Prelim Review: Optoelectronics

I. Intro to lasers

LASER: Light Amplification by stimulated Emission of radiation
↳ Lasers need ① gain medium ② cavity/Feedback/resonator

Note: Laser efficiency is often given in $\frac{mW}{mA}$

Confinement Factor: $\Gamma = \dfrac{\int_{-d/2}^{d/2} dy\, \bar{E}\cdot\bar{E}^* dy}{\int_{-\infty}^{\infty} \bar{E}\cdot\bar{E}^* dy}$

Determining threshold gain: Light intensity ($I \propto |E|^2$) changes as an exponential function of distance. The characteristic length scale of growth or decay is given by the gain or loss. At threshold, gain and loss balance exactly:

$$I = I_0 e^{\overset{\swarrow g \text{ or } \alpha}{a} z} \Rightarrow I_0 e^{\Gamma g L - \alpha_i L} R_1 e^{\Gamma g L - \alpha_i L} R_2 = I_0$$

where $L \equiv$ cavity length
$g \equiv$ gain
$\alpha_i \equiv$ intrinsic loss
$R_1, R_2 \equiv$ Mirror Losses

$$\Rightarrow \boxed{g_{th} = \frac{\alpha_i}{\Gamma} + \frac{1}{2\Gamma L} \ln \frac{1}{R_1 R_2}}$$

$$\Rightarrow \boxed{g_{th} = \frac{\alpha_i + \alpha_m}{\Gamma} = \frac{\alpha_{tot}}{\Gamma}}$$

Quantum Efficiency: Ratio of Photon current to electron current

$$QE = \frac{\text{Photons/sec}}{\text{Electrons/sec}} = \frac{P/h\nu}{I/q} = \boxed{\frac{\alpha_m}{\alpha_i + \alpha_m}}$$

Quality Factor:

$$Q = \frac{\text{Energy Stored}}{\text{Energy dissipated/cycle}} = \boxed{\frac{\omega_0}{\Delta\omega} = \omega_0 \underbrace{\tau_p}_{\text{Photon life time}} = Q}$$

↳ The photon life time can be expressed in terms of cavity loss:

$$\boxed{\frac{1}{\tau_p} = \alpha_{tot} \frac{c}{n}} \Rightarrow Q = \frac{\omega_0 n}{c\,\alpha_{tot}} \rightarrow \text{rewrite } g_{th}:$$

$$\boxed{g_{th} = \frac{\omega}{Q} \frac{n}{\Gamma c}} \propto \frac{1}{Q}$$

Note: The losses of a cavity are inversely proportional to $\frac{1}{Q}$ where Q is an associated quality factor. The total quality factor of a cavity is $\frac{1}{Q} = \frac{1}{Q_i} + \frac{1}{Q_m}$ where Q_i is the quality factor resulting from intrinsic loss & Q_m is from mirror loss.

Since $QE = \frac{\alpha_m}{\alpha_i + \alpha_m}$, it follows that:

$$QE = \frac{Q_m}{Q}$$

Thus a larger Q means lower g_{th} but also lower QE

Note: $QE = \frac{dP}{dI}\frac{q}{\hbar\omega}$ ← directly from definition (optical Power)

II. Semiconductors

Semiconductors have fixed bandgap ⇒ interact w/ particular wavelength of light: $\boxed{E_g = \frac{hc}{\lambda_g}}$ ⇒ $\lambda_g = \frac{hc}{E_g}$

⚠ Review Quantum

Energy of electron in Conduction Band: $E_e = E_c + \frac{\hbar^2 k^2}{2m_e}$

Energy of Hole in Valence Band: $E_h = E_v - \frac{\hbar^2 k^2}{2m_h}$

Conservation of energy requires: $E_e - E_h = h\nu$

Momentum must be conserved. In general $k_{h\nu} << k_e, k_h$ so,

$\boxed{E_g + \frac{\hbar^2 k^2}{2m_r^*} = h\nu}$ where $\frac{1}{m_r^*} = \frac{1}{m_e^*} + \frac{1}{m_h^*}$

Note: Due the momentum mismatch in indirect bandgap semiconductors, absorption is more difficult. → Phonons provide momentum for absorption in indirect bandgap semiconductors.

Electron Concentration: $\boxed{n = \int_{E_c}^{\infty} dE\, f_n(E)\rho(E)}$ where $f(E)$ = Fermi Dirac Dist, $\rho(E)$ = DOS

Deriving Density of States: We begin by noting that in a crystalline solid, electron wavefunctions obey periodic boundary conditions. This leads to a discrete set of wavevectors $(k_x, k_y, k_z) \rightarrow \left(m\frac{2\pi}{L_x}, n\frac{2\pi}{L_y}, \ell\frac{2\pi}{L_z}\right)$.

It follows that a single state occupies a volume $\frac{(2\pi)^3}{L_xL_yL_z}$.

Next, the volume (in k space) of an infinitesimal shell of thickness dk is given by $4\pi k^2 dk$. The number of states in this thin shell is:

$$\frac{4\pi k^2 dk}{\frac{(2\pi)^3}{L_xL_yL_z}} = \frac{k^2 dk}{2\pi^2/L_xL_yL_z}$$

The density of states in 3D is obtained by normalizing by volume V and multiplying by 2 (pauli-exclusion):

$$\rho(k)dk = \frac{2k^2 dk}{2\pi^2 V/L_xL_yL_z} = \frac{k^2 dk}{\pi^2}$$

Finally we note that $E(k) = E_c + \frac{\hbar^2 k^2}{2m_e}$ (for electrons) and find:

$$\boxed{\rho_e(E) = \frac{m_e^*}{\hbar\pi^2}\sqrt{\frac{2m_e^*(E-E_c)}{\hbar^2}}\, dE}$$ And $$\boxed{\rho_h(E) = \frac{m_h^*}{\hbar\pi^2}\sqrt{\frac{2m_h^*(E_v-E)}{\hbar^2}}}$$

↳ For holes

Electron concentration in 3D:

$$\boxed{\begin{aligned} F_n \ll E_c &\Rightarrow n = N_c e^{\frac{F_n - E_c}{kT}} \\ F_n \gg E_c &\Rightarrow n = N_c \frac{4}{3\sqrt{\pi}}\left(\frac{F_n - E_c}{kT}\right)^{3/2} \end{aligned}} \quad N_c = 2\left(\frac{\pi m_e k T}{2\pi^2 \hbar^2}\right)^{3/2}$$

Quantum Mechanics Review

Schrodinger's Equation: $\hat{H}\Psi(\bar{r},t) = i\hbar \frac{\partial}{\partial t}\Psi(\bar{r},t)$

↳ Hamiltonian: $\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}(\bar{r},t)$ where $\hat{p} = i\hbar\bar{\nabla}$

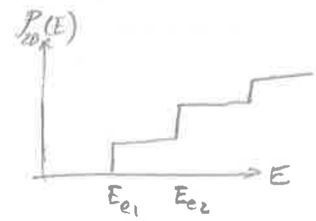
Quantum Well: Assume: $\Psi(\bar{r},t) = \Psi(\bar{r})e^{i\omega t}$ where $\Psi(\bar{r}) = e^{i(k_x x + k_y y)}\Psi(z)$

Plug in and solve for $\Psi(z)$

↳ $\boxed{E_n = \frac{\hbar^2}{2m_e^*}\left(\frac{n\pi}{L}\right)^2}$ $\Rightarrow$ $E = \frac{\hbar^2(k_x^2 + k_y^2)}{2m_e^*} + \frac{\hbar^2}{2m_e^*}\left(\frac{n\pi}{L}\right)^2$

