Semiconductor Devices Review

Fermi-dirac distribution : $\boxed{f(E) = \dfrac{1}{1+e^{(E-E_F)/kT}}}$

↳ Boltzmann Approximation : $E-E_F \gg kT \Rightarrow \boxed{f(E) \approx e^{-(E-E_F)/kT}}$

Electron concentration : $n = \int_{E_c}^{\text{Top of Conduction Band} \approx \infty} f(E)\,\rho_c(E)\,dE$ where $\rho_c(E) \equiv$ Density of states in conduction band

↳ For $E-E_F \gg kT$: $\boxed{n \approx N_c e^{-(E_c-E_F)/kT}}$ where $\boxed{N_c = 2\left[\dfrac{2\pi m_n kT}{h^2}\right]^{3/2}}$

↳ For holes : $\boxed{p \approx N_v e^{-(E_F-E_v)/kT}}$ where $N_v = 2\left[\dfrac{2\pi m_p kT}{h^2}\right]^{3/2}$

↳ For Si at 300k : $N_c \simeq 2.8\times10^{19}\ cm^{-3}$ and $N_v \simeq 1.04\times10^{19}\ cm^{-3}$

Pn Product : $np = n_i^2$ where $\boxed{n_i = \sqrt{N_c N_v}\, e^{-E_g/kT}}$ → Intrinsic carrier concentration

↳ Intrinsic Semiconductor : No dopants → $n = p = n_i$

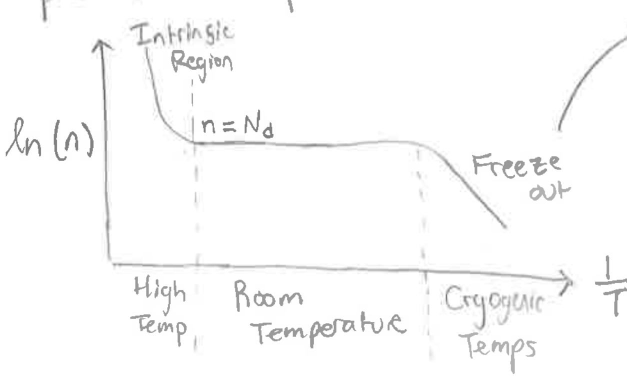
⚠ Charge Neutrality requires that $n + N_a = p + N_d$

Acceptor ion concentration (N_a); Donor ion concentration (N_d)

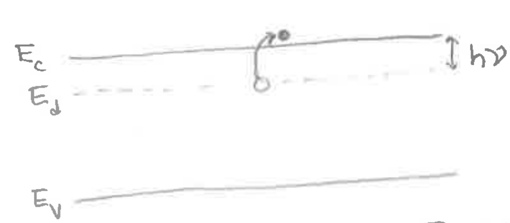
↳ $N_d - N_a \gg n_i \Rightarrow$ $n \simeq N_d - N_a$; $p \simeq n_i^2/n$

↳ $N_a - N_d \gg n_i \Rightarrow$ $p \simeq N_a - N_d$; $n \simeq n_i^2/p$

Temperature dependence of electron concentration :



→ can use Freeze out For IR detector :



Operate semiconductor in Freezeout so non of donor ions are ionized. Incident photons excite electron from donor level to conduction band

Motion & Recombination

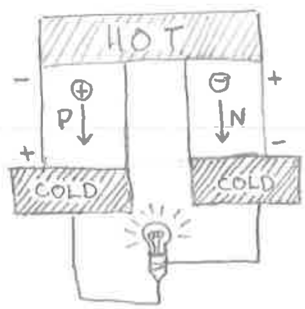
Average kinetic Energy $= \frac{\text{total KE}}{\text{number of electrons}} = \frac{\int f(E) D(E)(E-E_c) dE}{\int f(E) D(E) dE}$

↳ Equipartition Theorem: $\boxed{\langle KE \rangle = \frac{3}{2} kT} = \frac{1}{2} m_n v_{th}^2 \Rightarrow v_t = \sqrt{\frac{3kT}{m_n}}$

↳ Motion is random due to scattering → Net velocity is zero

↳ Scattering occurs on average after the mean free time ~0.1 ps

↳ Temperature gradients can cause current to flow!

