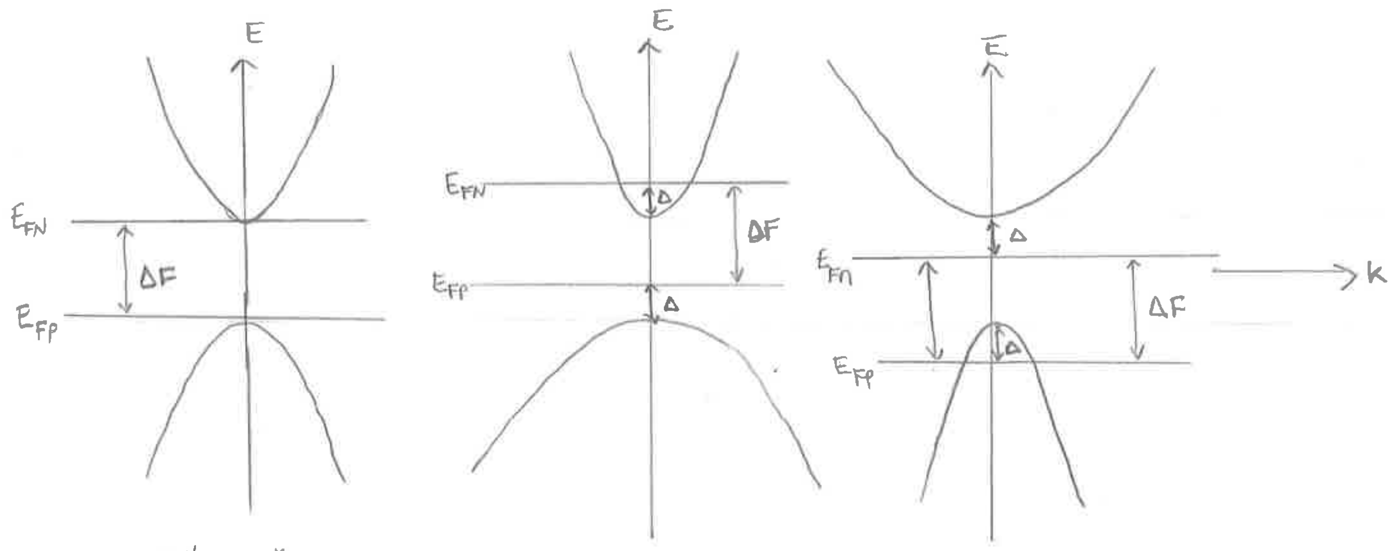
Trend 1: P-type Well ⇒ high Transparency electron concentration

If the quantum well is n-type, the reverse will be true: the well bands will stay largely fixed relative to the N-type region while the p-type region's bands are lowered significantly compared to the well bands. This leads to trend 2:

Trend 2: N-type Well ⇒ low $N_{tr}$

What about if the well is intrinsic? In this case things seem a bit ambiguous. The conduction/valence bands might slant under bias leading to other interesting effects (like QCSE).

Next, we can determine the electron concentration at transparency ($\Delta F = E_g$) for the specific case where $N = P$. Since $\Delta F = E_g$, the functional forms for N & P will depend strongly on the effective mass in the conduction and valence band:




$m_n^* = m_h^*$ $\qquad$ $m_n^* < m_h^*$ $\qquad$ $m_n^* > m_h^*$

~~It is important to note that a very specific doping must be chosen in the well such that N=P at the transparency condition. Knowing these exact dopant levels is not strictly necessary.~~ No doping

Let us consider the First case in which $m_n^* = m_h^*$. Recall that the effective density of states of the conduction & Valence band are related to the effective mass in the respective band by:

$$N_c \propto (m_n^*)^{3/2} \qquad \text{and} \qquad N_v \propto (m_h^*)^{3/2}$$

Since $m_n^* = m_h^*$, $N_C = N_V$. The energy band diagram is thus completely symmetric for electrons and holes. It follows that the quasi Fermi levels fall directly on the band edges, ie

$$F_c - E_c = 0 = E_v - F_v$$

Calculating the exact transparency electron concentration in this case is non-trivial. A value could be obtained by numerically computing

$$N_{tr} \approx \int_{E_c}^{\infty} dE\, \rho_n(E)\, f(E - \underset{F_n}{E_c})$$

As will become increasingly clear, having $m_n^* = m_h^*$ results in the lowest transparency carrier concentration of the three cases. This is the ideal situation!