Note: Quantum wells are 2D structures $\Rightarrow$ 2D Density Of States

$$\boxed{\rho_{2D}(E) = \frac{m_e}{\pi\hbar^2 L_z} = \text{const}}$$



NOTE: This is really $\rho_{2D}(E) = \sum_i \frac{m_e}{\pi\hbar^2 L_z}\Theta(E - E_i)$

The electron concentration in 2D (for $E_{fn} > kT$) is

$$n_{2D} \simeq (E_{Fn} - E_{e_1})\frac{m_e^*}{\pi\hbar^2 L_z}$$

⚠ Review Derivation of DOS in 1D & 0D

Derivation of Fermi's Golden Rule

We begin w/ the solution to SE for a two level system:

$$\varphi_n(\bar{r},t) = \varphi_n(\bar{r})e^{i E_n/\hbar\, t} = |n\rangle e^{i E_n/\hbar\, t} \quad \text{and} \quad \hat{H}_0|n\rangle = E_n|n\rangle$$

We assume that light is a weak perturbation:

$$\hat{H}'(\bar{r},t) = \hat{H}'(\bar{r})e^{-i\omega t} + \hat{H}'^{+}(\bar{r})e^{i\omega t}$$

Such that the total perturbed Hamiltonian is: $\hat{H} = \hat{H}_0 + \hat{H}'$

We assume that the solutions to SE w/ the perturbed Hamiltonian are a linear combination of the unperturbed wavefunction:

$$\Psi(\bar{r},t) = \sum_{n=1}^{\infty} a_n(t)|n\rangle e^{i\frac{E_n}{\hbar}t}$$

We substitute this solution into SE:

$$\left(\cancel{\hat{H}_0} + \hat{H}'\right) \sum_{n=1} a_n |n\rangle e^{iE_n/\hbar\, t} = i\hbar \sum_{n=1} \left(\frac{d}{dt} a_n\right) |n\rangle e^{iE_n/\hbar\, t} + \cancel{i\hbar \sum_{n=1} a_n |n\rangle \left(\frac{iE_n}{\hbar}\right) e^{iE_n/\hbar\, t}}$$

Leaving, $$\hat{H}' \sum_{n=1} a_n |n\rangle e^{i\frac{E_n}{\hbar}t} = i\hbar \sum_{n=1} \left(\frac{d}{dt} a_n\right) |n\rangle e^{i\frac{E_n}{\hbar}t}$$

We can simplify this expression by taking the inner product w/ some state $|m\rangle$ which is orthogonal to $|n\rangle$ (and part of same basis):

$$\sum_{n=1} a_n \langle m | \hat{H}' | n\rangle e^{iE_n/\hbar\, t} = i\hbar \frac{da_m}{dt} e^{i E_m/\hbar\, t}$$

$$\Rightarrow \quad \frac{da_m}{dt} = \frac{1}{i\hbar} \sum_{n=1} a_n(t) \hat{H}'_{mn} e^{i\omega_{mn}t} \quad \text{where} \quad \begin{cases} \hat{H}'_{mn} = \langle m | \hat{H}' | n \rangle \\ \omega_{mn} = \dfrac{E_m - E_n}{\hbar} \end{cases}$$

Next, we make a small modification: let's rewrite $\hat{H} = \hat{H}_0 + \lambda \hat{H}'$ and slowly "turn on" $\hat{H}'$ (ie start w/ λ small. Likewise, we rewrite $a_n(t) = a_n^{(0)}(t) + \lambda a_n^{(1)}(t) + \lambda^2 a_n^{(2)}(t) + \cdots$ (⚠ Review this)

Matching powers of λ we find:

$$\frac{da_m^{(0)}}{dt} = 0 \quad \text{and} \quad \frac{da_m^{(1)}}{dt} = \frac{1}{i\hbar} \sum_{n=1} a_n^{(0)}(t) \hat{H}'_{mn} e^{i\omega_{mn}t}$$

We assume all electrons start in a state $|i\rangle$ such that $a_i^{(0)}(t) = 1$ and $a_m^{(0)}(t) = 0$ for $m \neq i$. It follows that:

$$\frac{da_f^{(1)}}{dt} = \frac{1}{i\hbar} \hat{H}'_{fi} e^{i\omega_{fi}t} = \frac{1}{i\hbar}\left(\hat{H}'_{fi} e^{-i\omega t} + \hat{H}'^{+}_{fi} e^{i\omega t}\right) e^{i\omega_{fi}t}$$

since the harmonic time dependence has been made explicit, we integrate to find $a_f^{(1)}(t)$:

$$a_f^{(1)}(t) = \frac{1}{i\hbar} \frac{e^{i(\omega_{fi}-\omega)t} - 1}{i(\omega_{fi} - \omega)} \hat{H}'_{fi} + \frac{1}{i\hbar} \frac{e^{i(\omega_{fi}+\omega)t} - 1}{i(\omega_{fi} + \omega)} \hat{H}'^{+}_{fi}$$

It follows that the probability of finding an electron in a final state $|f\rangle$ is

$$\left|a_f^{(1)}(t)\right|^2 = \frac{|\hat{H}'_{fi}|^2}{\hbar^2} \frac{4\sin^2\left(\frac{\omega_{fi}-\omega}{2}t\right)}{(\omega_{fi} - \omega)^2} + \frac{|\hat{H}'^{+}_{fi}|^2}{\hbar^2} \frac{4\sin^2\left(\frac{\omega_{fi}+\omega}{2}t\right)}{(\omega_{fi} + \omega)^2}$$

Noting that $\frac{\sin^2(\Delta x t)}{\Delta x} = t^2 \sin^2(\Delta x t) \sim \frac{\pi t}{2}\delta(\Delta x)$, we end up with Fermi's golden Rule by taking a time derivative:

Absorption

Fermi's Golden Rule: $$\boxed{W_{i\to f} = \frac{d}{dt}|a_f^{(1)}(t)|^2 = \frac{2\pi}{\hbar}|\hat{H}'_{fi}|^2\delta(E_f - E_i - \hbar\omega) + \frac{2\pi}{\hbar}|\hat{H}'^{+}_{fi}|^2\delta(E_f - E_i + \hbar\omega)}$$

stimulated Emission

When more that two energy levels are present, Fermi's Golden Rule is modified to include the density of states:

$$\rightarrow \boxed{\frac{2\pi}{\hbar}|H'_{fi}|^2 \rho_f(E_f)\,\delta(E_f - E_i - \hbar\omega)}$$

⚠ Review Maxwell's Equations, gauge theory

Derivation of Matrix Elements:

Coulomb Gauge

① Force $= \frac{\partial \bar{P}}{\partial t} + e\bar{E} = \frac{\partial \bar{P}}{\partial t} - e\frac{\partial \bar{A}}{\partial t} = \frac{\partial}{\partial t}(\bar{P} - e\bar{A}) = \frac{\partial}{\partial t}\bar{\mathcal{P}}$

② Recall: $\hat{H} = \frac{p^2}{2m} + V(\bar{r}) \Rightarrow \hat{H} = H_0 + H' = \frac{1}{2m_0}(\bar{P} - e\bar{A})^2 + V(\bar{r})$

③ Assume weak interactions, ie $\bar{A}\cdot\bar{A} \sim 0 \Rightarrow H' = -\frac{e}{2m_0}(\bar{P}\cdot\bar{A} - \bar{A}\cdot\bar{P})$

④ $\bar{P}$ and $\bar{A}$ commute $\Rightarrow \boxed{H' = -\frac{e}{m_0}\bar{A}\cdot\bar{P}}$