↳ 

Temperature gradient causes diffusion of higher energy Majority carriers near hot reservoir towards cold reservoirs

⚠ Worth drawing out this band diagram

Drift velocity: $F = \frac{dp}{dt} \simeq \frac{\Delta p}{\Delta t} \Rightarrow \Delta p = F \Delta t \Rightarrow m_p v = qE\tau_{mp}$ ← Mean Free time for holes

$$\Rightarrow \boxed{v_{Drift,p} \simeq \frac{q\tau_{mp}}{m_p} E = \mu_p E}$$

↳ Hole Mobility: $v_{Drift,p} \simeq \mu_p E \rightarrow \boxed{\mu_p \equiv \text{hole Mobility} = \frac{q\tau_{mp}}{m_p}}$

↳ Electron Mobility: $v_{Drift,n} = -\mu_n E \rightarrow \boxed{\mu_n = \frac{q\tau_{mn}}{m_n}}$

↳ For Si @ 300k: $\mu_n = 1400 \frac{cm^2}{V \cdot s}$, $\mu_p = 470 \frac{cm^2}{V \cdot s}$

For GaAs @ 300k: $\mu_n = 8500 \frac{cm^2}{V \cdot s}$, $\mu_p = 400 \frac{cm^2}{V \cdot s}$

⚠ Mean free path: $\lambda_p \sim \tau_{mp} v_{th}$ (or for electrons $\lambda_n \sim \tau_{mn} v_{th}$)

Scattering Mechanisms:

(1) Phonon Scattering: electrons scatter off of phonon "particles"

↳ $\mu_{phonon} = \frac{q \tau_{ph}}{m} \propto T^{-3/2}$

(2) Impurity Ion Scattering: repulsive or attractive forces between electrons and impurity/dopant ions deflects electrons.

↳ $\mu_{impurity} \propto \frac{T^{3/2}}{N_a + N_d}$

(3) Total scattering rate/Mobility: $\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}}$; $\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}}$

Velocity Saturation:

In small devices, electric fields can be large which excite electrons to high velocities ⟶ If electron energy exceeds optical phonon energy, electrons will release energy as optical phonon ⟶ leads to maximum energy and hence saturated velocity

↳ $v_{sat} = \sqrt{\frac{2E_{opt}}{m}} \sim 10^7 \text{ cm/s}$

Drift Current & Conductivity:

↳ Drift current: $J_{p,drift} = q p v_{drift,p} = q p \mu_p E$

$J_{n,drift} = -q n v_{n,drift} = q n \mu_n E$

$\Rightarrow J_{tot} = J_{p,drift} + J_{n,drift} = \boxed{(q p \mu_p + q n \mu_n) E = J_{tot}}$

↳ Conductivity: $J = \sigma E \Rightarrow \boxed{\sigma = q p \mu_p + q n \mu_n}$

Diffusion Current:

$J_{n,diff} = q D_n \frac{dn}{dx}$; $J_{p,diff} = -q D_p \frac{dp}{dx}$

Negative to match correct direction that charge flows

Total Current:

$J_n = J_{n,drift} + J_{n,diff} = q n \mu_n E + q D_n \frac{dn}{dx}$

$J_p = J_{p,drift} + J_{p,diff} = q p \mu_p E - q D_p \frac{dp}{dx}$

$\Rightarrow J = J_n + J_p = \sigma E + q D_n \frac{dn}{dx} - q D_p \frac{dp}{dx}$

Einstein Relationship: $D_n = \frac{kT}{q}\mu_n$; $D_p = \frac{kT}{q}\mu_p$

Electron-Hole Generation and Recombination:

$n_0 \equiv$ equilibrium carrier concentration

↳ Excess carrier concentration: $\begin{cases} n \equiv n_0 + n' \\ p \equiv p_0 + p' \end{cases}$

↳ Charge neutrality: $n' = p'$

↳ Recombination → n' & p' will naturally return to 0

$$\frac{dn'}{dt} = -\frac{n'}{\tau} = -\frac{p'}{\tau} \Rightarrow \frac{n'}{\tau} = \frac{p'}{\tau} \equiv \text{Recombination Rate}$$

This seems weird...