In the remaining two cases $m_n^* \neq m_h^*$, thus the energy for electrons and holes is asymmetric. Since the quasi-fermi level splitting is equal to $E_g$, the Fermi levels must necessarily fall either above or below the two band edges.

Consider the case where $m_n^* < m_h^*$ (ie the usual case). In this case $N_c < N_v$. Recall that the electron concentration is given "generally" by:

$$n = N_c\, g_c(E - F_n)$$

where $g(E-F_n)$ has either an $\exp[(F_n - E_c)/kT]$ or $\left(\frac{F_n - E_c}{kT}\right)^{3/2}$ dependence. The hole concentration takes a similar form. Notice that $N_c < N_v$ however $n = p$. In order for this to be true, $g_c(E-F_n)$ must be greater than $g_v(E-F_p)$. This is only possible if $F_n$ and $F_p$ are both located above the conduction and valence band edges, respectively. Furthermore, since $\Delta F = E_g$, the gap between $F_n$ and $E_c$ must equal the gap between $F_v$ and $E_v$. It follows that:

$$n = N_c \frac{4}{3\sqrt{\pi}}\left(\frac{F_n - E_c}{kT}\right)^{3/2} = N_c \frac{4}{3\sqrt{\pi}}\Delta^{3/2}$$

and

$$p = N_v \exp\left(-\frac{F_v - E_v}{kT}\right) = N_v e^{-\Delta}$$

where $\Delta = \frac{F_n - E_c}{kT} = \frac{F_v - E_v}{kT}$

Here $\Delta$ is an unknown that must be found in order to compute the transparency concentration. We solve for $\Delta$ by enforcing $n \simeq p$:

$$n \simeq p \Rightarrow N_c \frac{4}{3\sqrt{\pi}}\Delta^{3/2} = N_v e^{-\Delta} \Rightarrow \boxed{\frac{4}{3\sqrt{\pi}}\Delta^{3/2} = \frac{m_h^*}{m_n^*} e^{-\Delta}}$$

Solving this transcendental equation yields $\Delta$ which can then be plugged back in to $n$ to find the transparency concentration:

$$\boxed{N_{tr} = N_c \frac{4}{3\sqrt{\pi}}\Delta^{3/2}}$$

It is very interesting to note that in our expression above, $\Delta$ depends only on the ratio of the effective masses $m_h^*/m_n^*$. In particular, as the ratio is reduced towards 1, $\Delta$ also decreases. Thus in order to reduce $N_{tr}$, we want to make the effective masses as similar as possible

The final case, $m_n^* > m_h^*$ is very similar to the 2nd case.
Instead, we must solve:

$$\frac{m_n^*}{m_h^*} e^{-\Delta} = \frac{4}{3\sqrt{\pi}} \Delta \quad \text{where} \quad \Delta = \frac{E_c - F_n}{kT} = \frac{E_v - F_v}{kT}$$

Which is in turn used to find the transparency carrier concentration:

$$\boxed{N_{tr} = N_c e^{-\Delta}}$$

## Transparency in Quantum Wells

The previous analysis was for double-heterostructures which are assumed to be large enough that the discrete energy levels are insignificant. In the case of a quantum well, the well area is shrunk down such that it is only ~10 nm wide. In this case, confinement effects become significant.

The thought process from the previous section is still applicable. The primary difference, however, is that a 2D density of states must be used which leads to new expressions for n and p.