Same Polarization

⑤ Assume planewave solution: $\bar{A} = \hat{e}\frac{A_0}{2}e^{i\bar{k}\cdot\bar{r} - i\omega t} + c.c. \Rightarrow \bar{E} = -\frac{\partial \bar{A}}{\partial t} = -i\omega\bar{A}$

Also assume wavelength of light >> electron characteristic length ($\bar{k}\cdot\bar{r} \sim 0$)

$\Rightarrow H' = -\frac{eA_0}{2m_0}\hat{e}\,e^{-i\omega t}\cdot\bar{P}$ <u>Dipole Approx</u>

⑥ Matrix Element: $H'_{ba} = \langle b|H'|a\rangle = \boxed{-\frac{eA_0}{2m_0}\hat{e}\cdot\langle b|\bar{P}|a\rangle}$

Alternative Form: $\boxed{H'_{ba} = -e\bar{E}\cdot\bar{r}_{ba}}$ "dipole Approximation"

Note: We often write: $|\hat{e}\cdot\bar{P}_{ba}| = \frac{m_o}{6}E_p$ where E_p is a material const

$\hookrightarrow \Rightarrow |\bar{r}_{ba}| = \frac{|\hat{e}\cdot\bar{P}_{ba}|}{m_o\omega} = \frac{E_p}{6\omega}$

Absorption rate: $$R_o(\hbar\omega) = \frac{2}{V}\underbrace{\sum_k}_{\text{All states that satisfy}} \frac{2\pi}{\hbar}|\mathcal{H}'_{ba}|^2 \cdot \delta(E_b - E_a - \hbar\omega)$$

$E_b - E_a = \hbar\omega$

$= (E_c - E_v) + \frac{\hbar^2 k^2}{2m_r}$

+ joint DOS

$$= \frac{2\pi}{\hbar}|\mathcal{H}'_{ba}|^2 \overbrace{\frac{2}{V}\sum_k \delta(E_b - E_a - \hbar\omega)}^{\text{Density of states!}}$$

$$= C_o|\hat{e}\cdot\bar{P}_{cv}|^2 \rho_r(\hbar\omega - E_a) \quad \text{where } C_o = \frac{\pi e^2}{n_r c\varepsilon_o m_o^2\omega}$$

Inter-band Absorption: $$R_{ab} = \frac{2}{V}\sum_k \frac{2\pi}{\hbar}|\mathcal{H}'_{ba}|^2 \delta(E_b - E_a - \hbar\omega) \underbrace{f_v(E_a)[1 - f_c(E_b)]}_{\text{account for occupation prob}}$$

Interband Emission: $$R_{ba} = \frac{2}{V}\sum_k \frac{2\pi}{\hbar}|\mathcal{H}'_{ba}|^2 \delta(\overbrace{E_a - E_b}^{\text{Note: Error in notes here}} + \hbar\omega) f_c(E_b)[1 - f_v(E_a)]$$

Net Absorption Rate:
(gain spectra)

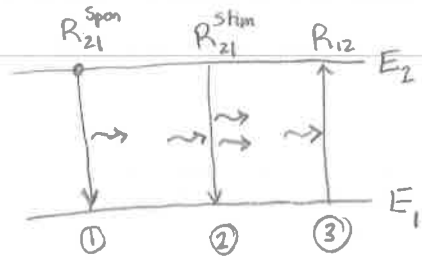
$$\boxed{\alpha(\hbar\omega) = -\frac{2}{V}\sum_k \frac{2\pi}{\hbar}|\mathcal{H}'^{+}_{ba}|^2 \delta(E_b - E_a - \hbar\omega) f_g(\hbar\omega - E_g)}$$

$$= R_{ab} - R_{ba}$$

$\hookrightarrow$ Note: Use fact that ① $\delta(x)$ is even so $\delta(E_b - E_a - \hbar\omega) = \delta(E_a - E_b + \hbar\omega)$ and ② $|\mathcal{H}'_{ba}|^2 = |\mathcal{H}'^{+}_{ab}|^2$

Bernard-Durafourg gain Condition: $\boxed{E_g < \hbar\omega < \Delta F}$

Excess Electron/hole Energy: $$\boxed{\begin{aligned} E_b - E_c &= (\hbar\omega - E_g)\frac{m_r^*}{m_e^*} \\ E_v - E_a &= (\hbar\omega - E_g)\frac{m_r^*}{m_h^*} \end{aligned}}$$



Einstein AB Coefficients:

① $R_{21}^{spon} = A_{21} f_2 (1-f_1)$

② $R_{21}^{stim} = B_{21} f_2 (1-f_1) P(E_{21})$

③ $R_{12} = B_{12} f_1 (1-f_2) P(E_{21})$

Basic Idea: Absorption/Emission rates are proportional to electron and photon occupation probabilities

Where $P(E_{21}) = N(E_{21}) n_{ph} \rightarrow n_{ph} \equiv$ Bose-Einsten dist $= \dfrac{1}{e^{\hbar\omega/kT} - 1}$

↳ $N(E_{21}) \equiv$ Photon Density of states

$$= \frac{2}{V} \int \frac{4\pi k^2 dk}{\left(\frac{2\pi}{L}\right)^2} \delta(E - \hbar\omega) \qquad \omega = \frac{kc}{n} \qquad \Rightarrow \boxed{N(E) = \frac{8\pi n^2 E^2}{h^3 c^3}}$$

At thermal Equilibrium: $\boxed{N(E_{21}) = \text{Photon DOS} = \frac{A}{B}}$ (Assuming $B_{12} = B_{21} = B$

$\boxed{R_{12} = R_{21}^{spon} + R_{21}^{stim}}$

Spontaneous Emission Rate: $\boxed{r_{21}^{spon}(E_{21}) = \frac{8\pi n^2 E_{21}^2}{h^3 c^3} C_0 |\hat{e} \cdot \bar{P}_{cv}|^2 \rho_r(\hbar\omega - E_g) \, f_c / f_g}$

⚠ Review derivation

Note: $n_{sp} = \dfrac{f_e}{f_g} \equiv$ "spontaneous emission Factor"

Interband & Intersubband Selection Rules

Bloch's Theorem: $|\psi\rangle = u(\bar{r}) \dfrac{e^{i\bar{k}\cdot\bar{r}}}{\sqrt{V}}$; $u(\bar{r}+\bar{R}) = u(\bar{r})$ quickly varying

⚠ Polarization is determined by $\hat{e} \cdot \langle b|\bar{P}|a\rangle$

$$\Rightarrow H'_{ba} = -\frac{eA_0}{2m_0} \hat{e} \cdot \int d\bar{r}\, u_c^*(\bar{r}) \frac{e^{-i\bar{k}_c\cdot\bar{r}}}{\sqrt{V}} \left[-i\hbar\bar{\nabla}\left(u_v(\bar{r}) \frac{e^{i\bar{k}_v\cdot\bar{r}}}{\sqrt{V}}\right)\right] e^{i\bar{k}_{op}\cdot\bar{r}}$$

$\approx$ term 1 + $\cancel{\text{term 2}} \rightarrow 0$ since $\boxed{\langle c|v\rangle = 0}$

$$\simeq -\frac{eA_0}{2m_0 V} \hat{e} \cdot \int u_c^* (-i\hbar\bar{\nabla}) u_v(\bar{r}) d\bar{r} \underbrace{\int d\bar{r}\, e^{i(\bar{k}_v - \bar{k}_c + \bar{k}_{op})\cdot\bar{r}}}$$

$$\boxed{= \delta(\bar{k}_v - \bar{k}_c + \bar{k}_{op}) = k\text{-Selection Rule}}$$

Transition between E_{hm} and E_{en} : $\mathcal{H}'_{ba} = \frac{eA_0}{2m_0} \hat{e} \cdot \bar{P}_{cv} \delta_{kk'} \cdot \delta_{mn}$ equivalent subband

conserve momentum

Note : In 2D $g_{max}(\hbar\omega) = C_0 |\hat{e} \cdot \bar{P}_{cv}|^2 \frac{m_r^*}{\pi \hbar^2 L_z}$ $(\hbar\omega < E_{h2}^{e2}$

2D Electron Concentration : $N_{2D} = \int dE \, \rho_e^{2D}(E) \, f_c^n(E)$

$F_c >> E_{e1}$ $N \simeq \frac{m_e^*}{\pi \hbar^2 L_z}(F_c - E_{e1})$; $F_c << E_{e1}$ $N \simeq \left(\frac{m_e kT}{\pi \hbar^2 L_z}\right) e^{-\frac{E_{e1} - F_c}{kT}}$

Intersubband transitions : $|a\rangle = u_c(\bar{r}) \frac{e^{i\bar{k}_t \cdot \bar{r}}}{\sqrt{A}} \varphi_1(z)$

$|b\rangle = u_c(\bar{r}) \frac{e^{i\bar{k}_t \cdot \bar{r}}}{\sqrt{A}} \varphi_2(z)$

$\hookrightarrow$ Do: $\mathcal{H}'_{ba} = \langle b|\mathcal{H}'|a\rangle \propto \langle b| -\bar{E} \cdot e\bar{r} |a\rangle = -e\bar{E} \cdot \underbrace{\langle b|\bar{r}|a\rangle}_{\text{Evaluate this integral}} = -\bar{E} \cdot \bar{\mu}_{ab}$

In 2D : $\bar{\mu}_{21} = \frac{-16}{9\pi^2} e L_z \hat{z}$ And $\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \varepsilon_0} \delta(\hbar\omega - E_{21}^e)(N_1 - N_2)|\mu_{21}|^2$

$\hookrightarrow$ Note: for $E_{e1} < F < E_{e2}$, $\alpha(\hbar\omega) \propto N_1$ while for $E_{e2} < F$ $\alpha(\hbar\omega) = \text{const}$

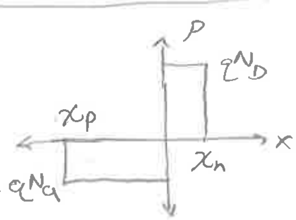
Lorentzian Broadening : $\delta(\Delta E) \rightarrow \boxed{g(\Delta E) = \frac{1}{\pi} \frac{\frac{\Gamma}{2}}{\Delta E^2 + \left(\frac{\Gamma}{2}\right)^2}}$

$\alpha(\hbar\omega) \rightarrow \frac{\pi\omega}{n c \varepsilon_0} g(\Delta E)(N_1 - N_2)|\mu_{21}|^2$

$E_{e2} - E_{e1}$

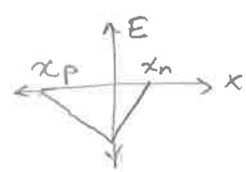
$\hookrightarrow$ With Broadening : $\boxed{\alpha_{max} = \frac{\pi\omega}{n c \varepsilon_0} \frac{1}{\pi} \frac{1}{\Gamma/2} \frac{m_e^*}{\pi \hbar^2 L_z} E_{21}^e |\mu_{21}|^2}$

PN Junction Review (see EE230A notes)



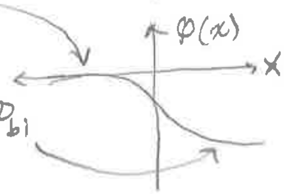
Step Junction Approx : $N_a x_p = N_d x_n$

Electric Field : $\bar{\nabla} \cdot \bar{D} = \rho$, $\bar{D} = \epsilon \bar{E}$ $\Rightarrow$ $\bar{D} \sim \int \rho \, dx$



Electric Potential : $\varphi(x) = \int E \, dx$ $\Rightarrow$ $\frac{-q^2 N_a}{2\epsilon_p}(x + x_p)^2$

Built-in Potential $\equiv \varphi_{bi}$ $\frac{q^2 N_a}{2\epsilon_n}(x - x_n)^2 - q\varphi_{bi}$



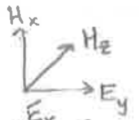
<u>I-V Characteristics:</u> $I = I_0\left(e^{qV/kT}-1\right) = q\overbrace{\left(\frac{D_n}{L_n}n_{p_0} + \frac{D_p}{L_p}p_{n_0}\right)}^{I_0}\left(e^{qV/kT}-1\right)$

⚠ Review, especially hetero junctions

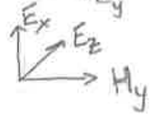
Waveguides, effective Index, etc

<u>Wave Equation:</u> $(\bar{\nabla}^2 + \omega^2\mu\varepsilon)\bar{E} = 0$

<u>TE Mode</u> (Slab Waveguide):



<u>TM Mode</u> (Slab Waveguide):



<u>Solving Slab waveguide Problems</u> (TE Mode):

① <u>Wave Equation:</u> $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \omega^2\mu\epsilon\right)E_y = 0$ ② Assume $E_y = e^{ik_z z}\varphi(x)$

③ From boundary conditions, guess $\varphi(x) = \{ C_0 e^{-\alpha(|x|-\frac{d}{2})}$ if $|x| \geq \frac{d}{2}$; $C_1\cos(k_x x)$ if $|x| \leq d/2$ (Even Modes); $C_1\sin(k_x x)$ if $|x| \leq d/2$ (odd Modes)$\}$

④ Substitute solutions into WE $\rightarrow \alpha^2 + k_x^2 = \omega^2(\mu_1\epsilon_1 - \mu\epsilon)$

⑤ Use Boundary conditions: E_y continuous at $x = \frac{d}{2} \Rightarrow C_0 = C_1\cos\left(k_x\frac{d}{2}\right)$
and $H_z = \frac{1}{i\omega\mu}\frac{\partial E_y}{\partial x}$ continuous at $x = \frac{d}{2} \Rightarrow \frac{\alpha}{\mu}C_0 = \frac{1}{\mu_1}C_1 k_x \sin\left(\frac{k_x d}{2}\right)$

⑥ Divide Results: $\alpha = \frac{\mu}{\mu_1}\tan\left(\frac{k_x d}{2}\right)$
⑦ Use w/ $\alpha^2 + k_x^2 = \omega^2(\mu_1\epsilon_1 - \mu\epsilon)$
} Transcendental Equation → Solve for k_x

⑧ Use k_x to find α & k_z → Use k_z to solve for effective index

<u>Single Mode Condition:</u> $\boxed{V < \pi}$ where $\boxed{V = k_0 d\sqrt{n_1^2 - n^2}}$

↳ ⚠ $\sqrt{\left(\frac{\alpha d}{2}\right)^2 + \left(\frac{k_x d}{2}\right)^2} = \frac{\omega d}{2}\sqrt{\mu_1\varepsilon_1 - \mu\varepsilon} < \frac{\pi}{2}$ ← Why is this the requirement

<u>Confinement Factor</u> (Symmetric Waveguide): $\Gamma \simeq \frac{V^2}{2+V^2}$ {Assumes Δn not too large}

<u>Effective Index:</u> $k_z = n_{eff}k_0$

<u>Modes Supported by Cavity:</u> $f_m = \underset{\text{Integer}}{m}\frac{c}{2n_{eff}L} \Rightarrow \Delta f = \frac{c}{2n_{eff}L}$ = <u>Mode Spacing</u>