↳ $np = n_i^2 \Rightarrow$ Generation = Recombination ; $np > n_i^2 \Rightarrow$ net Recombination

$np < n_i^2 \Rightarrow$ Net Generation

Quasi-Equilibrium: $np \neq n_i^2 \Rightarrow$ Semiconductor NOT in equilibrium

↳ Quasi-fermi levels: $n = N_c e^{-(E_c - E_{fn})/kT}$

$$p = N_v e^{-(E_{Fp} - E_v)/kT}$$

$$\Rightarrow np = N_c N_v e^{-(E_g - E_{fn} + E_{fp})/kT} = n_i^2 e^{(E_{fn} - E_{fp})/kT} = \boxed{n_i^2 e^{\Delta f/kT}}$$

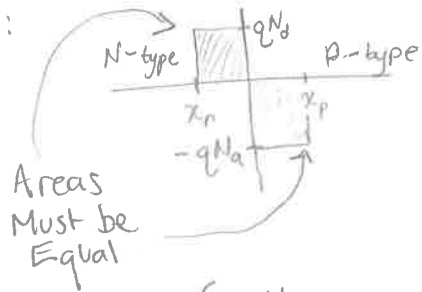
PN Junctions

depletion layer: $n \simeq 0$; $p \simeq 0$

Built in Potential: $\phi_{bi} = B - A = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2}$

Poisson's Equation: $\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_s}$ $\quad (\Leftarrow \bar{\nabla} \cdot \bar{D} = \rho)$

Charge distribution:



$\rightarrow$ Electric field continuity Requires:

$N_a |x_p| = N_d |x_N|$

Electric Potential:
$$V(x) = \begin{cases} -\frac{qN_a}{2\varepsilon_s}(x_p - x)^2 & (0 \le x \le x_p) \\ \phi_{bi} - \frac{qN_d}{2\varepsilon_s}(x - x_N)^2 & (-x_N \le x \le 0) \end{cases}$$

Depletion Layer Width: $W_{dep} = x_p - x_N = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$ $\rightarrow$ Largely determined by lighter dopant

$\hookrightarrow$ Under bias: $W_{dep} = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V)}{qN}}$ where $\frac{1}{N} = \frac{1}{N_a} + \frac{1}{N_d}$; $V > 0$ for FWD bias, $V < 0$ for Rev bias

Depletion Layer capacitance: $C_{dep} = A\frac{\varepsilon_s}{W_{dep}}$

$\hookrightarrow \Rightarrow \frac{1}{C_{dep}^2} = \frac{2(\phi_{bi} + V_r)}{qN\varepsilon_s A^2} \longrightarrow \begin{cases} \text{Slope gives } N \\ x\text{-intercept gives } \phi_{bi} \end{cases}$

Breakdown under reverse bias:

$\hookrightarrow$ As reverse bias increased, electric field across junction increases until breakdown occurs $\rightarrow V_B = \frac{\varepsilon_s E_{crit}}{2qN} - \phi_{bi}$

$\hookrightarrow$ Tunneling breakdown: goes as $\sim \exp(-H/E)$ where $H \sim E_g^{3/2} m^{*\,1/2}$, Critical field is $\sim 10^6$ V/cm, occurs when N is very high and V_B is quite low

↳ Avalanch breakdown: high energy electrons in conduction band traveling through depletion layer can excite new electrons up from valence band. The corresponding whole in the valence band can excite a new electron-hole pair as it accelerates through the depletion layer starting the process all over again

Forward Bias: Carrier Injection: under forward bias, the potential barrier at the junction is reduced, allowing electrons on the n-side and holes on the p-side to diffuse across the junction → current flows

↳ Minority Carriers density $\left\{ \boxed{\begin{aligned} n(x_p) &= n_{po} e^{qV/kT} = \frac{n_i^2}{N_a} e^{qV/kT} \\ p(x_n) &= p_{no} e^{qV/kT} = \frac{n_i^2}{N_d} e^{qV/kT} \end{aligned}} \right.$

⚠ Minority carries densities are elevated by factor $e^{qV/kT}$

→ Excess minority carries density: $\left\{ \boxed{\begin{aligned} n'(x_p) &= n_{po}\left(e^{qV/kT} - 1\right) \\ p'(x_n) &= p_{no}\left(e^{qV/kT} - 1\right) \end{aligned}} \right. \begin{aligned} &= n(x_p) - n_{po} \\ &= p(x_n) - p_{no} \end{aligned}$