Let us reconsider the 2nd case $m_n^* < m_h^*$ for a quantum well. In this case, the electron and hole concentrations take the form:

$$n = \left(\frac{m_n^* kT}{\pi \hbar^2 L_z}\right) \frac{F_c - E_c}{kT} = N_c^{2D} \Delta$$

and

$$p = \left(\frac{m_h^* kT}{\pi \hbar^2 L_z}\right) e^{-\frac{F_v - E_v}{kT}} = N_v^{2D} e^{-\Delta}$$

Assuming $n = p$, $\Delta$ is found by solving:

$$N_c^{2D} \Delta = N_v^{2D} e^{-\Delta} \Rightarrow \boxed{\Delta = \frac{m_h^*}{m_n^*} e^{-\Delta}}$$

The expression for the third case is very similar.

In the case of the quantum well, the "ideal" case, $m_n^* = m_h^*$

→

can be treated more precisely than the 3D Double-heterostructure case since the density of states is constant. For the quantum well w/ $m_n^* = m_h^*$, the transparency carrier concentration is given by:

$$N_{tr}^{2D}\Big|_{ideal} = \frac{m_n^*}{\pi \hbar^2 L_z} \int_{E_{e1}}^{\infty} \frac{dE}{1 + e^{E-E_{e1}/kT}} = \boxed{\frac{m_n^*}{\pi \hbar^2 L_z} k_B T \ln 2 = N_{tr}}$$

## Transparency Current

Foremost, Recall how the current in a DH or QW laser is derived: First, by definition, the rate of change of the carrier concentration in the well is given by

$$\frac{dn}{dt} = -\frac{n}{\tau} \quad \text{where} \quad \frac{1}{\tau} \equiv \text{Recombination Rate} = An + Bn^2 + Cn^3$$

assuming $p = n$. The total current is directly obtained from $\frac{dn}{dt}$:

$$I = \left|\frac{dQ}{dt}\right| = \frac{d}{dt}\left(\frac{qnV}{\eta_i}\right) = q\frac{dn}{dt}V = \boxed{\frac{q}{\eta_i}\frac{n}{\tau}V = I}$$

V is the "active volume," ie the volume of the quantum well or well region of the DH. and $\eta_i$ is the IQE, ie the fraction of charge captured by the well.

Substituting $N_{tr}$ or $N_{tr}^{2D}$ for n in this expression gives the transparency or *threshold* current:

$$\boxed{I_{tr} = \frac{q}{\eta_i}\frac{N_{tr}}{\tau}V_{active}}$$

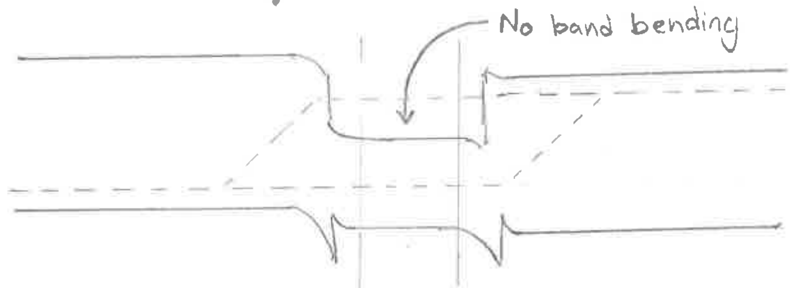
This expression leads to a number of important observations about QWs as compared to DHs.

Foremost, $V_{active}$ can be considerably smaller for QWs compared to DHs which leads to a lower threshold current (and thus a better wall plug efficiency). Furthermore, the effective masses in QWs can be tuned using strain, leading to even lower $I_{th}$.

## The unhandled Condition: $n \neq p$

Up until now, we have only handled the simple case $n = p$. But what about the case where $n \neq p$ due to doping conditions?

The answer to this question lies in the band diagram:




Notice that the well region has a region in which the bands are flat. Bands bend when net charge accumulates in a region, generating an electric field.

Because there is no bending in the center of the well, charge neutrality holds. As a result, $n$ and $p$ are related by

$$n + N_A = p + N_D$$

Where $N_A$ is the acceptor concentration and $N_D$ is the donor concentration. Substituting the aforemention expressions for $n$ and $p$ (depending on the relative sizes of $m_n^*$ & $m_h^*$) and solving for $\Delta = (F_n - E_c)/kT$ yields the transparency concentration.