↳ n_{eff} depends on $f \Rightarrow \boxed{\Delta f = \frac{c}{2n_{eff}L}\frac{1}{1 + \frac{f}{n_{eff}}\frac{dn_{eff}}{df}}}$

⚠ Review Laser Types: Gain-Guided, Ridge Waveguide index-guided, Burried Heterostructure

Rate Equations

Quantities: N = electron concentration, V = active Volume, I = current

η_i = internal quantum efficiency, $S = N_p$ = Photon density, $\frac{\alpha_m}{\alpha_m + \alpha_i}$

V_p = Optical Volume = V/Γ, τ_p = photon lifetime, η_o = quantum efficiency

τ = carrier lifetime, $v_g = \frac{c}{n_{eff}}$

<u>Rate Equations:</u>

① $$\frac{dN}{dt} = \frac{\eta_i I}{qV} - \frac{N}{\tau} - v_g g(N) S$$

R_{sp} = Spontaneous emission Rate

② $$\frac{dS}{dt} = \Gamma v_g g(N) S + \Gamma \beta R_{sp} - \frac{S}{\tau_p}$$

Note: <u>Recombination Rate</u>: $$\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}} = AN + BN^2 + CN^3$$

radiative, non-radiative, Non radiative, radiative, Auger

<u>Simplifying Assumptions</u>: $R_{sp} \simeq 0$, steady state $\Rightarrow \frac{dN}{dt} = 0 = \frac{dS}{dt}$

$$g_{th} = \frac{\alpha_m + \alpha_i}{\Gamma} = \frac{1}{\Gamma}\frac{1}{\tau_p}\frac{1}{v_g}$$

→ Above Threshold ($S>0$): $g(N) = g_{th}$ → gain is "clamped"

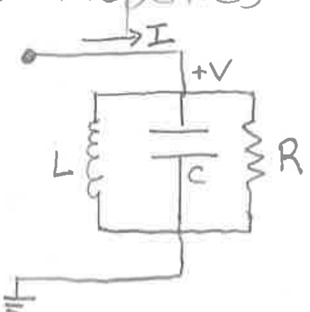
$$S = \frac{1}{v_g g_{th}} \frac{\eta_i}{qV}(I - I_{th})$$

$$P_{out} = \frac{\hbar\omega}{q}\eta_i \eta_o (I - I_{th}) = \frac{S V_p}{\tau_p}\hbar\omega \frac{\alpha_m}{\alpha_m + \alpha_i}$$

→ At threshold: $N_{th} = \frac{\eta_i I_{th}}{qV}\tau$

→ Below Threshold ($S=0$): $N = \frac{\eta_i I}{qV}\tau$

A.C. Properties of a laser



$I = I_L + I_c + I_R$ and $I_c = C\frac{dV}{dt}$

$$\Rightarrow \frac{d}{dt}\begin{pmatrix} V \\ I_L \end{pmatrix} = \begin{pmatrix} \frac{-1}{RC} & \frac{-1}{C} \\ \frac{1}{L} & 0 \end{pmatrix}\begin{pmatrix} V \\ I_L \end{pmatrix} + \begin{pmatrix} 0 \\ I_c \end{pmatrix} \Rightarrow$$

determinant

$\omega_R = \frac{1}{\sqrt{LC}}$

$\gamma = \frac{1}{RC}$ damping

Small Signal Analysis: $\frac{1}{\tau_{\Delta N}} = d(AN + BN^2 + CN^3) = A + 2BN + 3N^2$

$$\frac{d}{dt}(dN) = \frac{\eta_i}{qV} dI + \frac{1}{\tau_{\Delta N}} dN - (v_g g\, ds + v_g s\, dg)$$

$$\frac{d}{dt}(ds) = \Gamma(v_g g\, ds + v_g s\, dg) - \frac{ds}{\tau_p} + \Gamma \frac{dN}{\tau_{\Delta N}}$$

$\rightarrow R_{sp} = BN^2$

$\Rightarrow d(\beta R_{sp} = \beta B N^2)$

$= 2\beta BN = \frac{1}{\tau'_{\Delta N}}$

Assume Logarithmic Gain Model:

$$dg = \underbrace{\frac{\partial g}{\partial N}}_{=a} dN + \underbrace{\frac{\partial g}{\partial s}}_{\text{small}} ds \leftarrow g \rightarrow g(N,s)$$

$$g(N,s) = \frac{g_o}{1+\epsilon s} \ln\left(\frac{N}{N_{tr}}\right)$$

Yields linearized system:

$$\frac{d}{dt}\begin{pmatrix} dN \\ ds \end{pmatrix} = \begin{pmatrix} -\gamma_{NN} & -\gamma_{NS} \\ \gamma_{SN} & -\gamma_{SS} \end{pmatrix}\begin{pmatrix} dN \\ ds \end{pmatrix} + \begin{pmatrix} \frac{\eta_i}{qV} dI \\ 0 \end{pmatrix}$$

$\gamma_{NN} = \frac{1}{\tau_{\Delta N}} + v_g a s$

$\gamma_{NS} = v_g g - a_p v_g s$

$\gamma_{SN} = \Gamma v_g a s + \frac{\Gamma}{\tau'_{\Delta N}}$

$\gamma_{SS} = \Gamma v_g g - \frac{1}{\tau_p} + \Gamma s a_p$

⚠ To find resonant Frequency of laser, we assume time-harmonic dependence And then set determinant of matrix equal to zero $\rightarrow$ solve for omega

Relaxation Oscillation Frequency: $\boxed{\omega_R^2 = \frac{v_g a S}{\tau_p}}$ where $a = \frac{\partial g}{\partial N}$

$\hookrightarrow$ 2nd Order Efficiency: $\frac{P_1}{I_1} = \left(\eta_i \eta_o \frac{\hbar\omega}{q}\right) H(\omega) \rightarrow H(\omega) = \frac{\omega_R^2}{\omega_R^2 - \omega^2 + j\omega\gamma}$

$\hookrightarrow$ $|H(\omega_R)|^2 = \frac{\omega_R^2}{\gamma^2}$

$\hookrightarrow$ 3dB frequency: $\omega_{3dB} \simeq 1.5\,\omega_R \longleftarrow |H(\omega_{3dB})|^2 = \frac{1}{2}$ solve for ω_{3dB}

$\hookrightarrow$ $\gamma = \gamma_{NN} + \gamma_{SS} = \frac{1}{\tau_{\Delta N}} + v_g a S + \Gamma v_g S a_p$

often we write: $\boxed{\gamma = k f_R^2 + \gamma_o}$ k-factor $k = 4\pi^2 \tau_p \left(1 + \frac{\Gamma a_p}{a}\right)$ Nonlinear gain

Gain Models: Linear: $g(N) = a(N - N_{tr})$

logarithmic: $g(N) = g_o \ln N/N_{tr}$

Transparency Condition: $\boxed{\Delta F = E_g = F_c - F_v}$

⚠ Under transparency, $F_c - E_c = E_v - F_v$ ⇒ Use to find transparency concentration!

If $m_e^* < m_h^*$ ⇒ $N \simeq N_c \frac{4}{3\sqrt{\pi}} \left(\frac{F_c - E_c}{k_B T} \right)^{3/2} = N_c \frac{4}{3\sqrt{\pi}} \Delta^{3/2}$

⇒ $P \simeq N_v e^{-F_v - E_v / kT} = N_v e^{-\Delta}$

Assume $N = P$ ⇒ $\boxed{\frac{4}{3\sqrt{\pi}} \Delta^{3/2} = \left(\frac{m_h^*}{m_e^*} \right)^{3/2} e^{-\Delta}}$ ⇒ Solve for Δ & use to find N_{tr}

In a 2D quantum well:

$$N = \left(\frac{m_e^* kT}{\pi \hbar^2 L_z} \right) \frac{F_c - E_c}{kT} \quad \text{and} \quad P = \left(\frac{m_h^* k_B T}{\pi \hbar^2 L_z} \right) e^{-F_v - E_v / kT}$$

$$\Rightarrow \boxed{\Delta = \frac{m_h^*}{m_e^*} e^{-\Delta}}$$

Transparency Current: $I_{tr} = \frac{q N_{tr}}{\tau} \underbrace{d \cdot A}_{\text{Active Volume}}$

⚠ Quantum wells have smaller volume than DHs ⇒ smaller transparency current

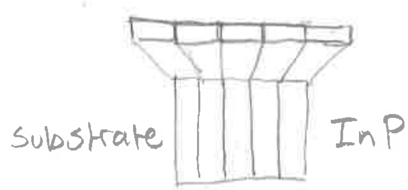
⚠ QWs can be strained in order to lower transparency concentration

Ideal QW transparency concentration: $\boxed{N_{tr}^{ideal} = \frac{m_e^*}{\pi \hbar^2 L_z} k_B T \ln 2}$ ← Review how to get this

Strained Semiconductors:

Trend: $a \uparrow E_g \downarrow$ and $a \downarrow E_g \uparrow$

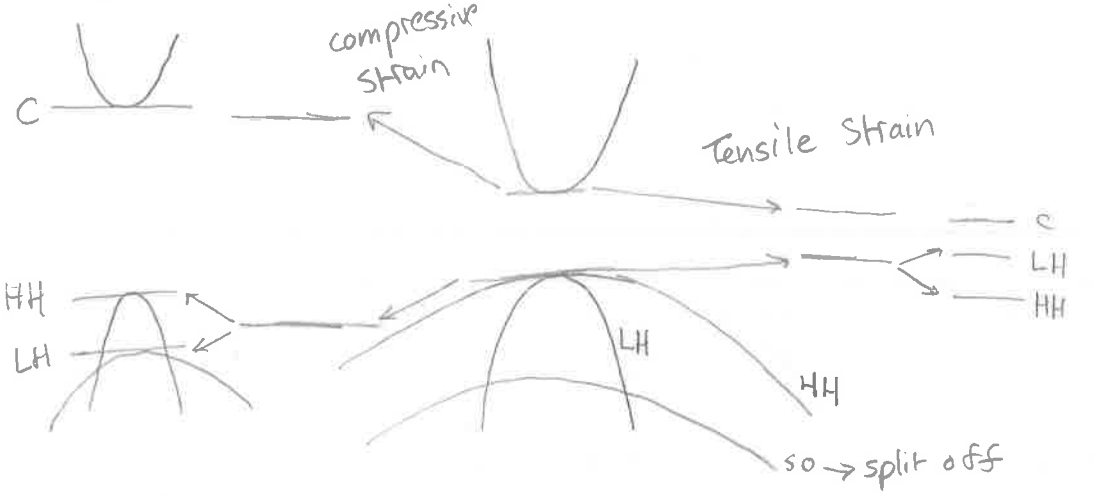
↳ tensile strain → E_g increases; compressive strain → E_g decreases



Hydrostatic Strain: Uniform Strain

Biaxial Strain: strain along 2 of 3 axes

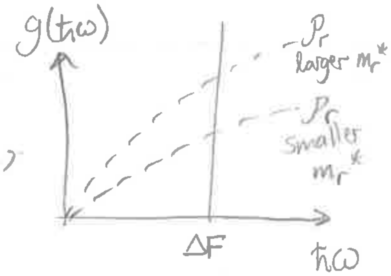
Effects of strain:



Note: "Heavy Hole" and "light hole" effective masses correspond to direction $\perp$ to quantum well

⚠ Want compressive strain for low threshold laser

⚠ Want tensile strain for amplifier



Polarization Selection Rules: Where does

	TE $\hat{x}, \hat{y}$	TM, $\hat{z}$	2TE + TM
C → HH	$\frac{3}{4}(1+\cos^2\theta)\, M_b^2$ compressive QW	$\frac{3}{2}\sin^2\theta\, M_b^2$	$3\, M_b^2$
C → LH	$\left(\frac{5}{4}-\frac{3}{4}\cos^2\theta\right) M_b^2$	$\left(\frac{1}{2}+\frac{3}{2}\cos^2\theta\right) M_b^2$ tensile QW	$3\, M_b^2$
Sum Rule HH + LH	$2\, M_b^2$	$2\, M_b^2$	$6\, M_b^2$

$\theta \rightarrow$ Angular factor: $\cos^2\theta = \dfrac{k_z^2}{k_z^2 + (k_x^2 + k_y^2)} \rightarrow$ At Band Edge

$k_x = k_y = 0$

$\Rightarrow \theta = 0,$

$\cos\theta = 1$

Note: This table is just the Matrix elements corresponding to different transitions

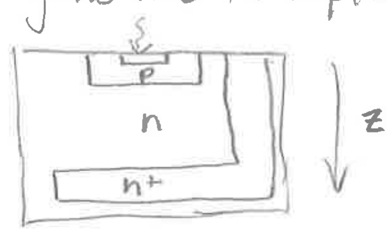
e.g. $|\hat{x}\cdot\bar{P}_{c-HH}|^2 = \frac{3}{4}(1+\cos^2\theta) M_b^2$ where $H'_{c-HH} = \dfrac{-eA_0}{2m_0}\,\hat{e}\cdot\bar{P}_{c-HH}$

Note: <u>In Bulk</u>: $|\hat{x}\cdot\bar{P}_{c-HH}|^2 = |\hat{y}\cdot\bar{P}_{c-HH}|^2 = |\hat{z}\cdot\bar{P}_{c-HH}|^2 = |\hat{x}\cdot\bar{P}_{c-LH}|^2 = M_b^2$

Polarization Dependence $\longrightarrow$ Review Chuang 4.1, 4.2, 9.5

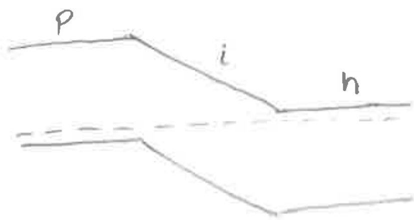
Detectors

Reverse Biased PN Junction: Only e-h pairs generated in depletion region are collected efficiently. E.g.



↳ Note: different wavelengths of light are absorbed at different depths
$\Rightarrow$ Use to make color-sensitive photo detectors

P-i-n Junction: Photodiode depletion width can be increased by adding intrinsic region $\Rightarrow$ higher collection efficiency.



I-V characteristics

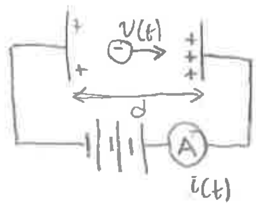
different from I_o for PN Junction

$$\boxed{I_{dark} = I_s\left(e^{\frac{qV}{kT}} - 1\right)}$$

↳ Photocurrent: $\boxed{I_{ph} = \eta \frac{q}{\hbar\omega} P_{opt}}$ $\rightarrow$ $\begin{cases} \eta = \text{Quantum Efficiency } [\%] \\ R = \text{Responsivity} = \eta \frac{q}{\hbar\omega} \ [A/W] \end{cases}$

↳ Ramo's Theorem: $i(t) = \frac{qV(t)}{d}$ (current only exists while electron is moving)

↳ Proof:



$dW = \text{Force} \times \text{Displacement} = qE\,dx = q\frac{V}{d}dx$
$= \text{Work by Power supply}$
$= i(t)V\,dt$

$\Rightarrow i(t)V\,dt = q\frac{V}{d}dx \Rightarrow i(t) = \frac{q}{d}\frac{dx}{dt} = \frac{qV(t)}{d}$ //

↳ Note: For each photon absorbed, only one charge is detected
→ Can be shown by computing total current using Ramo's theorem & integrating.

p-i-n Temporal Response: 2 contributions: transit time & RC

① Transit Time: $\tau_t = d/v_h$ (hole mobility limited)

② RC Time: $\tau_{RC} = RC = R\frac{\epsilon A}{d}$

③ Total: $\tau = \tau_t + \tau_{RC} = \frac{d}{v_h} + R\frac{\epsilon A}{d}$

→ An optimum depletion width occurs when $\tau_t = \tau_{RC} \Rightarrow \boxed{d_{opt} = \sqrt{R\epsilon A v_h}}$

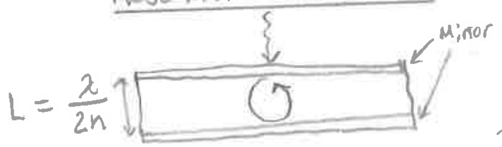
Bandwidth-Efficiency Tradeoff: $\boxed{\text{Efficiency } \eta = \eta_i\left(1 - e^{-\alpha d}\right)}$

(Assuming transit-time limited) Bandwidth $\boxed{f_{3dB} = \frac{v_h}{2\pi}\frac{1}{d}}$

→ In the limit of small d (low efficiency): $\boxed{\eta f_{3dB} \simeq \frac{\alpha \eta_i v_h}{2\pi}}$

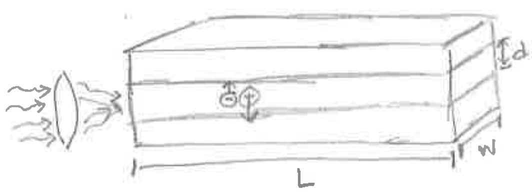
→ 3 improvements:

Resonant Enhanced:



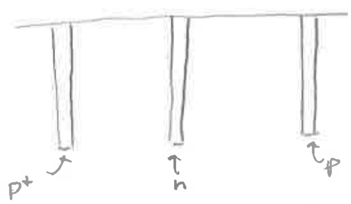
Note: high efficiency & f_{3dB} but spectrally dependent

Waveguide Detector:



$f_{3dB} = \frac{1}{2\pi}\frac{d}{R\epsilon L w}$

Trench Thingy



Expensive to Manufacture

Photoconductor: Reverse biased n-i-n junction → As long as a hole exists in the intrinsic region, electrons will continue to conduct

⇒ Photoconductive gain ≃ # electron trips per hole trip

→ Current: $\boxed{\Delta I \simeq P_{opt}\underbrace{\left[\eta\frac{q}{\hbar\omega}\right]}_{①}\underbrace{\frac{\tau_n}{\tau_t}}_{②}}$

① p-i-n

② Photoconductive gain → Reduce bandwidth by same amount

Derivation: $\Delta I = A\Delta J$ where $J = \sigma E = (nq\mu_n + pq\mu_p)E$

Vary J: $\Delta J \simeq \delta n \cdot q(\mu_n + \mu_p)E$ since $\delta n = \delta p$

In steady state: $\frac{d\delta n}{dt} = 0 = G_o - \frac{\delta n}{\tau_n} \Rightarrow \delta n = G_o \tau_n$ where $G_o = \frac{P_{opt}}{\hbar\omega}\eta\frac{1}{Lwd}$

Assume $\mu_n \gg \mu_p \Rightarrow \Delta J \simeq P_{opt}\frac{\eta}{\hbar\omega}\frac{1}{Lwd}\tau_n q \mu_n \frac{v_n}{\mu_n}$; $A = Lw$; $\tau_t = \frac{v_n}{d}$

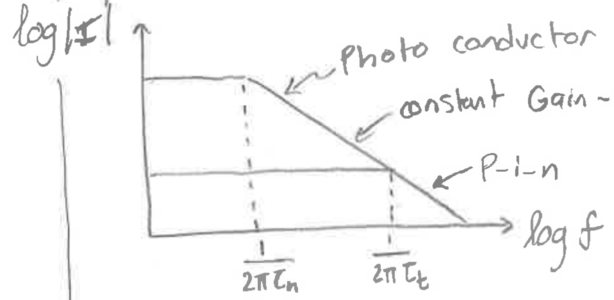
$\Rightarrow \Delta I = P_{opt}\left(\eta\frac{q}{\hbar\omega}\right)\frac{\tau_n}{\tau_t}$

n-i-n bandwidth : $\boxed{f_{3dB} = \frac{1}{2\pi}\frac{1}{\tau_n}}$

Note: BJT gain works the same way:

$$\text{Gain} = \beta = \frac{I_c}{I_B} = \left(\frac{\tau_B}{\tau_t}\right)$$

Frequency Response : (n-i-n vs p-i-n)



From the rate Equation: $\frac{dN}{dt} = \eta \frac{P_{opt}}{\hbar\omega}\frac{1}{Lwd} - \frac{N}{\tau_n}$

Assume solution $N = N_0 + N_1 e^{j\omega t}$ & $P_{opt} = P_0 + P_1 e^{j\omega t}$

$$\Rightarrow j\omega N_1 e^{j\omega t} = \left[P_0 + P_1 e^{j\omega t}\right]\eta \frac{1}{\hbar\omega}\frac{1}{Lwd} - \frac{N_0}{\tau_n} - \frac{N_1}{\tau_n}e^{j\omega t}$$

We consider AC & DC terms separately:

DC (steady state):

$$N_0 = \tau_n P_0 \left(\eta \frac{q}{\hbar\omega}\right)\frac{1}{lwd} = \tau_n G_0$$

AC:

$$N_1 = \frac{\eta P_1}{\hbar\omega(lwd)} \frac{1}{j\omega + 1/\tau_n}$$

Frequency Response → $\frac{I_1}{P_1} = \frac{J_1 lw}{P_1} = \frac{N_1 q v_n lw}{P_1} = \boxed{\frac{\eta q}{\hbar\omega}\left(\frac{\tau_n}{\tau_t}\right)\frac{1}{1 + j\omega\tau_n}}$

Noise

Poisson Statistics: $P(n) = \frac{\bar{n}^n}{n!}e^{-\bar{n}}$ where $\bar{n} = \langle n \rangle$ = average

Properties: $\langle (n-\bar{n})^2 \rangle = \bar{n}$ (variance equals mean)

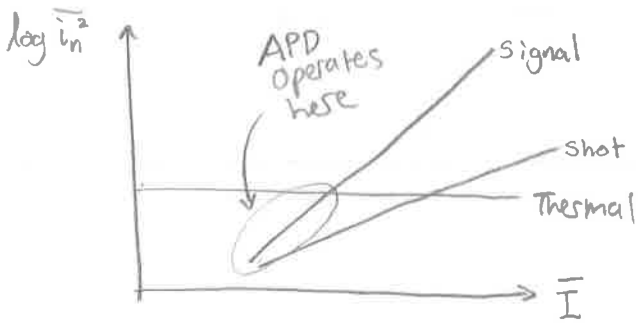
Shot Noise: $\boxed{\overline{i_n^2} = 2q\bar{I}d\nu}$ Arrises as a result of the discrete nature of charge.

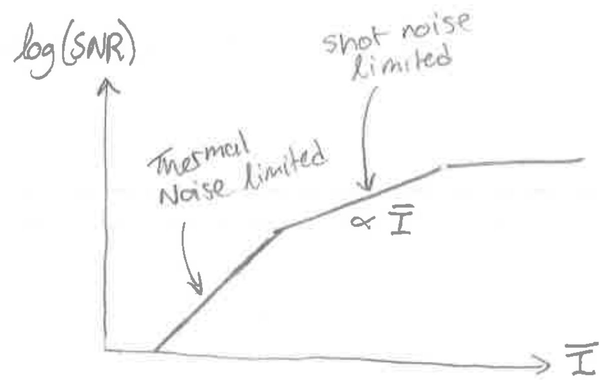
Thermal Noise: $\boxed{\overline{i_{th}^2} = \frac{4kT\,d\nu}{R}}$

⚠ Review Derivation of noise formulas!!

Signal-to-Noise Ratio (SNR): $\begin{cases} \text{Signal}: \overline{I}^2 \\ \text{Noise}: \overline{i^2_{n,shot}} + \overline{i^2_{n,therm}} + \cdots \end{cases}$

$$\Rightarrow SNR = \frac{\overline{I^2}}{2q\overline{I}\Delta\nu + \frac{4kT}{R}\Delta\nu}$$



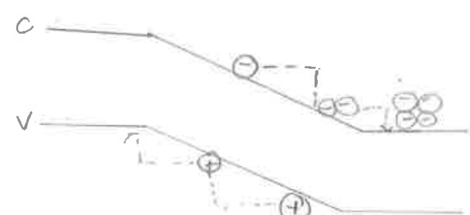


Note: Photoconductors are at a disadvantage since signal $\sim I_{ph}$ but $\overline{i^2_{n,shot}} \sim 2q(I_{ph} + I_{dark} + \cdots)$.

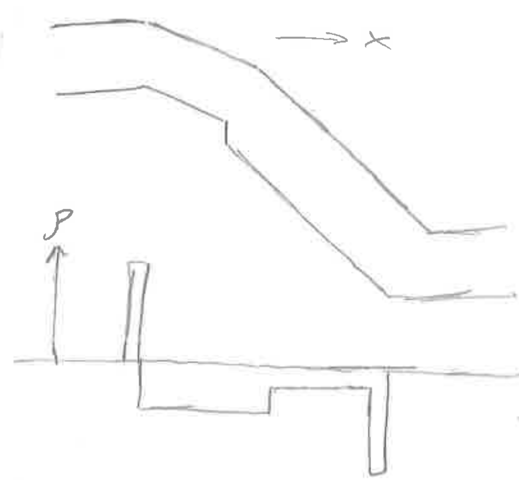
Avalanche Photodiode:

↳ Impact ionization: excited electrons excite more electrons

↳ holes can also cause impact ionization which can lead to a runaway process.



↳ Separate-absorption-multiplication (SAM)



Governing Equations:

$$\frac{dJ_n(x)}{dx} = \alpha_n J_n(x) + \beta_p J_p(x)$$

$$-\frac{dJ_p(x)}{dx} = \alpha_n J_n(x) + \beta_p J_p(x)$$

And $J = J_n(x) + J_p(x)$

Multiplication Factor: $\boxed{M = \frac{J}{J_n(0)}}$ ← This is what we care about! It is the gain

If we Assume α_n & β_p are const: $\boxed{M = \frac{1-k}{e^{-(1-k)\alpha_n W} - k}}$ where $\boxed{k = \frac{\beta_p}{\alpha_n}} < 1$

⚠ Ideally we want $k \sim 0$; $k = 1$ is bad $\Rightarrow$ instability

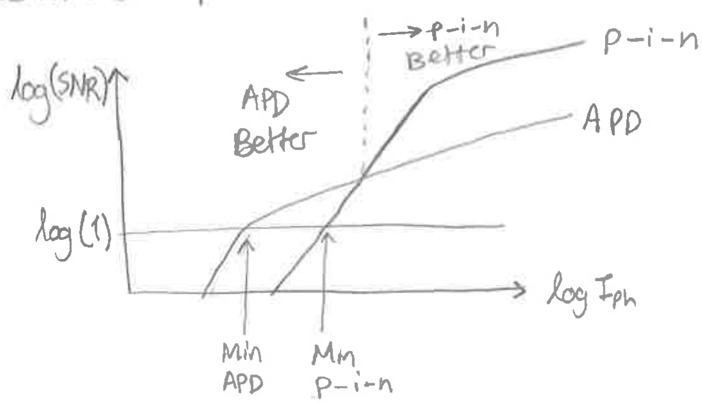
Noise Factor: $\boxed{F = \frac{\langle M^2 \rangle}{\langle M \rangle^2} = k \underbrace{\langle M_n \rangle}_{\text{APD Gain}} + (1-k)\left(2 - \frac{1}{\langle M_n \rangle}\right)}$

APD SNR: $$SNR = \frac{\frac{1}{2} I_{ph}^2 \langle M \rangle^2}{2q(I_{ph} + I_{dark})F\langle M \rangle^2 \Delta\nu + 4kT\Delta\nu / R}$$

?

$\Rightarrow$ APD is a good choice at low currents, ie good choice if thermal noise dominates!

Comparison of APD & p-i-n :



Semiconductor Review

Maxwell's Equations:

$$\bar{\nabla}\times\bar{E} = -\frac{\partial}{\partial t}\bar{B} \quad ; \quad \bar{\nabla}\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\bar{\nabla}\cdot\bar{D} = \rho \quad ; \quad \bar{\nabla}\cdot\bar{B} = 0$$

where $\bar{D} = \bar{\bar{\varepsilon}}\cdot\bar{E}$; $\bar{B} = \bar{\bar{\mu}}\cdot\bar{H}$

Continuity Equation: $\bar{\nabla}\cdot\bar{J} + \frac{\partial}{\partial t}\rho = 0$

Boundary Conditions:

$$\hat{n}\times(\bar{E}_1-\bar{E}_2) = 0 \quad ; \quad \hat{n}\times(\bar{H}_1-\bar{H}_2) = \bar{J}_s$$

$$\hat{n}\cdot(\bar{D}_1-\bar{D}_2) = \sigma \quad ; \quad \hat{n}\cdot(\bar{B}_1-\bar{B}_2) = 0$$

Review Derivation

↳ Derive from integral form of Maxwell's equations

Quasi-electrostatics: $\frac{\partial}{\partial t} \simeq 0 \Rightarrow \bar{\nabla}\times\bar{E} = 0$; $\bar{\nabla}\cdot\bar{D} = \rho \Rightarrow \bar{E} = -\bar{\nabla}\varphi$

↳ For moderate frequencies, we can "correct" the total current density by adding the displacement current density to the "conduction" current density.

$$\boxed{\bar{J}_{tot} = \bar{J}_{con} + \frac{\partial}{\partial t}(\varepsilon\bar{E})}$$

Poisson's Equation: $\bar{\nabla}\cdot(\varepsilon\bar{\nabla}\varphi) = -\rho \leftarrow \rho = q(p-n+C_0)$, $C_0 = N_D^+ - N_A^-$

Current Continuity Equation for Carriers:

$$\frac{\partial n}{\partial t} = R + \frac{1}{q}\bar{\nabla}\cdot\bar{J}_n \quad ; \quad \frac{\partial p}{\partial t} = R - \frac{1}{q}\bar{\nabla}\cdot\bar{J}_p$$

where $R = G_n - R_n = G_p - R_p$ is the total generation rates for electrons & holes

Carrier Transport Equations:

Mobility; diffusion coefficient

$$\bar{J}_n = q\mu_n n\bar{E} + qD_n\bar{\nabla}n$$

$$\bar{J}_p = q\mu_p p\bar{E} - qD_p\bar{\nabla}p$$