↳ Current Continuity Equations:

$$\left\{ \begin{aligned} &\frac{d^2 p'}{dx^2} = \frac{p'}{L_p^2} \quad \text{where } L_p = \sqrt{D_p \tau_p} \equiv \text{diffusion length for holes} \\ &\frac{d^2 n'}{dx^2} = \frac{n'}{L_n^2} \quad \text{where } L_n = \sqrt{D_n \tau_p} \equiv \text{diffusion length for } e^- \end{aligned} \right.$$

Solved using B.C.: $p'(\infty) = 0$; $p'(x_N) = p_{No}\left(e^{qV/kT} - 1\right)$

PN Diode IV Characteristics: Injected Minority carriers diffuse away from junction ⇒ Net current

↳ Recall: $J_{p,drift} = -qD_p \frac{dp'(x)}{dx}$ → $J_{tot,drift} = qD_n \frac{dn'(x)}{dx} - qD_p \frac{dp'(x)}{dx}$

↳ Plugging in: $J_{tot,drift} = \left(q \frac{D_p}{L_p} p_{No} + q \frac{D_n}{L_n} n_{po}\right)\left(e^{qV/kT} - 1\right)$

Since $I = \int d\bar{a} \cdot \bar{J} \approx AJ \Rightarrow$ $\boxed{I = I_0\left(e^{qV/kT} - 1\right) \quad \text{where } I_0 = Aqn_i^2\left(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a}\right)}$

↳ $I_0 \equiv$ Reverse saturation Current ⟶ As $V \to -\infty$, $I \to -I_0$
ie current saturates to I_0 under reverse bias

Charge storage: Under Forward bias, diodes store charge which is proportional to $p_N'(0)$ and $n_p'(0)$.

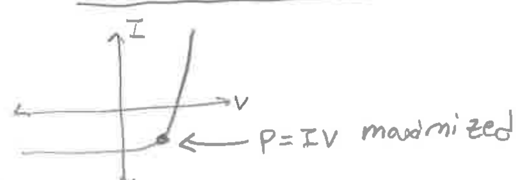
$$\Rightarrow Q \propto I \Rightarrow I = Q/\tau_s \quad \text{where } \tau_s \equiv \text{Charge storage time.}$$

↳ This gives rise to diffusion capacitance.

↳ $$C = \frac{dQ}{dV} = \frac{d}{dV}(I\tau_s) = \tau_s \frac{dI}{dV} = \boxed{\tau_s \frac{I_{DC}}{kT/q} = C_{diff}}$$

Solar Cells: Photons absorbed w/in a diffusion length of the junction can generate minority carriers which are swept across the junction by the built-in potential ⟶ short circuit current

$$I = I_0\left(e^{qV/kT} - 1\right) - I_{sc}$$



↳ External load must be matched to cell such that it operates at it's optimum.

⚠ $\frac{hc}{\lambda} \sim \frac{1.24}{\lambda}$ (um)

↳ Short Circuit Current & Open Circuit Voltage:

Minority carrier concentration modified: $\frac{d^2p'}{dx^2} = \frac{p'}{L_p^2} - \frac{G}{D_p}$ ← rate of generation due to incident light

with boundary conditions: $p'(0) = 0$, $p'(\infty) = \text{const} \Rightarrow \frac{d^2p'}{dx^2} = 0$.

$$\Rightarrow p'(\infty) = L_p^2 \frac{G}{D_p} = \tau_p G$$

Solving the DEQ: $p'(x) = \tau_p G(1 - e^{-x/L_p})$

So, $J_p = -q D_p \frac{dp'}{dx} = q \frac{D_p}{L_p} \tau_p G e^{-x/L_p}$

And $I_{sc} = A J_p(0) = \boxed{A q L_p G = I_{sc}}$

Quantum Review

Schrodinger's Equation is:

$$\hat{H}\Psi(\bar{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\bar{r},t)$$

Where the Hamiltonian operator is given by:

$$\hat{H} = \frac{p^2}{2m} + V(r,t) = +\frac{(-i\hbar\nabla)^2}{2m} + V(r,t)$$

$$= \frac{-\hbar^2\nabla^2}{2m} + V(r,t)$$

Thus:

$$\boxed{\left[\frac{-\hbar^2\nabla^2}{2m} + V(r,t)\right]\Psi(\bar{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\bar{r},t)}$$

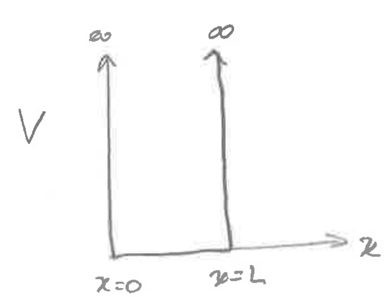
For simplicity, assume that $\Psi(\bar{r},t)$ has time harmonic dependence, ie $\Psi(\bar{r},t) = \Psi(\bar{r})e^{-i\omega t}$ and $V(\bar{r},t) = V(\bar{r})e^{-i\omega t}$

Schrodinger's Equation thus simplifies to the time-independent form:

$$\left[\frac{-\hbar^2\nabla^2}{2m} + V(\bar{r})\right]\Psi(\bar{r}) = E\Psi(\bar{r}) \quad \text{where } \underline{E = \hbar\omega}$$

To solve this equation we must specify a potential $V(\bar{r})$. Let us first consider an infinite square well:

$$V(\bar{r}) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{else} \end{cases}$$



For this set of Boundary conditions, we must find $\psi(x)$ for the case that $V=0$ and enforce boundary conditions such that $V \to \infty$ outside the well:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) = E\,\psi(x) \Rightarrow \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + E\psi = 0$$

Consider the trial solution, $\psi(x) = e^{ax}$:

$$\Rightarrow \frac{\hbar^2}{2m}a^2 + E = 0 \Rightarrow a^2 = \frac{-2mE}{\hbar^2} \Rightarrow a = \pm i\sqrt{\frac{2mE}{\hbar^2}} = \pm ik$$

Thus

$$\psi(x) = Ae^{ikx} + Be^{-ikx} \quad \text{where} \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

Alternatively, we can write:

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

Since $V \to \infty$ at the boundary of, we require that

$$\psi(x=0) = \psi(x=L) = 0;$$

$\psi(x=0)=0$ gives $B=0$ while $\psi(x=L)=0$ gives

$$A\sin(kL) = 0 \Rightarrow kL = \pi n \Rightarrow k = \frac{\pi n}{L} \quad \text{for } n \in \mathbb{I}$$

There is a final condition we must apply: $|\psi(x)|^2$ gives the probability of measuring an electron at a location x. The wavefunction must thus be properly normalized:

$$\int_0^L dx\,|\psi(x)|^2 = 1 \Rightarrow \int_0^L dx\,A^2\sin^2(kx) = 1$$

$$\Rightarrow A^2\left[\frac{x}{2} - \frac{\sin(2kx)}{4k}\right]_0^L = A^2\left[\frac{L}{2} - \frac{\sin(2\frac{\pi n}{L}L)}{4k}\right] = A^2\frac{L}{2} = 1$$

$$\Rightarrow A = \sqrt{\frac{2}{L}}$$

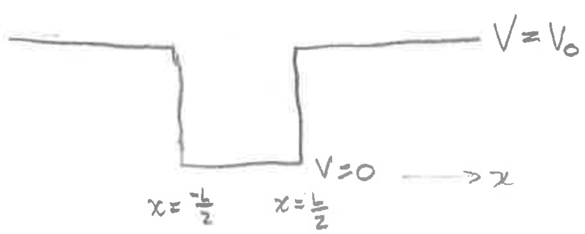
Thus the wavefunction which satisfies the infinite square well potential is:

$$\boxed{\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}$$

Furthermore, this implies that the electron energy is quantized since,

$$k = \frac{n\pi}{L} = \sqrt{\frac{2m^* E}{\hbar^2}} \Rightarrow E = \frac{\hbar^2 \pi^2}{2m^*}\left(\frac{n}{L}\right)^2 \quad \text{for } n \in \mathbb{Z}$$

Next, consider a non-infinant square-well:



$$V(x) = \begin{cases} 0 & |x| < \frac{L}{2} \\ V_0 & |x| > \frac{L}{2} \end{cases}$$

Here we must solve three forms of SE: